MEI Position Paper

Curriculum and assessment in mathematics in England at KS4 and KS5

Contents

Executive Summary

The disruption to our national examination system caused by the pandemic prompted MEI to consider the effectiveness of mathematics curricula and assessments at KS4 and KS5.

This position paper details our analysis of England's current systems of curriculum and assessment, with proposals for how they might be improved.

We believe these proposals can help more young people to master they mathematics they need to meet their aspirations for further study and employment, and equip them to use and understand mathematics and statistics with confidence in all aspects of their lives.

Key points relating to each section are summarised below. Detailed proposals are given within each section of the paper.

1. Accrediting Qualifications (proposals 1 – 3)

There is a lack of coherence between the intended curriculum of the current GCSE and A Level mathematics qualifications and the curriculum as it is implemented in schools and colleges. The result is that students' mathematics education has not improved to the extent intended by the most recent qualification reforms. At AS and A Level, not all subject criteria are being met, particularly those relating to the analysis of data and statistics, and the use of technology to help analyse and solve quantitative problems.

An expert curriculum and assessment body for mathematics should be established with the aim of ensuring coherence across curricula, teaching, professional development and assessment. A long-term planning and development process is needed to ensure that teaching and learning resources, professional development and assessments are designed to enable the curriculum to be implemented as intended.

2. Mathematics to age 16 (proposals 4 – 9)

Mathematics is vitally important for future work, study and economic well-being. It is by nature a subject where each level of understanding is dependent on a secure foundation of earlier study, meaning that each assessment requires a suitable level of challenge if it is to provide useful information about what students know, understand and can do.

In GCSE Mathematics, thresholds for lower grades are too low. Students awarded lower grades, including grades 4 and 5 on the higher tier, are not given a proper opportunity to demonstrate what they know, understand and can do. This is demotivating for students and means grades do not give a clear indication of the mathematics a student has mastered.

There are potential benefits from including an element of teacher assessment in mathematics alongside external examinations.

The content of foundation tier GCSE Mathematics has a large overlap with what would reasonably be considered 'essential maths'. Students need to achieve mastery of this to form a solid foundation for further study of mathematics, and to use basic mathematics in everyday life. Consequently, we propose that all students should take a GCSE focused on essential

maths when ready, prior to a further GCSE based on content for progression to A level Mathematics.

Well-designed assessment could enable many more students to master ‘essential maths’ and also improve students’ attitudes to mathematics.

3. 16 to 18 mathematics (proposals 10 – 17)

We want to move to a position where all students are keen to continue with mathematics post-16 and can make an informed choice of the best pathway to meet their needs and aspirations.

We face a persistent problem that most students who achieve grade 3 in GCSE Mathematics at age 16 do not go on to achieve a grade 4 by age 18, and the proportion of students who achieve below grade 3 gaining a grade 4 by age 18 is extremely low.

In considering ‘essential maths’, an expert curriculum and assessment body should review whether a single assessment model is suitable for all young people up to the age of 18 sitting the examination when ready, or whether a different assessment model should be designed for the assessment of ‘essential maths’ for post-16 students.

At level 3,Core Maths is now available in 30% of schools and colleges that offer A level Mathematics. This represents significant progress, but there are still large numbers of young people who cannot access these qualifications.

It is important that students have the option to study either Core Maths or AS Mathematics. Action is required to ensure schools and colleges routinely offer AS Mathematics and that it is the norm for 16 to18 institutions offering level 3 courses to offer a Core Maths qualification. Appropriate funding support should be available to allow this.

Grade boundaries for A level Mathematics in 2019 examinations were too low across the whole range of grades. Ofqual should work with the exam boards to check that suitable target grade boundaries for AS and A level Mathematics are included in each Assessment Strategy and should hold the exam boards accountable for achieving grade boundaries close to these.

4. Technology in mathematics assessment (proposals 18 – 20)

The use of digital technology is now central to many applications of mathematics. Using technology in the assessment of mathematics allows direct assessment of contemporary approaches to solving mathematical problems, including the use of software to model and analyse data and geometry.

Engagement with the large data sets in AS and A level Mathematics falls short of the intentions expressed in the AS and A level Mathematics specifications. Implementation of the more general requirement for technology to permeate the study of AS and A level mathematics also falls short of the intentions in the specifications.

There are significant opportunities to assess the use of technology and appropriate software to help solve mathematical problems. There should be government support for trialling the use of technology and software in mathematics assessments at A level to bring such assessments up to date.

Introduction

MEI is an independent national charity committed to improving mathematics education. Our support for mathematics education includes the development of curriculum specifications and schemes of assessment, providing professional development for teachers, directly supporting students and creating teaching and learning resources. Most of our work is directed towards the mathematics education of 11 to 18 year-olds, addressing both academic and technical pathways, and mathematics in other subjects.

MEI has been prompted to consider mathematics assessment due to several factors:

- Examinations in summer 2020 and 2021 were cancelled due to the COVID-19 pandemic and replaced by alternative assessments. This highlighted the dependence of the English assessment system on end of course examination. Several jurisdictions make use of assessments during the course and/or have some form of teacher assessment - we considered examples of these in our earlier discussion paper, *Assessment in mathematics in England*[1].

- The systems replacing assessment by examination in 2020 and 2021 led to grade inflation. There are plans to return to former standards by 2023, but guarding against similar problems in the future is important and requires wider consideration of how to improve the assessment system in the longer term.

- The appropriateness of students sitting high-stakes external examinations at age 16 when they will continue in education for another two years has been questioned by groups such as Rethinking Assessment[2].

- Changes to curriculum and funding post-16 have led to a narrowing of the 16 to 18 curriculum[3], something recognised in an Education Policy Institute (EPI) report commissioned by the Royal Society[4]. There are benefits to a system whereby students take more subjects post-16; this is common in many other countries and would enable more, or even all, students to study mathematics to age 18, as is common in many other countries.

In July 2021, MEI published a discussion paper, *Assessment in mathematics in England*[1]. The aim of the discussion paper was to prompt debate and seek feedback on the advantages and disadvantages of the current approach and how it might be improved. Following feedback and further consideration stimulated by our discussion paper, this paper presents a set of proposals for the assessment of mathematics at KS4 and 16 to 18 in England.

[1] https://mei.org.uk/reports/mei-discussion-paper-on-assessment-in-mathematics
[2] https://rethinkingassessment.com
[3] Three A level subjects studied over two years is now the standard curriculum model offered to post-16 students following an academic pathway. Before the curriculum and funding changes, students normally studied four subjects to AS standard in year 12, then chose 3 of the 4 to take to full A level in year 13.
[4] A narrowing path to success? Sept 2021, Robinson and Bunting, EPI

Our ideas and proposals are organised in four main sections:

5. Accrediting qualifications
6. Mathematics to age 16
7. 16 to 18 mathematics
8. Technology in mathematics assessment

We appreciate that many of the proposals would take some time to implement and have indicated those that can be implemented relatively quickly.

Although the proposals are focused on secondary and 16 to 18 mathematics, MEI recognises that changes to the mathematics curriculum and qualifications would be part of a wider reform process. Maths plays a central role in the curriculum and the NCETM and Maths Hubs have helped transform the professional development and teaching of maths over recent years. This is already influencing what happens in other subjects and there is also scope for mathematics to lead the way in transforming assessment.

1. Accrediting qualifications

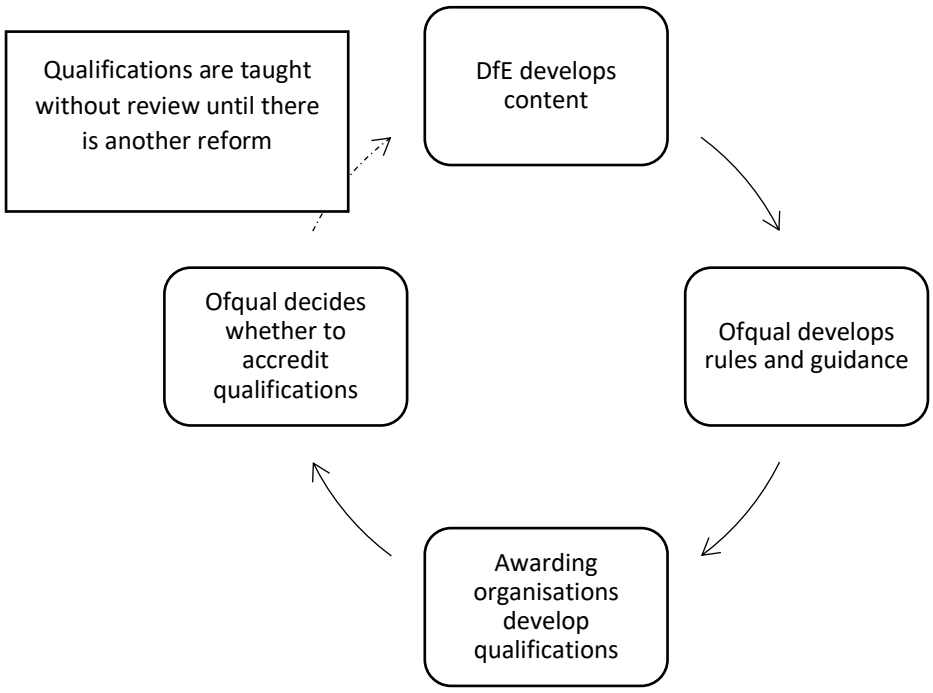
The system used to accredit new mathematics qualifications for first teaching in 2015 (GCSE) and 2017 (A level) took place in stages. Different bodies were responsible for choosing the content and for setting the rules for the assessments, as shown in Figure 1, below. Our view is that this has contributed to a lack of coherence between the intended curriculum and the assessments, with the outcome that students' mathematics education has not improved to the extent intended.

For example:

- At GCSE, a student can achieve a grade 4 'standard pass' in mathematics by gaining fewer than 25% of the marks. This does not demonstrate that the student is equipped to use mathematics in future study, work and life, nor that they have a sound grasp of the mathematical content in the curriculum.

- At A level, the nature of the current assessment means that the aims of the DfE subject criteria for AS and A level Mathematics[5], particularly those relating to analysis of data and statistics and the use of technology to anaylse and solve quantitative problems, are not being met.

Figure 1: The current qualification development and accreditation process



- The current system does not include built-in expert review of the implementation of qualifications in relation to the intended curriculum, nor a method for trialling any new approaches and how these should be implemented, to inform future reforms.

[5] https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/516949/GCE_AS_and_A_level_subject_content_for_mathematics_with_appendices.pdf

- This means that the system is one of intermittent reform rather than evolution, which makes it difficult to ensure that change results in improvement.

There is communication between the stakeholders at each stage of the cycle but no possibility of review of content or assessment rules once development of qualifications has started; the only possibility for adjusting qualifications is to start the cycle again by developing new content and rules for assessment and then developing new qualifications.

The UNESCO International Bureau of Education glossary explains curriculum coherence as:

> *A characteristic of curriculum indicating the extent to which the curriculum aims and content, as well as textbooks, teaching methods, and assessment are all aligned and reinforce one another. Some research findings suggest that a high level of curriculum coherence is associated with high performing systems. (Adapted from: Oates 2010)*[6]

Ofqual notes the interplay between curriculum, teaching, learning and assessment.

> *Qualifications do not operate in a vacuum, independently of other educational concerns. The four pillars of education – curriculum, teaching, learning and assessment – need to operate in synergy with each other. It is especially important that assessment design decisions – however sensible from a validity perspective – do not impact unduly upon curriculum, teaching or learning in such a way as to threaten the acquisition of the very learning outcomes that the qualification is supposed to certify.*[7]

It is important that the process of curriculum and qualification development takes place in a coherent way, with curriculum, teaching and learning taking priority. It is also important that curriculum and qualification development takes account of the likely future mathematical needs of the population; at the time of writing this is being investigated by the Royal Society's Mathematical Futures Programme[8].

Proposal 1

An expert curriculum and assessment body for mathematics should be established to oversee the development and implementation of mathematics curricula and qualifications at secondary and 16 to 18 level. The work of this body should take place on a regular cycle, over a number of years and oversee development and implementation with the aim of ensuring coherence across curricula, teaching, professional development and assessment. Its work should be informed by suitable organisations including the findings of the Royal Society's Mathematical Futures Programme[8].

This document outlines a number of issues that MEI would like the expert curriculum body to consider.

[6] http://www.ibe.unesco.org/en/glossary-curriculum-terminology
[7] An approach to understanding validation arguments, Ofqual, October 2017
[8] https://royalsociety.org/topics-policy/projects/mathematical-futures

Proposal 2

Any changes to qualifications and assessment should be supported by provision of teaching and learning resources and professional development for teachers specifically designed to ensure that changes can be implemented as intended.

Proposal 3

To allow time for the development process and successful implementation, a timescale for changes needs to be mapped out which includes time for

- implementation of the most recent reform;
- review of the most recent reform;
- trialling new approaches to teaching and assessment;
- preparing teaching and learning resources and professional development programmes for teachers in advance of the implementation of reforms.

It is likely that the timescale for change would be more than five years. An OECD working paper[9], referring to a 2010 study, gave examples of countries with a systematic curriculum review process of this kind.

> *Amongst the seven countries having a systematic curriculum review process in the 2010 study, many are top-performers in PISA (OECD, 2019[53]): Japan (ten year cycles), Singapore (six years with a mid-review at the 3-year-mark), Finland (ten year cycles), and Ontario (seven year cycles) (Sargent et al., 2010[52]). Evidence does not point towards an ideal length for the curriculum cycle*.

[9] Curriculum Reform: A literature review to support effective implementation. OECD Working Paper No. 239, 2020

2. Mathematics to age 16

MEI has three main concerns about the current GCSE Mathematics qualification:

A. Thresholds for some grades are too low. This, combined with the current heavy reliance on the use of statistics to set grade boundaries, means that students awarded these grades are not given a full opportunity to demonstrate what they know and can do.

B. GCSE Mathematics assessment is too dependent on examinations.

- The COVID-19 pandemic highlighted the vulnerability of this system in a national crisis.
- The pressure of examinations, which is especially high when they are the only means of assessment, prevents some students from demonstrating their knowledge and skills and can have an adverse impact on student well-being.
- Examinations are not effective at assessing several key aspects of mathematical performance – see Table 3.
- The large number of candidates each year means there will always be some candidates undergoing a personal crisis close to examination time, which can have a devastating effect on their performance.

C. It does not achieve its purposes.

Ofqual, the exam regulator, describes the aims of GCSE mathematics as follows.

> *GCSE specifications in mathematics should provide a broad, coherent, satisfying and worthwhile course of study. They should encourage students to develop confidence in, and a positive attitude towards mathematics and to recognise the importance of mathematics in their own lives and to society. They should also provide a strong mathematical foundation for students who go on to study mathematics at a higher-level post-16.*[10]

and identifies these three purposes of all GCSE qualifications.

> *To provide evidence of students' achievements against demanding and fulfilling content;*
>
> *To provide a strong foundation for further academic and vocational study and for employment; and*
>
> *To provide (if required) a basis for schools and colleges to be held accountable for the performance of all of their students*[11]

[10] GCSE Subject Level Conditions and Requirements for Mathematics, February 2017, Ofqual
[11] GCSE Subject Level Conditions and Requirements for Mathematics, February 2017, Ofqual

The first two of these purposes are not met by the current GCSEs because the assessments do not provide full evidence of what students know and can do and they allow students to gain grades without a sufficiently strong foundation of understanding.

A. Low grade thresholds

Mathematics is vitally important for future work, study and economic well-being. It is by nature a subject where each level of understanding is dependent on a secure foundation of earlier study, meaning that each assessment requires a suitable level of challenge if it is to provide useful information about what students know, understand and can do.

The current two-tier GCSE in Mathematics has low grade boundaries for the lower grades at each tier. This does not provide assurance that students have a confident understanding of mathematics at a level commensurate with the grade they have been awarded; students who achieve a grade with a low number of marks have often picked up odd marks here and there and have not shown the ability to complete tasks successfully or to solve problems.

The grade boundaries from the Pearson/Edexcel GCSE Mathematics examinations in June 2019 are shown in Table 1 as an example; this is the latest set of grade boundaries from the GCSE specification with the highest number of candidates. The first part of the table presents marks out of 240 and the second shows the boundaries as a percentage, rounded to the nearest whole number. As well as showing some low thresholds for some grades, it highlights that some candidates are awarded the same grade despite having very different examination experiences.

Similar grade boundaries are seen in other years and for other GCSE Mathematics exam boards' specifications: these low grade boundaries are a feature of the design of the current GCSE.

Table 1: June 2019 Pearson/Edexcel GCSE Mathematics grade boundaries[12]

Raw marks	Max mark	9	8	7	6	5	4	3	2	1	U
Foundation	240					184	149	111	73	36	0
Higher	240	198	167	137	108	80	52	38			0

Percentage (rounded to integer)	Max mark	9	8	7	6	5	4	3	2	1	U
Foundation	100					77	62	46	30	15	0
Higher	100	83	70	57	45	33	22	16			0

[12] Pearson, https://qualifications.pearson.com/en/support/support-topics/results-certification/grade-boundaries.html?Qualification-Family=GCSE

Students taking maths examinations are usually aware when they cannot do a question, so low grade boundaries are likely to indicate that students found the examination difficult while sitting it. Assessment should provide an opportunity for students to demonstrate what they understand and can do. Students entering an exam knowing that they will not be able to do most of the questions does not constitute good assessment practice.

A perception that mathematics is too difficult is a major reason for not wanting to continue with the subject post-16. Research from 200813, found that students offered the reasons shown in Table 1 for not wanting to continue with mathematics post-16. For all predicted GCSE grades below A*, the main reason is the perceived difficulty of the subject.

Table 2: Student reasons for not continuing with mathematics, by predicted grade

Predicted Grade	**A* (n=23)**	**A (n=96)**	**B (n=317)**	**C (n=540)**
Too difficult	22%	47%	62%	58%
Do not enjoy/ like it	17%	39%	39%	30%
Boring	13%	20%	13%	14%
Not needed for future degree/ career	35%	13%	10%	10%
Not useful in life	9%	6%	3%	2%
Prefer other courses	17%	3%	2%	3%
Note: these are percentages of those students who gave any reason; some gave more than one reason, so percentages may total to more than 100%.				

The way the papers are designed means that low boundaries for the lower grades are inevitable. If GCSE Mathematics is to assess students at age 16 a different model is needed to enable students to gain grades that reliably reflect what they can do. Such a model would need higher grade boundaries than the current model, particularly for lower grades.

Proposal 4

The expert mathematics curriculum and assessment body (see Proposal 1) should develop a system of assessment of the compulsory mathematics curriculum which allows students to show positive achievement in the assessments in order to achieve grades, rather than achieving grades for accumulating a small number of marks from fragments of questions. This is likely to involve changing the tiering arrangements currently used in GCSE mathematics.

[13] Margaret Brown, Peter Brown & Tamara Bibby (2008) "I would rather die": reasons given by 16-year-olds for not continuing their study of mathematics, Research in Mathematics Education, 10:1, 3-18

B. Role of examinations

Methods of assessment

In our earlier discussion paper on mathematics assessment[14], MEI considered the advantages of timed written examinations for assessing mathematics and also what timed written assessments cannot do well. Table 3, below, provides a summary.

Table 3

Advantages of timed written examinations	What current mathematics examinations don't do well
Cost effective when there are large numbers of candidates. Generally fair to most students. Assurance that the work is the candidate's own. Familiarity for stakeholders. Motivational for most students. The awarding of marks is reasonably accurate for mathematics examinations.	Enabling all students to demonstrate positive achievement. Criterion referenced assessment. Assessing consistent performance over time. Assessing performance on sustained or collaborative tasks. Assessing application of mathematics using technology. Assessment by outcome rather than by task.

Types of non-exam assessment used in mathematics include the following:

- Investigations
- Working with data
- Mathematical modelling tasks
- Teacher formative assessment, with external checks

Due to concerns about plagiarism and undue assistance, tasks which students do during their course no longer contribute to GCSE Mathematics grades. The 2021 teacher assessments made use of informal examinations, but these were not organised in a uniform way across the country. However, there is anecdotal evidence that students were motivated to work hard to achieve good teacher grades in 2021.

Teacher assessment could allow criterion referencing to be used as part of the assessment of mathematics, but the following would need to be in place for this to be successful and applied in a standard way across the country:

[14] https://mei.org.uk/reports/mei-discussion-paper-on-assessment-in-mathematics/

- Agreement of criteria to be assessed in this way
- Examples of assessments – these could be questions to be used as part of normal teaching or short tests – or other acceptable forms of evidence
- Agreement of the conditions under which students should be assessed in this way, including what help they could, and could not, receive
- A system of record keeping and moderation which is not too burdensome and time-consuming
- Training to enable teachers and moderators to assess students fairly and accurately

Proposal 5

The potential benefits of including some element of teacher assessment, alongside external examinations, should be investigated.

C. Purposes of GCSE mathematics

Two important purposes of GCSE Mathematics are to allow young people to demonstrate that they have mastered:

- the mathematics knowledge, skills and understanding necessary to apply basic mathematics in future study, work and life – called 'essential maths'[15] in this paper. Achieving at least a grade 4 in GCSE Mathematics is currently used as a proxy for this. However, the low grade thresholds allow students to achieve this level without a confident understanding of the essential maths they will need.
- the mathematics needed to be ready to study AS/A level Mathematics. Achieving a high grade at GCSE is taken as an indication of readiness for AS/A level Mathematics.

GCSE Mathematics is a tiered GCSE; the purpose of this is to allow all students to show what they can do in the examination. The nature of mathematics means that students are often unable to attempt examination questions which require a level of understanding they have not yet attained, whereas examination questions in other subjects can often enable different students to demonstrate different levels of understanding.

Research undertaken in Wales and Northern Ireland[16] found that there was a labelling effect associated with being allocated to foundation tier.

> *The messages conveyed to students about their 'ability' by tiers appear to be internalised and seen as fixed qualities rather than malleable concepts. Thus, as Elwood and Murphy (2002) have argued, when we consider the difficulty of moving between tiers, and the*

[15] The draft curriculum from MEI's Nuffield funded project to develop a new mathematics GCSE curriculum for post-16 resit students could inform the curriculum development for essential maths https://mei.org.uk/about-mei/what-we-do/current-projects-and-programmes/a-new-maths-gcse-curriculum-for-post-16-resit-students/

[16] Barrance, R. (2020), Tiering in the GCSE: A children's rights perspective. British Journal of Educational Research 46, 1210-1231. https://doi.org/10.1002/berj.3629

restrictions in the curriculum offered to foundation candidates, there is a danger that the 'label' of 'foundation' or 'higher' becomes a self-fulfilling prophecy.

The content of foundation tier GCSE Mathematics has a large overlap with what would reasonably be considered to be 'essential maths'. All students need to achieve mastery of this in order to be able to use mathematics successfully in their lives and to form a solid foundation for further study of mathematics. Consequently, we propose that **all** students should first be assessed on 'essential maths'.

The foundational importance of numeracy, and literacy, was recognised in the 2022 education white paper[17].

The cornerstones of a broad, academic, knowledge-rich curriculum are literacy and numeracy. From early years, right through a child's time in school, securing the basics of literacy and numeracy are non-negotiable as the gateway to further learning, attainment, and fulfilling experiences. That is why we have placed such an emphasis on standards of reading, writing and maths over the past decade – and why achieving world-class levels of literacy and numeracy across England is our mission over the next decade.

In 2018, the percentage of UK 15-year-olds reporting that they learnt mathematics by heart was the third highest of 64 jurisdictions internationally. This is likely to reflect repetitive 'teaching to the test' as opposed to deeper study of the subject. The less a student relies on memorisation the more likely they are to report high self-efficacy and positive attitudes towards mathematics[18].

Well-designed assessment could enable many more students to master 'essential maths' and also improve students' attitudes to mathematics.

Proposal 6

All young people should be given the opportunity to master essential maths by age 18. The benefits of this being a separate qualification, with assessment taking place when young people are ready, rather than at a fixed age, should be investigated by the expert curriculum and assessment body (see Proposal 1) as part of the design of the curriculum. To ensure that repeated assessment does not become burdensome for students or schools, students should not be entered for essential maths before the end of Year 9.

[17] Opportunity for all, Strong schools with great teachers for your child, March 2022
[18] OECD https://www.oecd.org/publications/ten-questions-for-mathematics-teachers-and-how-pisa-can-help-answer-them-9789264265387-en.htm

Proposal 7

The expert mathematics curriculum and assessment body (see Proposal 1) should oversee a trial of assessment of essential maths, giving consideration to the following matters.

- The feasibility of teacher assessment
- The feasibility of online assessment
- The possibility of criterion referencing
- The effect on teacher workload

Proposal 8

If essential maths is taken as a qualification before the age of 16, students who have gained the qualification should continue to study mathematics to at least age 16. There should be appropriate pathways available to reflect their aspirations and allow as many students as possible to continue with mathematics to level 3.

The changes to assessment during the COVID-19 pandemic have catalysed a number of groups to think about the examination system. AQA, one of the English exam boards, has started an online hub for data and thinking about the future of assessment and qualifications. An online report on the past, present and future of GCSEs[19] summarises survey evidence about the value of GCSEs as follows:

- *The value of GCSEs is determined by their 'currency' with the public and how students feel about their own GCSE qualifications*
- *An annual survey of public attitudes towards GCSEs published by Ofqual suggests they are well understood, trusted, considered good preparation for further study and are perceived to develop a broad range of skills for students*
- *A survey of around 1,000 young people who took their GCSEs in 2016 and 2017 found that a large majority are glad they took GCSEs, feel pride in their GCSEs, and report that their GCSE grades helped inform decisions about what to do next and to move forward to the next stage. A majority of respondents also report that preparing for their GCSE exams helped motivate them and that taking GCSE exams helped prepare them for exams they took in subsequent years.*
- *However, it is also clear that there is variation among young people regarding their experience of GCSEs and the value of GCSEs to them. Across multiple measures, respondents who averaged grades D to G or 3 to 1, report less value and satisfaction from their GCSEs.*

[19] What Next for GCSEs? The past, present and future of GCSEs, October 2021, James Lloyd, Head of Policy and Public Affairs, AQA https://www.aqi.org.uk/publications/what-next-for-gcses/

The widespread trust for GCSEs leads us to propose that, as long as GCSE continues to be the main qualification at age 16, a separate qualification in essential maths should be a GCSE. Clearly, this would require changes to the current regulatory system for GCSE.

Students who are resitting GCSE Mathematics under the current system are at a similar stage of learning mathematics as target students for an essential maths qualification; over 90% of students taking GCSE Mathematics in November (mainly resit students) take the foundation tier[20]. Essential maths should be available at foundation tier only to ensure that all students are given the opportunity to show sufficient understanding of the foundations for further mathematical learning before proceeding further.

Proposal 9

The expert curriculum and assessment body (see Proposal 1) should consider the future design of a GCSE in essential maths based on a reformed version of the content of current foundation tier GCSE Mathematics. A further GCSE would be based on the content needed for progression to A level Mathematics.

Essential maths would focus on the mathematics understanding and skills all students need and be taken by students when ready from the end of year 9 onwards. It would be designed to ensure that they had achieved mastery of this content before progressing further. For students who succeed in gaining essential maths at grade 4 or 5 before the age of 16, this would be followed by further study of mathematics.

It should be the norm for students to continue mathematics until the age of 18 and there should be appropriate pathways available to reflect their aspirations and allow as many students as possible to continue with mathematics to level 3. Many students would succeed in essential maths at the end of year 9 or 10 and could then aim for a further GCSE based on the content needed for progression to A level Mathematics. Reforming Mathematics GCSE in this way would have the following benefits:

- It is likely that fewer students would need to resit GCSE Mathematics post-16.
- Students who gained the essential maths qualification at grade 4 or 5 would be well equipped to study Core Maths and ready for quantitative study in other level 3 subjects.
- Students who undertook additional learning in mathematics after gaining essential maths grade 4 or 5 would be ready to study A level Mathematics and would have a firm foundation of essential maths skills.
- All students achieving a grade 4 or 5 would have a proven foundation in essential maths for future life and for further study of mathematics.

It would be possible to develop an alternative essential maths GCSE for post-16 students resitting mathematics, in a similar way to the development of an alternative GCSE English qualification for

[20] https://www.gov.uk/government/statistics/entries-for-gcse-november-2021-exam-series/provisional-november-2021-exam-entries-gcse-english-language-and-mathematics#gcse-mathematics-by-tier

such students[21]. The mathematical content could be the same but questions could be more focused on applications of mathematics, informed by the research MEI undertook into a new mathematics GCSE curriculum for post-16 resit students[22].

Despite calls from some quarters for 'dropping GCSEs', there is a strong case for retaining qualifications in mathematics at this stage because of the importance of mathematics in supporting a wide range of post-16 pathways and employment.

We have more to say about 16 to 18 mathematics in the next section, but the availability of clear mathematics pathways up to the age of 18 and an expectation that all students continue with mathematics to age 18 could have a positive impact on student attitudes to mathematics pre-16.

One possible outcome of a serious review of assessment at age 16 might be that a national assessment system remains for (say) English and mathematics, with alternative arrangements to mark attainment in other subjects. This might allow a system to be devised which works well for assessing mathematics, rather than the current one-size-fits-all approach. The tiering arrangement and exam-only approach of the current GCSE Mathematics qualifications have come about from consideration of the whole GCSE system; we suggest there is now an opportunity to decide what works best for mathematics.

[21] https://qualifications.pearson.com/en/news-policy/qualifications/edexcel-gcses/english/relaunch-english-language-learning-with-gcse-english-language-2021.html

[22] A new mathematics GCSE curriculum for post-16 resit students, Jan 2020, Davies et al https://mei.org.uk/about-mei/what-we-do/current-projects-and-programmes/a-new-maths-gcse-curriculum-for-post-16-resit-students/

3. 16 to 18 mathematics

Between age 16 and 18, many young people may not be working towards a mathematics qualification, but most are learning and using maths in some way in their education, whether on an academic or technical pathway.

Figure 2, below, sets out post-16 maths participation for young people aged 19 in 2016, when GCSEs were graded A* to G. These data[23] were originally produced for Adrian Smith's review of 16 to 18 mathematics[24] and count participation in terms of mathematics qualifications; the widths of the bands are proportional to numbers of students. One of the most striking things about this diagram is that most students with grades B and C, who have successfully completed GCSE Mathematics, do not continue to work towards a mathematics qualification post-16.

Figure 2: Post-16 maths participation for different GCSE maths grades

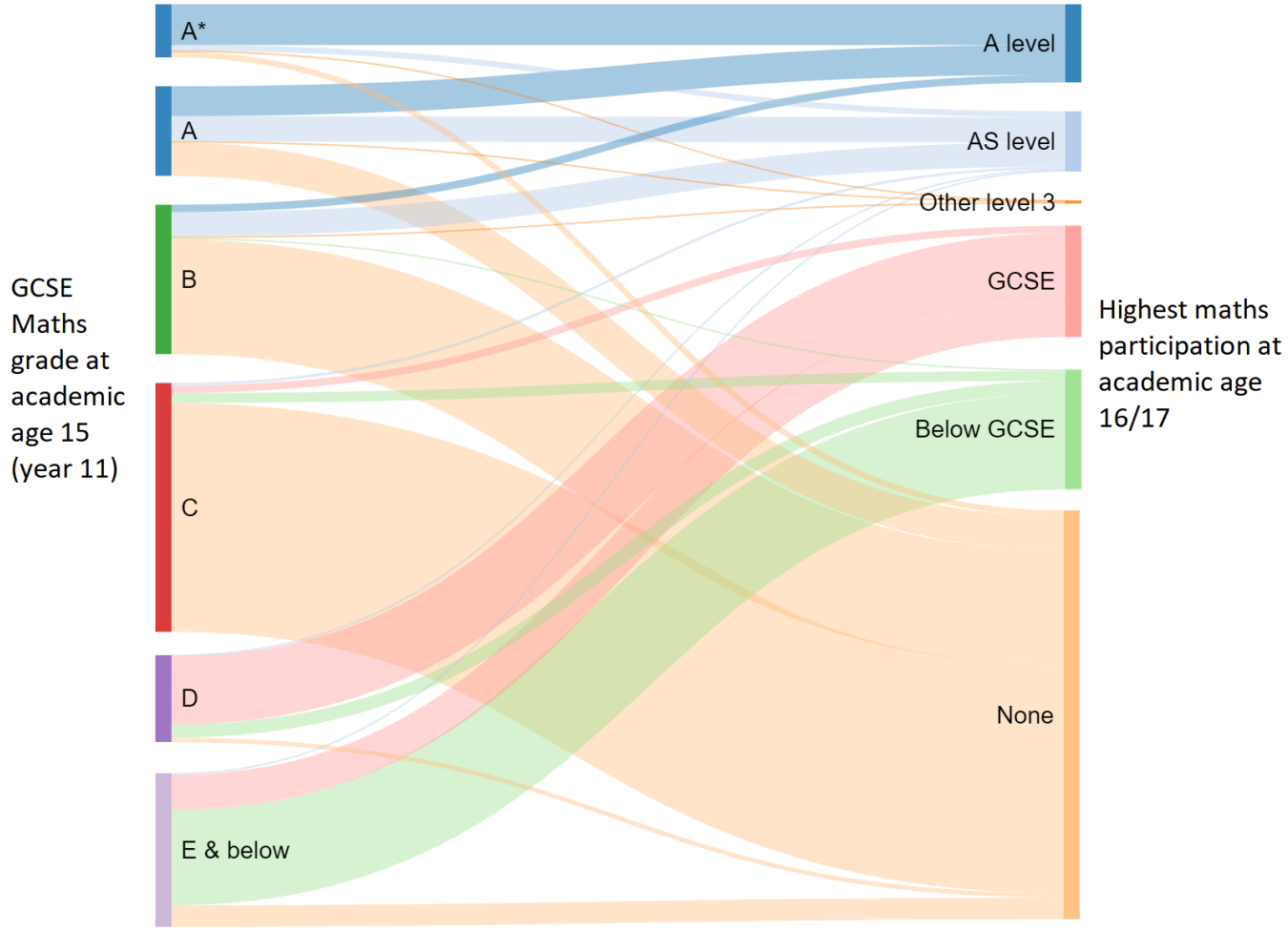


Figure 2 does not illustrate the variety of the mathematics being studied or used by those in the 'None' category, but these students are not working towards a mathematics qualification to support the mathematics that is useful for their aspirations. We want to move to an assessment system where students do not seek to drop mathematics at age 16 but are keen to continue with

[23] Post-16 maths participation in 2015 to 2016, Ad-hoc notice, July 2017, DfE. The DfE has not published similar data since.

[24] Professor Sir Adrian Smith's review of post-16 mathematics education for 16-to 18-year-olds in England

the subject and can make an informed choice of the best pathway to meet their needs and aspirations. A consideration of 16 to 18 technical qualifications is beyond the scope of this paper, however, the move to T levels is resulting in students studying a single, very large, qualification and it is often difficult to study mathematics alongside it.

Level 2 16 to 18 mathematics

As shown in Figure 2, most students who have not achieved a level 2 pass at KS4 (grade 4 or above in the current system) are continuing with mathematics, aiming to achieve at least GCSE grade 4 (or equivalent).

MEI is concerned that the current provision for 16 to 18 students who have not achieved grade 4 in GCSE Mathematics is not fit for purpose. Part of the reason for this is the insistence that 17- and 18-year-olds aim at achieving grade 4 using the current GCSE model.

National statistics[25], summarised in table 4, below, show that most students who achieved grade 3 in GCSE Mathematics at age 16 do not go on to achieve at least grade 4 by age 18, and the proportion of students who achieved below grade 3 GCSE Mathematics at age 16 who progress to achieving a grade 4 by age 18 is extremely low.

Table 4: 16 to 18 GCSE maths results 2018-19

	Number of students in scope for 16-19 funding requirement for maths	**Number of students achieving grade 4 GCSE Maths (or equivalent) or better by age 18**	**Percentage success rate**
Grade 3 GCSE Maths (or equivalent) at end KS4	64,766	24,060	37%
Below grade 3 GCSE Maths (or equivalent) at end KS4	77,722	1862	2%

Since the results in Table 4 were achieved there has been a change in policy whereby it is now possible for students with grade 3 or below in GCSE Mathematics to take level 2 Functional Skills instead of GCSE Mathematics. The COVID-19 pandemic has prevented results from examinations being available for this new policy, so it is yet to be seen whether there is an improved success rate at level 2 for these students. However, the MiFEC Interim Report 2, 'An analysis of policy enactment and practice'[26], found that

> *The difficulty of Level 2 Functional Skills mathematics and its unsuitability as a 'stepping-stone' to GCSE means a common strategy amongst the sample colleges is to move students straight from Functional Skills mathematics level 1 to GCSE. Teachers consider*

[25] https://www.gov.uk/government/statistics/a-level-and-other-16-to-18-results-2018-to-2019-revised
[26] https://www.nuffieldfoundation.org/project/mathematics-in-further-education-colleges-mifec

this to be a large step due to the additional content and focus on knowledge rather than skills and application but this is often outweighed by the argument that students find Functional Skills level 2 assessments too difficult and inaccessible.

Proposal 10

As part of their work on the design of essential maths (see proposals 6 – 9), the expert curriculum and assessment body (see Proposal 1) should consider whether the current assessment model is suitable for all young people up to the age of 18 sitting the examination when ready, or whether a different assessment model should be designed for post-16 students.

Mathematics at level 3

Raising mathematics participation in 16 to 18 education

MEI believes it is important, both to individuals and to society as a whole, that more students study mathematics to age 18; this view is echoed in a report from the Higher Education Policy Institute.

> *Reform A-Levels so that pupils taking an academic path continue a humanities subject, Mathematics and a foreign language until the end of school and so that they study more subjects in Years 12 and 13. This could also include making a science subject compulsory.*[27]

Richmond and Regan, in a report from independent think tank EDSK, observed that A levels were originally designed to provide a broader education than is currently offered in many centres.

> *The original proposals for A-levels in 1951 envisioned a system in which students took numerous subjects as far as possible through secondary education, gradually dropping some subjects as they progressed. It is therefore concerning that, despite being a high quality and rigorous programme in the modern era, A-levels are now an outlier by international standards in terms of the breadth of subjects studied. Similar problems are found with vocational qualifications, with students in England having the option of studying a single subject from the ages of 16 to 18. Not only do other countries reject this narrow conception of secondary education, but they also typically make subjects such as their native language and maths compulsory up to the age of 18 – again, a stark contrast to England that reflects poorly on our current approach. The meagre funding settlement for post-16 education has made matters worse by denying schools and colleges the resources they need to offer a rigorous and broad education to older students.*[28]

A standard A level curriculum offer of 3 A levels plus an AS, or qualification of equivalent size, supported by adequate funding, would allow an increase in uptake of mathematics qualifications.

[27] The Humanities in Modern Britain: Challenges and Opportunities, Gabriel Roberts, HEPI report 141 (2021)
[28] Re-assessing the future Part 2 – the final years of secondary education, Tom Richmond and Eleanor Regan, April 2021 https://www.edsk.org/publications/reassessing-the-future-part-2/

Core Maths

Since the data were collected for Figure 2, Core Maths qualifications have become available for students with a grade 4 or above in GCSE Mathematics but who do not choose to study AS/A level Mathematics[29].

Core Maths qualifications allow students to gain a level 3 qualification alongside other courses which strengthens and certificates their transferrable mathematics skills. Core Maths also helps them become better informed citizens, able to make sense of the quantitative information they will encounter in everyday life, work and further study. It is strongly supported by the British Academy and Royal Society[30] and increasingly recognised as an ideal option for students who have passed GCSE Mathematics at grade 4 or better, but who do not wish to study AS or A level Mathematics.

There were just over 12,000 entries to Core Maths qualifications in 2021 and numbers are gradually increasing. However, this still leaves over 200,000 students each year who continue in education with at least a grade 4 in Mathematics at GCSE but who do not gain a level 3 mathematics qualification[31]. At the time of writing, Core Maths is available in 30% of the schools and colleges which offer A level Mathematics. This represents significant progress, but there are still large numbers of young people who cannot access these valuable qualifications.

AS Mathematics

The number of candidates taking AS Mathematics has declined dramatically since 2018[32], as part of a sharp drop in entries for all AS examinations. This is due to two factors:

- Changes to post-16 funding have led to the large majority of institutions offering a three A level curriculum, rather than the former system in which students generally studied four subjects to AS level and continued with three to A level.

- A level examinations were made linear in the most recent reform, so the marks obtained in AS Mathematics no longer count towards A level Mathematics.

AS Mathematics is valuable:

- as a stepping-stone to A level Mathematics, particularly for students who are undecided at the start of year 12 whether to take A level Mathematics as one of their 3 A level subjects at the end of year 13, and so start with AS Mathematics as one of four subjects in year 12;

- as a qualification in its own right, particularly for students who do not need a full A level in Mathematics for progression to their chosen higher education programme but who would benefit from improving their algebraic skills and being introduced to calculus before embarking on their higher education programme.

It is important that post-16 students who are ready to progress to mathematics at a higher level, but who do not wish to choose Mathematics as an A level, have the option to study either Core

[29] https://mei.org.uk/teachers/core-maths/
[30] https://royalsociety.org/-/media/policy/Publications/2022/2022-01-26-core-maths-joint-statement.pdf
[31] https://explore-education-statistics.service.gov.uk/data-tables/permalink/9b189ab5-d679-4f15-ad04-3cd27d0a9c46
[32] From 2016 to 2020, AS Mathematics entries and AS entries overall dropped by 94% in England.

Maths or AS Mathematics. Action is required to ensure schools and colleges routinely offer AS Mathematics. In the current system, encouraging and enabling greater numbers of students to take Core Maths or AS Mathematics could greatly increase the number of students taking Mathematics post-16.

AS Further Mathematics

The number of candidates taking AS Further Mathematics has also declined, for the same reasons as AS Mathematics. However, it has not dropped to the same extent as other AS levels[33] and continues to provide valuable preparation for HE for many students. Nevertheless, many students who could benefit from taking AS Further Mathematics may not be able to do so due to their school or college having a general policy of not entering students for AS levels.

Proposal 11

All students aged 16-18 should have the opportunity to take a mathematics qualification suited to their needs and aspirations and be encouraged to do so.

Proposal 12

It should be the norm for post-16 institutions offering level 3 courses to offer a Core Maths qualification. Appropriate funding support should be available to allow this.

Proposal 13 – can be implemented quickly

While the current A level system continues, the uptake of AS qualifications in Mathematics and Further Mathematics should be encouraged by schools, colleges and HE.

Proposal 14

The expert mathematics curriculum and assessment body (see Proposal 1) should consider models in which achievement in AS Mathematics and AS Further Mathematics contributes to the respective A level results.

Proposal 15

The expert curriculum and assessment body (see Proposal 1) should consider the inclusion, alongside external examinations, of some other forms of assessment, including teacher assessment.

A level Mathematics thresholds

The first full sitting of the reformed specifications in A level Mathematics took place in 2019; the intention was that A levels should be no harder than before. Grade boundaries for A level

[33] From 2016 to 2020, AS Further Mathematics entries dropped by 83% in England.

Mathematics in the 2019 examinations were too low across the whole range of grades; see Table 5 below. Consequently, many students will have been discouraged by their experience in the examination because they found it more difficult than they expected. This will reinforce the notion that mathematics is hard, and there is a significant risk that this message will spread to younger students and may discourage them from choosing to study mathematics post-16.

Table 5: Grade boundaries for 2019 A level Mathematics qualifications, expressed as a percentage of the maximum mark[34]

	AQA	Edexcel	OCR A	OCR B (MEI)
A*	77	72	72	79
A	62	55	54	65
B	50	45	43	53
C	39	34	33	41
D	28	24	23	30
E	17	14	13	19

Numbers achieving intermediate grades in A level Mathematics

MEI is concerned that too few candidates were awarded grades B and C in 2019 compared with previous years – see Figure 3 below. This effect was repeated in 2020 under the calculated grades arrangement (which was subsequently abandoned). This effect is unfair on students, particularly on those students aiming at grades AAB or ABC for university entrance. It affects most of all those students whose third subject was mathematics, wisely advised to study mathematics for its value in supporting their HE course but finding out that they did not make the grade. An additional 5000 students would have expected to get at least grade C if they had sat A level Mathematics in 2018, rather than in 2019[35].

[34] Based on data published by the exam boards

[35] The percentage of candidates getting at least grade C dropped from 80.8 to 75.6; a drop of just over 5 percentage points. There were 91,895 candidates for A level Mathematics in 2019. 5% of 91,895 is 4595.

Figure 3: Cumulative proportion of UK candidature achieving A level Mathematics grades 2015 - 19 and 2020 algorithm

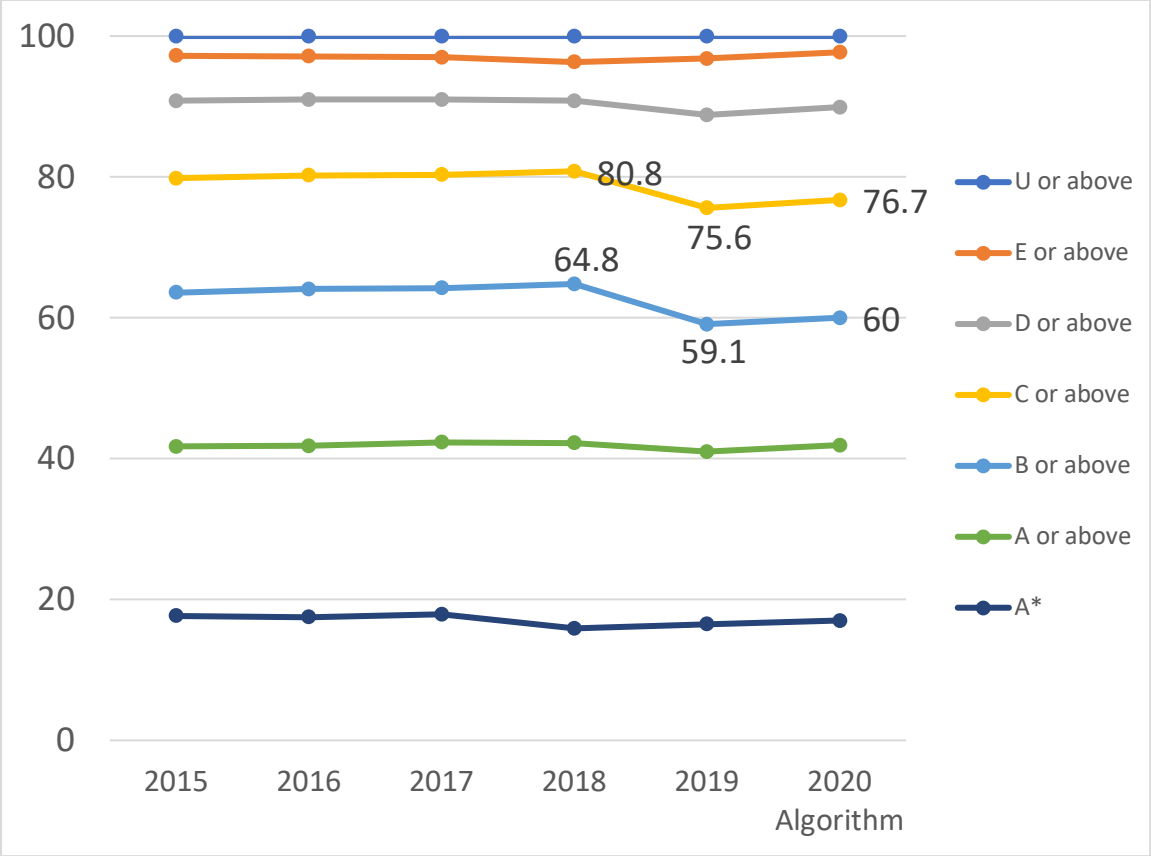
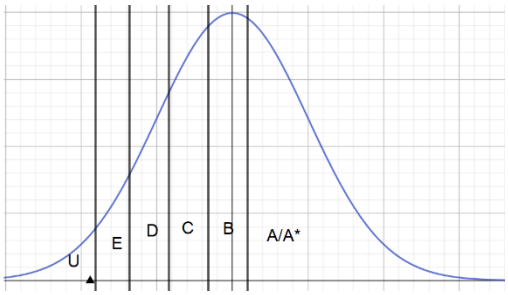
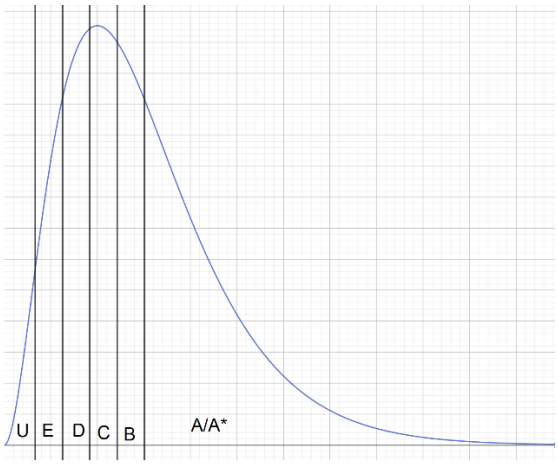


Figure 3 shows that exam boards do a good job of keeping the proportion obtaining grade A or above and grade E or above steady from year to year, but this has not worked consistently for grades B and C in 2019 or for the algorithm in 2020. This is due to the way that grades are decided at A level. The key grade boundaries are A/B and E/U. Once the marks for these grade boundaries are decided, the interval between them is divided equally to establish the other grade boundaries. The percentages obtaining grades B, C and D are dependent on the way that marks are distributed. This is illustrated in the modelling in Table 6 below. This leads us to make proposal 17, below.

In each model the percentage of candidates getting A and A* combined is 42% and the percentage failing to get a grade is 3%, in line with typical percentages for A level Mathematics. However, as can be seen in Table 6 and Table 7 below, if examinations get harder, the percentages getting grades B and C will reduce, even though the percentages at the key grade boundaries are maintained.

Table 6: Modelling A level grade distributions for similar candidates sitting examinations of different difficulties

<table>
<tr><th></th><th>42% A/A*, 3% U</th><th>Approx percentages at each grade</th></tr>
<tr><td>Normal (this is generally thought of as modelling a typical exam)</td><td> 
</td><td>
<table>
<tr><th>grade</th><th>% of candidates</th><th>Cumulative %</th></tr>
<tr><td>A/A*</td><td>42</td><td>42</td></tr>
<tr><td>B</td><td>20.5</td><td>62.5</td></tr>
<tr><td>C</td><td>17.4</td><td>79.9</td></tr>
<tr><td>D</td><td>11.4</td><td>91.3</td></tr>
<tr><td>E</td><td>5.7</td><td>97</td></tr>
<tr><td>U</td><td>3</td><td>100</td></tr>
</table>
</td></tr>
<tr><td>Positive skew

(this models an exam which candidates found generally difficult)</td><td> 
</td><td>
<table>
<tr><th>grade</th><th>% of candidates</th><th>Cumulative %</th></tr>
<tr><td>A/A*</td><td>42</td><td>42</td></tr>
<tr><td>B</td><td>14.3</td><td>56.3</td></tr>
<tr><td>C</td><td>15.7</td><td>72</td></tr>
<tr><td>D</td><td>14.8</td><td>86.8</td></tr>
<tr><td>E</td><td>10.2</td><td>97</td></tr>
<tr><td>U</td><td>3</td><td>100</td></tr>
</table>
Note: percentages will vary depending on the amount of skew
</td></tr>
<tr><td>Negative skew

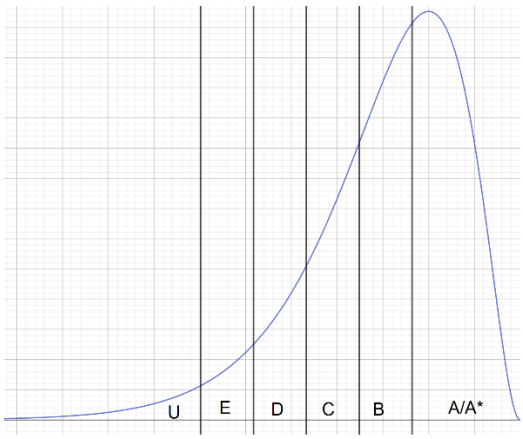
(this models an exam which candidates found generally easy)</td><td> 
</td><td>
<table>
<tr><th>grade</th><th>% of candidates</th><th>Cumulative %</th></tr>
<tr><td>A/A*</td><td>42</td><td>42</td></tr>
<tr><td>B</td><td>26.2</td><td>68.2</td></tr>
<tr><td>C</td><td>16.3</td><td>84.5</td></tr>
<tr><td>D</td><td>8.5</td><td>93</td></tr>
<tr><td>E</td><td>4</td><td>97</td></tr>
<tr><td>U</td><td>3</td><td>100</td></tr>
</table>
Note: percentages will vary depending on the amount of skew
</td></tr>
</table>

Table 7: Summary of cumulative percentages for higher grades from modelling

	Typical exam	Difficult exam	Easier exam
Grade A or above	42	42	42
Grade B or above	62.5	56.3	68.2
Grade C or above	79.9	72	84.5
Grade E or above	97	97	97

Proposal 16 – can be implemented quickly

Ofqual should work with the exam boards to check that suitable target grade boundaries for AS and A level Mathematics are included in each Assessment Strategy. Ofqual should hold the exam boards accountable for achieving grade boundaries close to these. In addition, exam boards should be required to publish their target grade boundaries for AS and A level Mathematics.

Proposal 17 – can be implemented quickly

To avoid unfair variation in the numbers of candidates achieving the grades often required for university entrance, the GCE Qualification Level Conditions and Requirements[36] should be adjusted so that key grade boundaries for A level Mathematics include grade C/D, as well as A/B and E/U. This would mean that examiners would use statistical information and professional judgement to decide these three key grade boundaries before the intermediate boundaries were set arithmetically.

[36] https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/1032771/6804-39_GCE_Qualification_Level_Conditions_and_Requirements.pdf

4. Technology in mathematics assessment

The use of digital technology is now central to many applications of mathematics. Using technology in the assessment of mathematics allows direct assessment of contemporary approaches to solving mathematical problems, including the use of software to model and analyse data and geometry. In our earlier paper, *Assessment in mathematics in England*[37], MEI focused on encouraging students' use of technology in the classroom, in line with the best practice, and identified four possible assessment models:

a. Technology is not used in examinations, but the understanding and skills gained from using technology in lessons help students understand the mathematics better.

b. Examinations include items that directly advantage students who have used technology in learning (this has been attempted with the large data sets in AS and A level Mathematics).

c. Direct assessment involving technology use is incorporated with a paper-based examination, or a suitable centre-based assessment contributes to the final grade.

d. Computer-based examinations which include use of appropriate software applications, changing the style of questions to assess the use of statistical and graphing tools.

Options a and b reflect the design of the current A level subject criteria for Mathematics; however, these have not resulted in widescale adoption of the use of technology in the classroom. Options c and d can ensure that students have better skills to apply mathematics and are prepared for the mathematics they will encounter in HE and the workplace.

The 2021 PISA assessments (postponed to 2022) will be computer-based. The framework for assessment[38] outlines some of the potential features of using computer-based assessment of mathematical literacy.

> *The main mode of delivery for PISA 2021 will be the computer-based assessment of mathematics (CBAM). The transition has been anticipated with both the 2015 and 2018 studies moving to computer-based delivery. In order to maintain trends across the studies, both the 2015 and 2018 assessments were computer neutral despite using a computer-based delivery mode. The transition to a full CBAM in 2021 provides a range of opportunities to develop the assessment of mathematical literacy to be better aligned with the evolving nature of mathematics in the modern world, while ensuring backward trends to previous cycles. These opportunities include new item formats (e.g. drag and drop); presenting students with real-world data (such as large, sortable datasets); creating mathematical models or simulations that students can explore by changing the variable values; curve fitting and using the best fit curve to make predictions. In addition to a wider range of question types and mathematical opportunities that the CBAM provides, it also allows for adaptive assessment.*

England can learn from PISA in its use of computer-based assessment in mathematics. Exam boards are investigating how computer-based assessment might offer an alternative mode of

[37] https://mei.org.uk/reports/mei-discussion-paper-on-assessment-in-mathematics/
[38] Pisa 2021 Mathematics Framework (Draft), November 2018, OECD

delivery with associated cost and efficiency benefits; there are significant opportunities to assess the use of technology and appropriate software to solve mathematical problems.

Proposal 18

There should be government support for trialling the use of suitable technology and software in mathematics assessments at A level to bring such assessments up to date.

The conditions and requirements for AS and A level Mathematics include the following statement, with a similar statement for Further Mathematics.

> The use of technology, in particular mathematical and statistical graphing tools and spreadsheets, must permeate the study of AS and A level mathematics.[39]

An additional statement about a specific use of technology for AS and A level Mathematics occurs in the requirements concerning large data sets.

> AS and A level mathematics specifications must require students to:
>
> - become familiar with one or more specific large data set(s) in advance of the final assessment (these data must be real and sufficiently rich to enable the concepts and skills of data presentation and interpretation in the specification to be explored)
> - use technology such as spreadsheets or specialist statistical packages to explore the data set(s)
> - interpret real data presented in summary or graphical form
> - use data to investigate questions arising in real contexts[39]

One piece of research into the teaching of the new A levels in Mathematics found that some centres were not engaging with this requirement for a number of reasons.

> *The introduction of the large dataset also generated significant uncertainty, in relation to the style of the questions as well as the extent to which students would need to be familiar with the dataset. A number of participating centres chose not to cover this element of the course or only to engage with it superficially, as they felt that the time and resources required to engage with it did not reflect the marks available; this trend increased over time.*[40]

Engagement with the large data sets in AS and A level Mathematics falls short of the intentions expressed in the AS and A level Mathematics specifications. Implementation of the more general requirement for technology to permeate the study of AS and A level mathematics also falls short of the intentions in the specifications. Appropriate development work and trialling is needed in advance of the next reform of level 3 mathematics qualifications, the education system in England will not be ready for the introduction of appropriate technology in the examinations.

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[39] GCE Subject Level Conditions and Requirements for Mathematics, April 2016, Ofqual

[40] Teaching and learning for 'moving goal-posts': Reformed A Levels in mathematics, Ben Redmond, Jennie Golding, Grace Grima, Proceedings of the British Society for Research into Learning Mathematics 40 (1) March 2020

Proposal 19

The expert curriculum and assessment body (see Proposal 1) should oversee an ongoing trial of computer-based assessment in Level 3 mathematics assessments, including the use of appropriate software for working with data, and graphing software.

Proposal 20

Computer-based assessment, including the use of appropriate software for working with data, and graphing software, should be incorporated into Level 3 Mathematics assessments as soon as the trial in Proposal 19 indicates that the appropriate systems can be put in place.