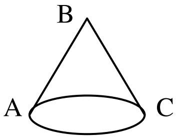
Introduction to matrices

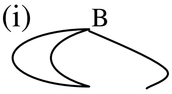
Exercise level 1

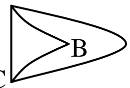
1. This diagram shows a map of the roads linking 3 towns A, B and C. The corresponding '*direct route*' matrix is shown beside it.

$$\begin{array}{c c} & \begin{array}{ccc} A & B & C \end{array} \\ \begin{array}{c} A \\ B \\ C \end{array} & \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 0 \end{bmatrix} \end{array}$$



For each of the following diagrams construct the *direct route* matrix.

(i) 

(ii) 

2. A café sells 3 main meals A, B, and C each day. On two days the sales of each type are shown in the matrix below.

$$\begin{array}{c c} & \begin{array}{cc} M & T \end{array} \\ \begin{array}{c} A \\ B \\ C \end{array} & \begin{bmatrix} 4 & 6 \\ 3 & 5 \\ 7 & 2 \end{bmatrix} \end{array}$$

If meal A costs £4, meal B costs £5 and meal C costs £3 construct a matrix showing the amount taken for each of the meals on each of the two days. Hence state the total amount taken for each meal over the two days.

3. $\mathbf{A} = \begin{pmatrix} 2 & -3 \\ -1 & 5 \end{pmatrix}$ $\mathbf{B} = \begin{pmatrix} -3 & -1 \\ 2 & 7 \end{pmatrix}$ $\mathbf{C} = \begin{pmatrix} 2 & 3 & -4 \\ -1 & 2 & 5 \end{pmatrix}$ $\mathbf{D} = \begin{pmatrix} -1 & -4 & 2 \\ -3 & 5 & 6 \end{pmatrix}$

Calculate, if possible,

(i) $\mathbf{A} + 2\mathbf{B}$ (ii) $\mathbf{C} - \mathbf{D}$ (iii) $3\mathbf{A} - 2\mathbf{C}$ (iv) $3\mathbf{D} - \mathbf{C}$

4. $\mathbf{A} = \begin{pmatrix} 2 & 1 \\ -3 & 4 \end{pmatrix}$ $\mathbf{B} = \begin{pmatrix} -1 & 3 & 2 \\ 5 & 1 & -2 \end{pmatrix}$ $\mathbf{C} = \begin{pmatrix} 3 & -1 \\ 1 & 2 \end{pmatrix}$ $\mathbf{D} = \begin{pmatrix} 4 & -1 \\ 2 & 5 \\ -3 & 1 \end{pmatrix}$

Calculate, if possible, the following
(i) **AB** (ii) **AC** (iii) **BC** (iv) **BD**

5. The matrices **A** and **B** are defined by

$$\mathbf{A} = \begin{pmatrix} 2 & 3 \\ 3 & 2 \end{pmatrix} \qquad \mathbf{B} = \begin{pmatrix} 0 & 3 \\ 3 & 0 \end{pmatrix}$$

   (i) Calculate

   (a) $\mathbf{A} + \mathbf{B}$ (b) $\mathbf{AB}$

   (ii) Show that $\mathbf{A} + \mathbf{B} - \mathbf{AB} = m\mathbf{I}$, where m is an integer and **I** is the 2 x 2 identity matrix.

6. The matrices **A**, **B** and **C** are given by $\mathbf{A} = \begin{pmatrix} 1 & 4 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 2 & -1 \end{pmatrix}$

   Find (i) $2\mathbf{A} + \mathbf{C}$ (ii) $\mathbf{AB}$ (iii) $\mathbf{BC}$

7. If $\mathbf{A} = \begin{pmatrix} 3 & 1 \\ x & 2 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 6 & 2 \\ 4 & y \end{pmatrix}$ find the values of x and y given that $\mathbf{AB} = \mathbf{BA}$.

8. $\mathbf{M} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ and $\mathbf{N} = \begin{pmatrix} 2 & -2 \\ -1 & 3 \end{pmatrix}$.

   Find $\mathbf{M}^2 - \mathbf{N}^2$ and $(\mathbf{M} + \mathbf{N})(\mathbf{M} - \mathbf{N})$ and explain why your results are not equal.