

# Mathematical Problem Solving

## AS/A Level example

### Example 19

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A geometric progression has first term  $a$  and common ratio  $r$ , and the terms are all different and  $a \neq 0$ . The first, second and fourth terms of the geometric progression form three consecutive terms of an arithmetic progression. Given that the geometric progression converges and has a sum to infinity of  $3 + \sqrt{5}$ , find the exact values of  $a$  and  $r$ .

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Looking at this problem it clearly has attribute A; there is very little guidance given beyond some initial information and a finish point. More hidden in the initial read-through but very apparent when actually solving the problem is attribute F. The problem requires several mathematical processes and depends on different parts of mathematics being brought together to reach the solution. It would be considered to be a problem solving task in an examination.

It is worth looking at how this problem may have appeared as an examination question in previous years:

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A geometric progression has first term  $a$  and common ratio  $r$ , and the terms are all different. The first, second and fourth terms of the geometric progression form the first three terms of an arithmetic progression.

- (i) Show that  $r^3 - 2r + 1 = 0$   
[3]
  
  - (ii) Given that the geometric progression converges, find the exact value of  $r$ .  
[5]
  
  - (iii) Given also that the sum to infinity of this geometric progression is  $3 + \sqrt{5}$ , find the value of the integer  $a$ .  
[4]
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