Mathematical Problem Solving
AS/A Level example

Example 18

The class are going to learn about quadratic functions.

Work with quadratic functions and their graphs; the discriminant of a quadratic function, including the conditions for real and repeated roots; completing the square; solution of quadratic equations including solving quadratic equations in a function of the unknown

DfE: Mathematics GCSE subject content and assessment objectives 2013

The teacher starts the lesson off with this problem.

Find the values of a such that the turning point (i.e. the vertex) of the parabola $y = x^2 + 2ax + 1$ is closest to the origin.

What is the locus of the turning point as a varies?

The discussion about this problem could follow the graph of the function being illustrated on a graph plotting function. The students should be able to see that it is about relating the turning point of the parabola to a, but at the start of the process may be unaware of how to do this. They may also need to think about how to find the distance of a point from the origin. The students should be able to make a guess at the equation of the locus but the teacher should make it clear that the result has to be shown mathematically.

The teacher can then move in to the main part of the lesson introducing different forms of quadratics and their uses, giving practise exercises in completing the square and finding vertices of parabolas.

Near the end of the lesson, the teacher can return to the original problem and the students, by completing the square, can find an expression for the vertex in terms of a (which should give them the equation of the locus of the vertex). To find the minimum distance requires the students to find an expression for the distance of the vertex from the origin ($\sqrt{a^4 - a^2 + 1}$), realise that this is a minimum when $a^4 - a^2 + 1$ is a minimum and complete the square again for $a^4 - a^2 + 1$.