

## Section 1: Introduction to matrices

## Solutions to Exercise level 1

$$1. \quad (i) \quad \begin{array}{l} A \\ B \\ C \end{array} \begin{array}{ccc} & B & C \end{array} \quad (ii) \quad \begin{array}{l} A \\ B \\ C \end{array} \begin{array}{ccc} & B & C \end{array}$$

$$\begin{array}{l} A \begin{pmatrix} 0 & 1 & 0 \end{pmatrix} \\ B \begin{pmatrix} 1 & 0 & 2 \end{pmatrix} \\ C \begin{pmatrix} 0 & 2 & 0 \end{pmatrix} \end{array} \quad \begin{array}{l} A \begin{pmatrix} 0 & 1 & 2 \end{pmatrix} \\ B \begin{pmatrix} 1 & 0 & 1 \end{pmatrix} \\ C \begin{pmatrix} 2 & 1 & 0 \end{pmatrix} \end{array}$$

2. Multiply the A row by 4, the B row by 5 and the C row by 3.

$$\begin{array}{cc} M & T \\ A \begin{pmatrix} 4 \times 4 & 4 \times 6 \end{pmatrix} \\ B \begin{pmatrix} 5 \times 3 & 5 \times 5 \end{pmatrix} \\ C \begin{pmatrix} 3 \times 7 & 3 \times 2 \end{pmatrix} \end{array} = \begin{array}{cc} M & T \\ A \begin{pmatrix} 16 & 24 \end{pmatrix} \\ B \begin{pmatrix} 15 & 25 \end{pmatrix} \\ C \begin{pmatrix} 21 & 6 \end{pmatrix} \end{array}$$

The total taken for meal A over the two days is £16 + £24 = £40.

The total taken for meal B over the two days is £15 + £25 = £40.

The total taken for meal C over the two days is £21 + £6 = £27.

$$3. \quad (i) \quad A + 2B = \begin{pmatrix} 2 & -3 \\ -1 & 5 \end{pmatrix} + 2 \begin{pmatrix} -3 & -1 \\ 2 & 7 \end{pmatrix} = \begin{pmatrix} 2 & -3 \\ -1 & 5 \end{pmatrix} + \begin{pmatrix} -6 & -2 \\ 4 & 14 \end{pmatrix}$$

$$= \begin{pmatrix} -4 & -5 \\ 3 & 19 \end{pmatrix}$$

$$(ii) \quad C - D = \begin{pmatrix} 2 & 3 & -4 \\ -1 & 2 & 5 \end{pmatrix} - \begin{pmatrix} -1 & -4 & 2 \\ -3 & 5 & 6 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & 7 & -6 \\ 2 & -3 & -1 \end{pmatrix}$$

(iii) cannot be done as A and C do not have the same order

$$(iv) \quad 3D - C = 3 \begin{pmatrix} -1 & -4 & 2 \\ -3 & 5 & 6 \end{pmatrix} - \begin{pmatrix} 2 & 3 & -4 \\ -1 & 2 & 5 \end{pmatrix}$$

$$= \begin{pmatrix} -3 & -12 & 6 \\ -9 & 15 & 18 \end{pmatrix} - \begin{pmatrix} 2 & 3 & -4 \\ -1 & 2 & 5 \end{pmatrix}$$

$$= \begin{pmatrix} -5 & -15 & 10 \\ -8 & 13 & 13 \end{pmatrix}$$

$$4. (i) \quad AB = \begin{pmatrix} 2 & 1 \\ -3 & 4 \end{pmatrix} \begin{pmatrix} -1 & 3 & 2 \\ 5 & 1 & -2 \end{pmatrix} = \begin{pmatrix} 3 & 7 & 2 \\ 23 & -5 & -14 \end{pmatrix}$$

$$(ii) \quad AC = \begin{pmatrix} 2 & 1 \\ -3 & 4 \end{pmatrix} \begin{pmatrix} 3 & -1 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 7 & 0 \\ -5 & 11 \end{pmatrix}$$

(iii)  $BC$  cannot be calculated as the matrices are not conformable (the number of columns in  $B$  is not the same as the number of rows in  $C$ )

$$(iv) \quad BD = \begin{pmatrix} -1 & 3 & 2 \\ 5 & 1 & -2 \end{pmatrix} \begin{pmatrix} 4 & -1 \\ 2 & 5 \\ -3 & 1 \end{pmatrix} = \begin{pmatrix} -4 & 18 \\ 28 & -2 \end{pmatrix}$$

$$5. (i) \quad A+B = \begin{pmatrix} 2 & 3 \\ 3 & 2 \end{pmatrix} + \begin{pmatrix} 0 & 3 \\ 3 & 0 \end{pmatrix} = \begin{pmatrix} 2 & 6 \\ 6 & 2 \end{pmatrix}$$

$$AB = \begin{pmatrix} 2 & 3 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} 0 & 3 \\ 3 & 0 \end{pmatrix} = \begin{pmatrix} 9 & 6 \\ 6 & 9 \end{pmatrix}$$

$$(ii) \quad A+B-AB = \begin{pmatrix} 2 & 6 \\ 6 & 2 \end{pmatrix} - \begin{pmatrix} 9 & 6 \\ 6 & 9 \end{pmatrix} = \begin{pmatrix} -7 & 0 \\ 0 & -7 \end{pmatrix} = -7 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = -7I$$

$$6. (i) \quad 2A+C = 2 \begin{pmatrix} 1 & 4 \\ 2 & -1 \end{pmatrix} + \begin{pmatrix} 2 & -1 \\ 2 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 8 \\ 4 & -2 \end{pmatrix} + \begin{pmatrix} 2 & -1 \\ 2 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 4 & 7 \\ 6 & -3 \end{pmatrix}$$

$$(ii) \quad AB = \begin{pmatrix} 1 & 4 \\ 2 & 2 \end{pmatrix} \begin{pmatrix} -3 \\ 5 \end{pmatrix} = \begin{pmatrix} -5 \\ 10 \end{pmatrix}$$

$$(iii) \quad BC = \begin{pmatrix} -3 \\ 2 \end{pmatrix} \begin{pmatrix} 2 & -1 \end{pmatrix} = \begin{pmatrix} -6 & 3 \\ 4 & -2 \end{pmatrix}$$

$$7. \quad AB = \begin{pmatrix} 3 & 1 \\ x & 2 \end{pmatrix} \begin{pmatrix} 6 & 2 \\ 4 & y \end{pmatrix} = \begin{pmatrix} 22 & 6+y \\ 6x+8 & 2x+2y \end{pmatrix}$$

$$BA = \begin{pmatrix} 6 & 2 \\ 4 & y \end{pmatrix} \begin{pmatrix} 3 & 1 \\ x & 2 \end{pmatrix} = \begin{pmatrix} 18+2x & 10 \\ 12+xy & 4+2y \end{pmatrix}$$

$$AB = BA \Rightarrow \begin{pmatrix} 22 & 6+y \\ 6x+8 & 2x+2y \end{pmatrix} = \begin{pmatrix} 18+2x & 10 \\ 12+xy & 4+2y \end{pmatrix}$$

$$22 = 18 + 2x \quad \Rightarrow x = 2$$

$$6 + y = 10 \quad \Rightarrow y = 4$$

$$\begin{array}{ll} \text{Check:} & 6x + 8 = 12 + 8 = 20 & 12 + xy = 12 + 8 = 20 \\ & 2x + 2y = 4 + 8 = 12 & 4 + 2y = 4 + 8 = 12 \end{array}$$

$$8. \quad M^2 = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix}$$

$$N^2 = \begin{pmatrix} 2 & -2 \\ -1 & 3 \end{pmatrix} \begin{pmatrix} 2 & -2 \\ -1 & 3 \end{pmatrix} = \begin{pmatrix} 6 & -10 \\ -5 & 11 \end{pmatrix}$$

$$M^2 - N^2 = \begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 6 & -10 \\ -5 & 11 \end{pmatrix} = \begin{pmatrix} -5 & 14 \\ 5 & -10 \end{pmatrix}$$

$$M + N = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 2 & -2 \\ -1 & 3 \end{pmatrix} = \begin{pmatrix} 3 & 0 \\ -1 & 4 \end{pmatrix}$$

$$M - N = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 2 & -2 \\ -1 & 3 \end{pmatrix} = \begin{pmatrix} -1 & 4 \\ 1 & -2 \end{pmatrix}$$

$$(M + N)(M - N) = \begin{pmatrix} 3 & 0 \\ -1 & 4 \end{pmatrix} \begin{pmatrix} -1 & 4 \\ 1 & -2 \end{pmatrix} = \begin{pmatrix} -3 & 12 \\ 5 & -12 \end{pmatrix}$$

$$(M + N)(M - N) = M^2 + NM - MN - N^2$$

Since matrix multiplication is not commutative,  $NM \neq MN$ .