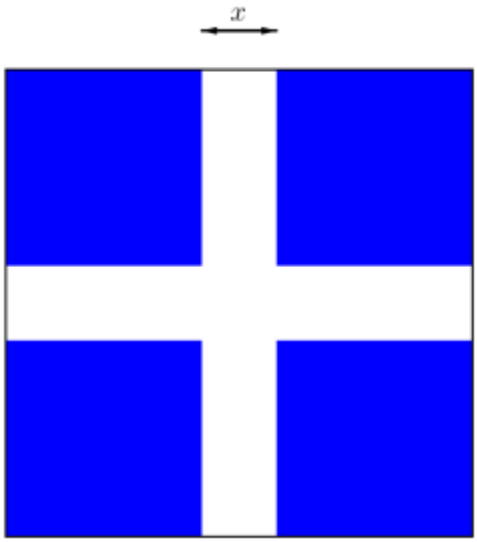
Mathematical Problem Solving
GCSE example

Example 12



This square flag measures 60 cm by 60 cm and has 4 lines of reflection symmetry.

Find the value of x such that $\frac{1}{9}$ of the flag is blue.

Student 1

The area of the whole square is

$60 \times 60 = 3600 \text{ cm}^2$

$\frac{1}{9} \times 3600 = 400$

The area of one blue square is 100 cm^2

The length of one side of a blue square is $\frac{(60-x)}{2}$

The area of a blue square is $\frac{(60-x)^2}{4} = 100$

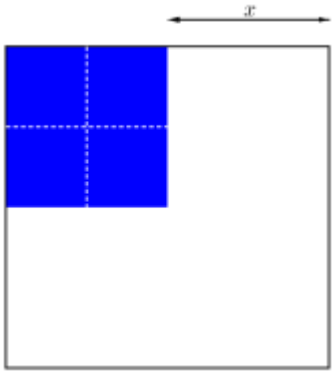
$(60-x)^2 = 400$

$60 - x = \pm 20$

$x = 60 \pm 20$

Student 2

The blue squares can be put together in one corner like this



If we have this area as $\frac{1}{9}$ of the original area then we have

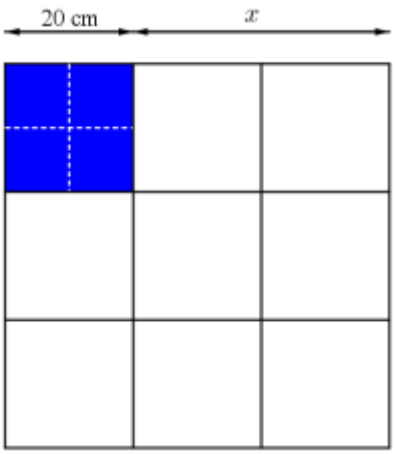
Student 1

Either $x = 80$ (impossible since the cross width is greater than the side length of the square)

or $x = 40$

So $x = 40$ cm

Student 2



So $x = 40$ cm

Both of the examples given are good methods. Working through the checklist of questions:

- Both students have the correct answer
- In both cases the calculations look secure and both methods are correct
- The reasoning in both cases is clear
- Both students have presented their solutions in clear steps. The calculations are clearer for student 1 than for student 2 although the method is not necessarily better.
- Both students have been efficient. Student 2 clearly has a more efficient method than student 1. There are fewer steps and the thought processes show more mathematical insight than just following a routine procedure.
- These two methods would be good to present to a class for comparison.
- Both methods could be used effectively in finding a general solution.

The assessment of each method should focus on the problem solving that has taken place and should provide feedback for each student to consider.

Student 1

The feedback should state that

- The student has the correct solution
- The algebra is accurate and clearly set out
- The method is efficient but there is a more efficient way

It should also pose the following questions to the student

- Did you consider moving the squares around?
- Could you have done this without having to solve a quadratic equation?
- Could you adapt your method to find the solution for any size of square flag with any fraction shaded blue?

Student 2

The feedback should state that

- The student has the correct solution
- The method is accurate but could have been described in more detail so someone reading it would have more idea of the thought processes used
- The method is very efficient

It should also pose the following questions to the student

- Could you adapt your method to find the solution for any size of square flag with any fraction shaded blue?