

Mathematical Problem Solving

AS/A Level example

Solution to example 15

Without using a calculator, show that

$$\sqrt{3} - \sqrt{2} = \sqrt{5 - 2\sqrt{6}}$$

Find better ways to write

(i) $\sqrt{12 - 2\sqrt{35}}$

(ii) $\sqrt{13 + 2\sqrt{42}}$

(iii) $\sqrt{21 + 6\sqrt{10}}$

$$RHS^2 = 5 - 2\sqrt{6}$$

$$LHS^2 = (\sqrt{3} - \sqrt{2})^2 = 3 + 2 - 2\sqrt{3}\sqrt{2} = 5 - 2\sqrt{6}$$

(i) $\sqrt{a} - \sqrt{b} = \sqrt{12 - 2\sqrt{35}}$

$$(\sqrt{a} - \sqrt{b})^2 = 12 - 2\sqrt{35}$$

$$a + b + 2\sqrt{ab} = 12 - 2\sqrt{35}$$

$$a + b = 12 \text{ and } ab = 35$$

The two numbers are obviously 7 and 5

$$\text{So } \sqrt{12 - 2\sqrt{35}} = \sqrt{7} - \sqrt{5}$$

(ii) $\sqrt{13 + 2\sqrt{42}}$

Here $a + b = 13$ and $ab = 42$. It will be $\sqrt{a} + \sqrt{b}$ as it is $13 + 2\sqrt{42}$ in the square root.

The two numbers are obviously 7 and 6

$$\text{So } \sqrt{13 + 2\sqrt{42}} = \sqrt{7} + \sqrt{6}$$

$$\text{(iii) } \sqrt{21 + 6\sqrt{10}}$$

In this example the square root is not in the desired form. To be able to use the method from parts (i) and (ii), $6\sqrt{10}$ can be rewritten as $2 \times 3\sqrt{10} = 2\sqrt{9 \times 10} = 2\sqrt{90}$

$$\text{So } \sqrt{21 + 6\sqrt{10}} = \sqrt{21 + 2\sqrt{90}}$$

Here $a + b = 21$ and $ab = 90$.

With a bit of thought it can be seen that 15 and 6 will work.

$$\text{So } \sqrt{21 + 6\sqrt{10}} = \sqrt{15} + \sqrt{6}$$