# Mathematical Problem Solving
## GCSE example

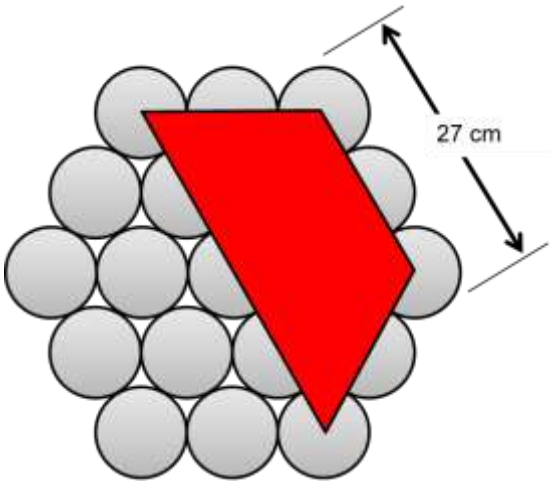
### Example 4

The class have been studying trigonometry and have previously covered the formula for the area of a triangle.

- **know and apply formulae to calculate: area of triangles**
- **know the formulae for: Pythagoras' theorem, $a^2 + b^2 = c^2$, and the trigonometric ratios, $\sin\theta = \frac{opposite}{hypotenuse}$, $\cos\theta = \frac{adjacent}{hypotenuse}$ and $\tan\theta = \frac{opposite}{adjacent}$; apply them to find angles and lengths in right-angled triangles and, where possible, general triangles in two and three dimensional figures**
- **know and apply Area $= \frac{1}{2}ab\sin C$ to calculate the area, sides or angles of any triangle.**

DfE: Mathematics GCSE subject content and assessment objectives 2013

In order to check the students' understanding of the methods used, the teacher presents the class with this problem on the board.




This diagram shows 19 identical circles arranged in a hexagon.

All of the vertices of the trapezium are in the centre of a circle.

**Find the area of the trapezium.**

As was the case with the *incomplete* problems, it is important that the teacher makes it clear that the task also develops problem-solving skills. This time, the students need to be made aware that most of the required information is contained in the question and that they are going to go through the same process of asking questions to find that information. Some additional information is needed to solve the problem but this

should consist of things that the students should know. The difference to the questions this time is that the students will have to come up with the answers to those questions themselves.

The procedure is the same:

The students should be given two minutes time to consider the problem without writing anything down. They should then spend two more minutes writing down as many questions as they can that would give key information for solving the problem. Students should then spend two more minutes comparing their questions with those of one or two other students. They need to be reminded to say why they think their question will reveal some important information.

The students should hopefully come up with some questions like these:

- What is the area of a trapezium?
- What is the radius/diameter of one of the circles?
- What are the side lengths of the trapezium?
- Which ones are the parallel sides?
- What is the height of the trapezium?
- Is it symmetrical?

There may well be other valid questions. For this example the teacher should encourage the students to answer their own questions. Plenty of hints will be needed and the solution will probably gradually grow from the discussion.

The teacher should encourage and model the use of accurate mathematical language wherever possible.

The responses below are examples of what may be said. It will be necessary for the teacher to help the students to use the correct vocabulary.

Working through the questions in order:

| Question | Answer |
| --- | --- |
| *What is the area of a trapezium?* | We learnt the formula for the area of a trapezium a few weeks ago. Can anyone remember what it is?<br><br>This isn't given in the problem as you are expected to know it.<br><br>*The teacher should persist until the correct formula is given with all of the terms in that formula correctly defined.*<br><br>$\frac{1}{2}h(a+b)$ |
| *What is the radius/diameter of one of the circles?* | What does the problem tell you about the circles?<br><br>*They are identical.*<br><br>*There are 19 of them.*<br><br>Which of those pieces of information is most useful in finding the radius/diameter?<br><br>Does the problem give you any measurements that can help you find the radius/diameter?<br><br>*27 cm is on the diagram.*<br><br>*The teacher should prompt the class until they calculate the correct diameter for a circle.*<br><br>Diameter $= 9$ cm, radius $= 4.5$ cm. |
| *What are the side lengths of the trapezium?* | How can we find the side lengths using the circles?<br><br>*Counting radii and diameters.*<br><br>What information in the problem means we can do this?<br><br>*All of the vertices of the trapezium are in the centre of a circle.*<br><br>Which side lengths can we find by this method?<br><br>*The teacher should make sure that the students find the lengths of every side of the trapezium.* |

| Question | Answer |
| --- | --- |
| *Which ones are the parallel sides?* | Which two sides do you think are parallel?<br><br>*The teacher should persuade the students that, as they are told that the shape is a trapezium and two of the sides are clearly not parallel, they can state the obvious!* |
| *What is the height of the trapezium?* | You aren't given the height of the trapezium anywhere in the question. What does that mean you are going to do?<br><br>*Work it out.*<br><br>*This part will take the most careful explaining by the teacher. It is important that the students come out of this response understanding that calculating the height of the trapezium is one of the key parts of solving the problem.*<br><br>*The teacher should encourage drawing a diagram and marking known lengths on it.*<br><br>What sort of triangle do we have?<br><br>What lengths do we know? |
| *Is it symmetrical?* | Is there any information given in the question or that we have found that tells us if it is symmetrical?<br><br>*It's a trapezium so it has parallel sides.*<br><br>*The two slant sides are the same length.*<br><br>*This is actually redundant information although it could be said that it needs to be known before Pythogoras's rule can be used to find the height.* |