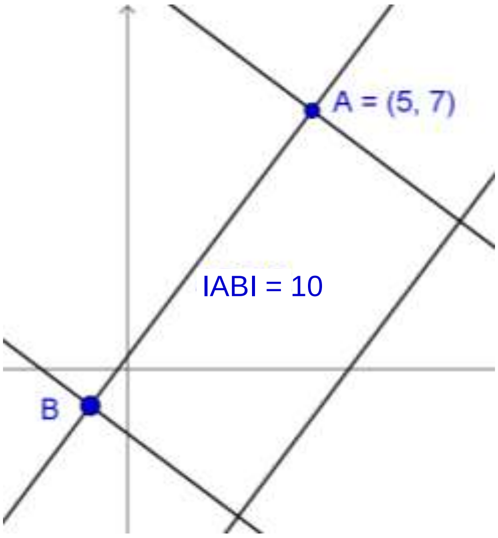
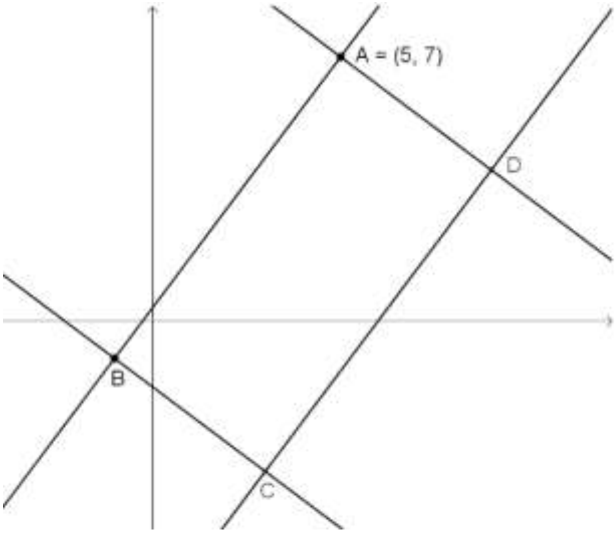
# Mathematical Problem Solving
## AS/A Level example

### Solution to example 1

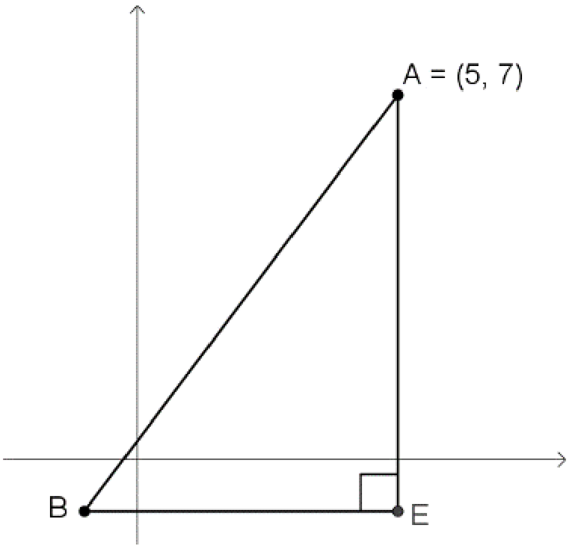



**Find the equations of the four straight lines in this diagram.**

Labelling the points of intersection $A$, $B$, $C$ and $D$ as shown:



For $AB = 10$ the right-angled triangle $AEB$ (shown below) must be a Pythagorean triple with $10$ as the hypotenuse since all of the points are at integer values. There is only one triple that has this property, a $\{6,8,10\}$ triangle.




B is either the point $(-1,-1)$ from $5-6$ and $7-8$ or the point $(-3,1)$ from $5-8$ and $7-6$. B is clearly below the $x$ axis so the point B is at $(-1,-1)$.

The gradient of $AB = \frac{8}{6} = \frac{4}{3}$. The gradient of $BC = -\frac{3}{4}$.

For integer coordinates we now have a $\{3,4,5\}$ triangle or one that is a multiple of $\{3,4,5\}$. Since $BC < AB$ (which the students should have established in their questioning), only a $\{3,4,5\}$ is possible.

This places point $C$ at $(3,-4)$ and, since $AD$ is parallel to $BC$ and $AB$ is parallel to $CD$, point $D$ at $(9,4)$.

The equations of the lines can now be found

The line through $AB$: gradient $= \frac{4}{3}$, through $(-1,-1)$ $\quad y + 1 = \frac{4}{3}(x+1)$

This simplifies to $4x - 3y + 1 = 0$.

The line through $CD$: gradient $= \frac{4}{3}$, through $(3,-4)$ $\quad y + 4 = \frac{4}{3}(x-3)$

This simplifies to $4x - 3y - 24 = 0$.

The line through $BC$: gradient $= -\frac{3}{4}$, through $(-1,-1)$ $\quad y + 1 = -\frac{3}{4}(x+1)$

This simplifies to $3x + 4y + 7 = 0$.

The line through $AD$: gradient $= -\frac{3}{4}$, through $(5,7)$ $\quad y - 7 = -\frac{3}{4}(x-5)$

This simplifies to $3x + 4y - 43 = 0$.