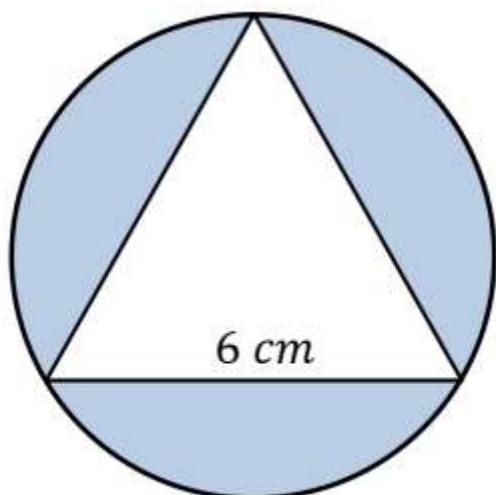


# Mathematical Problem Solving

## GCSE example

### Solution to example 14

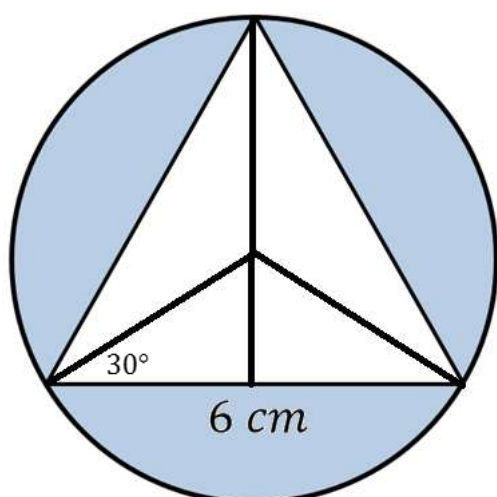


This diagram shows an equilateral triangle of side length 6 cm drawn inside a circle so that each corner touches the circumference of the circle.

What area of the circle is shaded?

The area of the equilateral triangle can be found using  $\text{Area} = \frac{1}{2}ab \sin C$

$$\text{Area of equilateral triangle} = \frac{1}{2} \times 6 \times 6 \times \sin 60 = 18 \times \frac{\sqrt{3}}{2} = 9\sqrt{3} \text{ cm}^2$$



The radius of the circle can be found by trigonometry

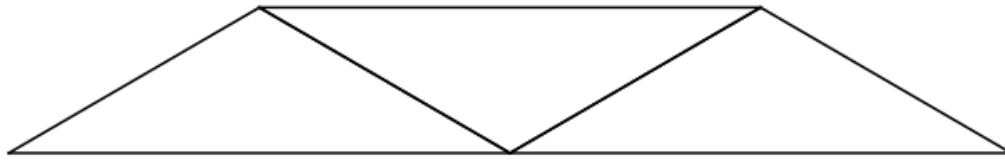
$$\cos 30 = \frac{3}{r}$$

$$r = \frac{3}{\cos 30} = \frac{6}{\sqrt{3}} = 2\sqrt{3}$$

$$\text{The area of the circle is } \pi \times (2\sqrt{3})^2 = 12\pi \text{ cm}^2$$

$$\text{The shaded area} = 12\pi - 9\sqrt{3} \text{ cm}^2.$$

The area of the equilateral triangle could also be found by dissecting it into three equal pieces and rearranging them to form a trapezium:



The area can be found using  $\text{Area} = \frac{1}{2}h(a + b)$

Area of trapezium (area of triangle)

$$= \frac{1}{2} \times \cos 30 \times \cos 60 \times (6 + 12)$$

$$= \frac{1}{2} \times \sqrt{3} \times 1 \times (6 + 12)$$

$$= 9\sqrt{3} \text{ cm}^2$$

The final stages of the calculation follow those above from the calculation of the radius of the circle onwards.