# Section 1: Introduction to matrices

## Solutions to Exercise level 2

1. In the second half of the season:
A win two and lose two
B win one and lose three
C win two and draw two
D win one, draw one and lose two
E win two, draw one and lose one

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array}\begin{array}{c} \begin{array}{ccc} W & D & L \end{array} \\ \begin{pmatrix} 2 & 0 & 2 \\ 1 & 0 & 3 \\ 2 & 2 & 0 \\ 1 & 1 & 2 \\ 2 & 1 & 1 \end{pmatrix} \end{array}$$

The results matrix for the whole season is

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array}\begin{array}{c} \begin{array}{ccc} W & D & L \end{array} \\ \begin{pmatrix} 2 & 1 & 1 \\ 2 & 1 & 1 \\ 0 & 4 & 0 \\ 1 & 1 & 2 \\ 1 & 1 & 2 \end{pmatrix} \end{array} + \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array}\begin{array}{c} \begin{array}{ccc} W & D & L \end{array} \\ \begin{pmatrix} 2 & 0 & 2 \\ 1 & 0 & 3 \\ 2 & 2 & 0 \\ 1 & 1 & 2 \\ 2 & 1 & 1 \end{pmatrix} \end{array} = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array}\begin{array}{c} \begin{array}{ccc} W & D & L \end{array} \\ \begin{pmatrix} 4 & 1 & 3 \\ 3 & 1 & 4 \\ 2 & 6 & 0 \\ 2 & 2 & 4 \\ 3 & 2 & 3 \end{pmatrix} \end{array}$$

A's points are $(4\times3)+(1\times1)=13$
B's points are $(3\times3)+(1\times1)=10$
C's points are $(2\times3)+(6\times1)=12$
D's points are $(2\times3)+(2\times1)=8$
E's points are $(3\times3)+(2\times1)=11$

The league positions are A, C, E, B, D.

2. (i) $\begin{pmatrix} 3 & -5 \\ 2 & x \end{pmatrix}+\begin{pmatrix} 1 & y \\ 3 & 2 \end{pmatrix}=\begin{pmatrix} 4 & 6 \\ 5 & -2 \end{pmatrix}$

$\begin{pmatrix} 4 & -5+y \\ 5 & x+2 \end{pmatrix}=\begin{pmatrix} 4 & 6 \\ 5 & -2 \end{pmatrix}$

$-5+y=6 \quad \Rightarrow y=11$

$x+2=-2 \quad \Rightarrow x=-4$

(ii) $\begin{pmatrix} 3 \\ -1 \end{pmatrix}+x\begin{pmatrix} -2 \\ y \end{pmatrix}=\begin{pmatrix} -5 \\ 11 \end{pmatrix}$

$\begin{pmatrix} 3-2x \\ -1+xy \end{pmatrix}=\begin{pmatrix} -5 \\ 11 \end{pmatrix}$

$3-2x=-5 \quad \Rightarrow x=4$

$-1+xy=11 \quad \Rightarrow -1+4y=11 \quad \Rightarrow y=3$

 

3. (a) (i) $(A+B)^2 = \left[\begin{pmatrix}1 & 0\\ 2 & -1\end{pmatrix}+\begin{pmatrix}-2 & 1\\ 3 & 0\end{pmatrix}\right]^2$

$$=\begin{pmatrix}-1 & 1\\ 5 & -1\end{pmatrix}\begin{pmatrix}-1 & 1\\ 5 & -1\end{pmatrix}$$

$$=\begin{pmatrix}6 & -2\\ -10 & 6\end{pmatrix}$$

(ii) $AB=\begin{pmatrix}1 & 0\\ 2 & -1\end{pmatrix}\begin{pmatrix}-2 & 1\\ 3 & 0\end{pmatrix}=\begin{pmatrix}-2 & 1\\ -7 & 2\end{pmatrix}$

(iii) $BA=\begin{pmatrix}-2 & 1\\ 3 & 0\end{pmatrix}\begin{pmatrix}1 & 0\\ 2 & -1\end{pmatrix}=\begin{pmatrix}0 & -1\\ 3 & 0\end{pmatrix}$

(b) $A^2=\begin{pmatrix}1 & 0\\ 2 & -1\end{pmatrix}\begin{pmatrix}1 & 0\\ 2 & -1\end{pmatrix}=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$

$B^2=\begin{pmatrix}-2 & 1\\ 3 & 0\end{pmatrix}\begin{pmatrix}-2 & 1\\ 3 & 0\end{pmatrix}=\begin{pmatrix}7 & -2\\ -6 & 3\end{pmatrix}$

$$A^2+AB+BA+B^2=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}+\begin{pmatrix}-2 & 1\\ -7 & 2\end{pmatrix}+\begin{pmatrix}0 & -1\\ 3 & 0\end{pmatrix}+\begin{pmatrix}7 & -2\\ -6 & 3\end{pmatrix}$$

$$=\begin{pmatrix}6 & -2\\ -10 & 6\end{pmatrix}$$

$$=(A+B)^2$$

4. $A=\begin{pmatrix}2 & 0\\ 1 & 2\end{pmatrix}$

$A^2=\begin{pmatrix}2 & 0\\ 1 & 2\end{pmatrix}\begin{pmatrix}2 & 0\\ 1 & 2\end{pmatrix}=\begin{pmatrix}4 & 0\\ 4 & 4\end{pmatrix}$

$A^2=hA+kI \Rightarrow \begin{pmatrix}4 & 0\\ 4 & 4\end{pmatrix}=h\begin{pmatrix}2 & 0\\ 1 & 2\end{pmatrix}+k\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$

$$\Rightarrow\begin{pmatrix}4 & 0\\ 4 & 4\end{pmatrix}=\begin{pmatrix}2h+k & 0\\ h & 2h+k\end{pmatrix}$$

$$\Rightarrow h=4, k=-4$$

5. Total value of sales for each garage is given by

$$\begin{matrix}G_1\\ G_2\\ G_3\end{matrix}\begin{pmatrix}3 & 1\\ 2 & 0\\ 4 & 1\end{pmatrix}\begin{pmatrix}8000\\ 10500\end{pmatrix}=\begin{matrix}G_1\\ G_2\\ G_3\end{matrix}\begin{pmatrix}3\times8000+1\times10500\\ 2\times8000+0\times10500\\ 4\times8000+1\times10500\end{pmatrix}=\begin{matrix}G_1\\ G_2\\ G_3\end{matrix}\begin{pmatrix}34500\\ 16000\\ 42500\end{pmatrix}$$