Introduction to matrices

Exercise level 2

1. Five football teams took part in a league competition during which they each played each other in the first half of the season and again in the second half of the season. The results matrix for the first half of the season is shown in the matrix below.

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array}\begin{array}{c} \begin{array}{ccc} W & D & L \end{array} \\ \begin{bmatrix} 2 & 1 & 1 \\ 2 & 1 & 1 \\ 0 & 4 & 0 \\ 1 & 1 & 2 \\ 1 & 1 & 2 \end{bmatrix} \end{array}$$

The results from the second half of the season are shown below:

B 3 : A 4	C 1 : B 0	E 1 : D 0	D 3 : C 3	E 0 : A 6
E 2 : C2	D 2 : A 0	E 5 : B 2	C 3 : A 1	D 2 : B 4

Construct a results matrix for the whole season. If a win gains 3 points and a draw 1 point, determine the league positions at the end of the season.

2. Find the values of x and y in each of the following matrix equations:

(i) $\begin{pmatrix} 3 & -5 \\ 2 & x \end{pmatrix} + \begin{pmatrix} 1 & y \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} 4 & 6 \\ 5 & -2 \end{pmatrix}$ (ii) $\begin{pmatrix} 3 \\ -1 \end{pmatrix} + x\begin{pmatrix} -2 \\ y \end{pmatrix} = \begin{pmatrix} -5 \\ 11 \end{pmatrix}$

3. If $\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 2 & -1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} -2 & 1 \\ 3 & 0 \end{pmatrix}$,

(i) Find $(\mathbf{A} + \mathbf{B})^2$

(ii) Find $\mathbf{AB}$ and $\mathbf{BA}$

(iii)Show that $(\mathbf{A} + \mathbf{B})^2 = \mathbf{A}^2 + \mathbf{AB} + \mathbf{BA} + \mathbf{B}^2$

4. The matrix $\mathbf{A} = \begin{pmatrix} 2 & 0 \\ 1 & 2 \end{pmatrix}$ and h and k are numbers such that $\mathbf{A}^2 = h\mathbf{A} + k\mathbf{I}$, where $\mathbf{I}$ is the identity matrix. Find h and k.

5. Three garages G_1, G_2, G_3 sell cars of two types A and B. The sales in one week are shown below together with a matrix showing the price to be paid for each type. Work out the total value of sales for the three garages in that week.

Sales $\begin{array}{c} \\ G_1 \\ G_2 \\ G_3 \end{array}\begin{array}{c} \begin{array}{cc} A & B \end{array} \\ \begin{pmatrix} 3 & 1 \\ 2 & 0 \\ 4 & 1 \end{pmatrix} \end{array}$ Prices $\begin{array}{c} \\ A \\ B \end{array}\begin{array}{c} \begin{array}{c} \end{array} \\ \begin{pmatrix} 8000 \\ 10500 \end{pmatrix} \end{array}$

