Introduction to Decision Mathematics

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Decision or Discrete?

Decision mathematics is also known as **finite mathematics** or **Discrete Maths**

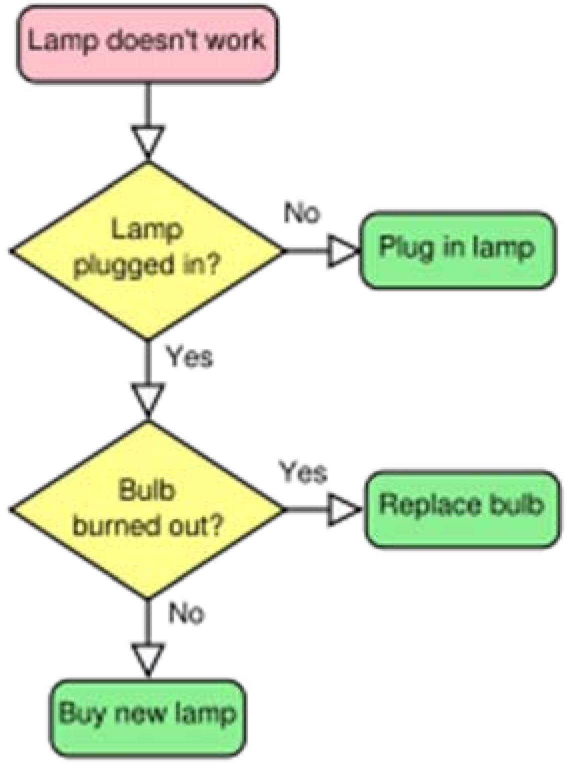
Why Decision Maths?

Decision mathematics has become popular in recent decades because of its applications to computer science.

Many of the problems involve Optimisation – finding an efficient solution – and hence methods are applicable to many real world situations.

Algorithms

Many problems in this area can be solved by applying an Algorithm – a clear, finite, terminating list of instructions.



Russian Peasant Multiplication

- Write the two numbers (A and B) you wish to multiply, each at the head of a column.
- Starting with A, divide by 2, discarding any fractions, until there is nothing left to divide. Write the series of results under A.
- Starting with B, keep doubling until you have doubled it as many times as you divided the first number. Write the series of results under B.
- Add up all the numbers in the B-column that are next to an odd number in the A-column. This gives you the result.

http://www.youtube.com/watch?v=L6wYOpjgDZw

Russian Peasant Multiplication

$$17\times 22 = 17\times(0\times 2^0 + 1\times 2^1 + 1\times 2^2 + 0\times 2^3 + 1\times 2^4)$$

$$= 17\times(2+4+16)$$

$$= 34+68+272$$

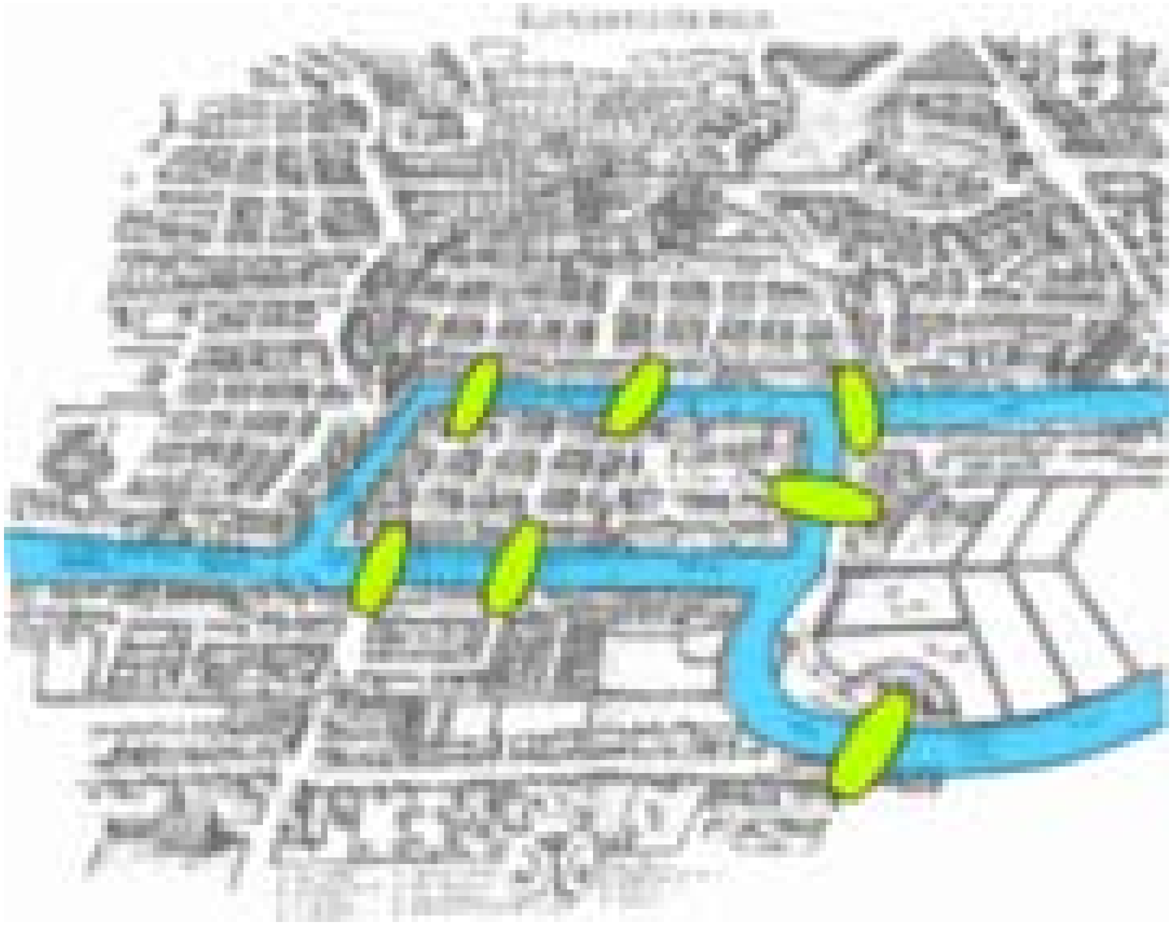
$$= 374$$

Königsburg’s 7 Bridges

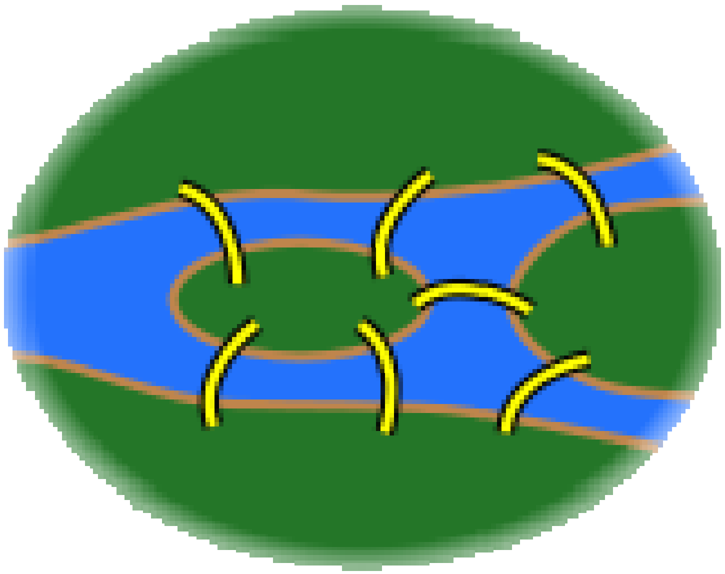
The city of Königsberg, Prussia (now Kaliningrad, Russia) is set on the Pregel River, and included two large islands which were connected to each other and the mainland by seven bridges. The question is whether it is possible to walk with a route that crosses each bridge exactly once.

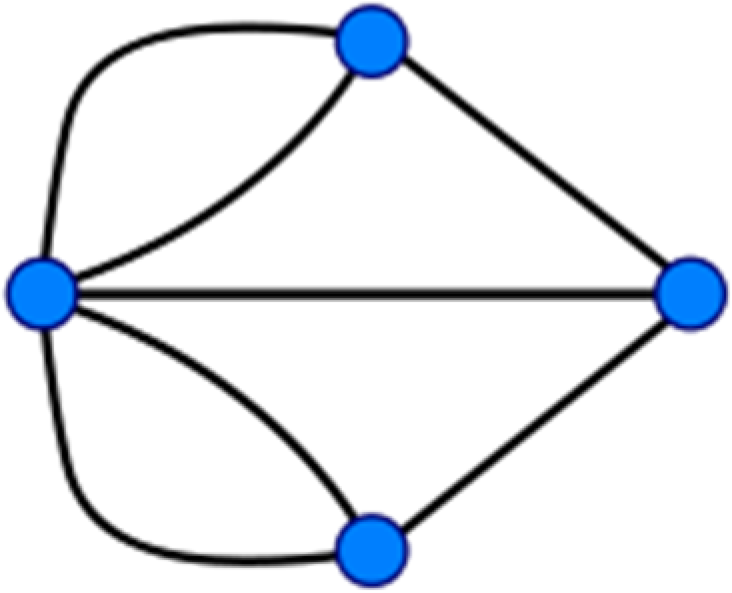
Leonard Euler investigated this in 1736.

Town Plan

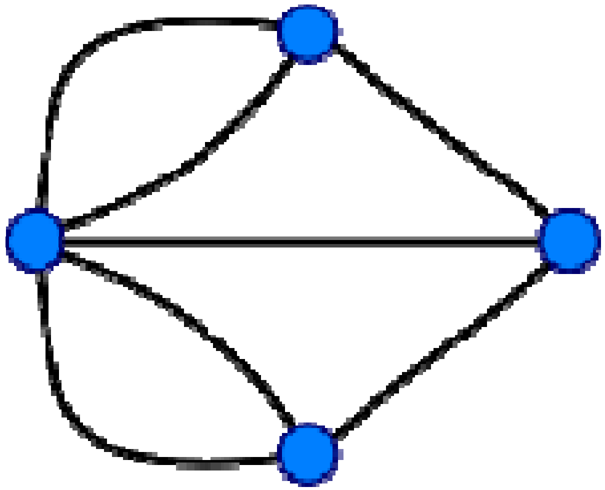


Modelling





A Graph



This graph has:

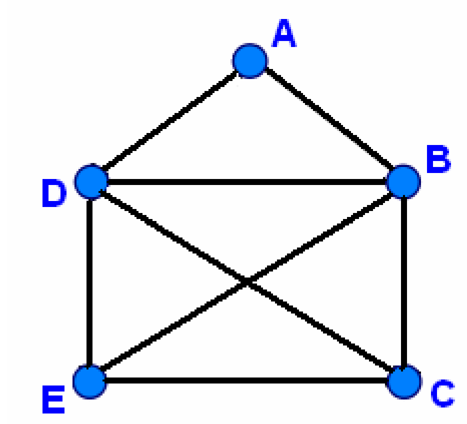
- Four vertices/nodes
- Seven edges/arcs

This graph is:

- connected
- not simple
- not complete
- not Eulerian
- not traversable

Another Famous Graph

http://www.metacafe.com/watch/477022/draw_a_envelope_without_moving_the_pen_off_the_paper



This graph has:

- Five vertices/nodes
- Eight edges/arcs

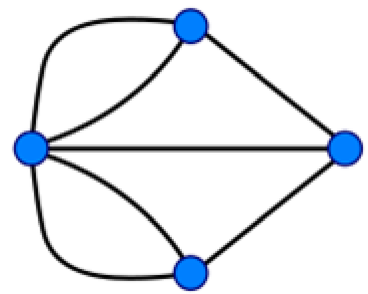
This graph is:

- connected
- simple
- not complete
- not Eulerian
- traversable

The Eulerian path shown is ECDBEDABC

The Solution

Euler realized that the problem could be solved in terms of the degrees of the nodes. The degree of a node is the number of edges touching it; in the Königsberg bridge graph, three nodes have degree 3 and one has degree 5. Euler proved that a circuit of the desired form is possible if and only if there are exactly two or zero nodes of odd degree. Such a walk is called an *Eulerian path* or *Euler walk*. Further, if there are two nodes of odd degree, those must be the starting and ending points of an Eulerian path. Since the graph corresponding to Königsberg has four nodes of odd degree, it cannot have an Eulerian path.



Networks

In the real world, graphs are more likely to be networks – the edges have a ‘weight’.

A very common network would be a map indicating the road distances between towns.

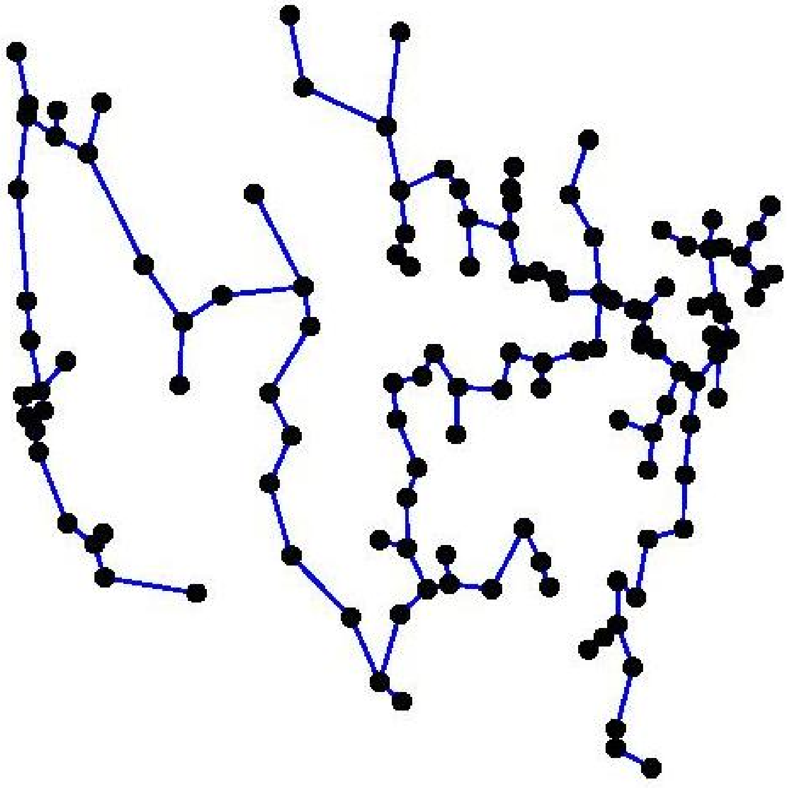
Famous Network Algorithms

- Prim’s Algorithm (1921-)
- Kruskal’s Algorithm (1928-)
- Dijkstra’s Algorithm (1930-2002)
- Floyd’s Algorithm (1921-)
- Ford-Fulkerson Algorithm (maxflow-mincut)

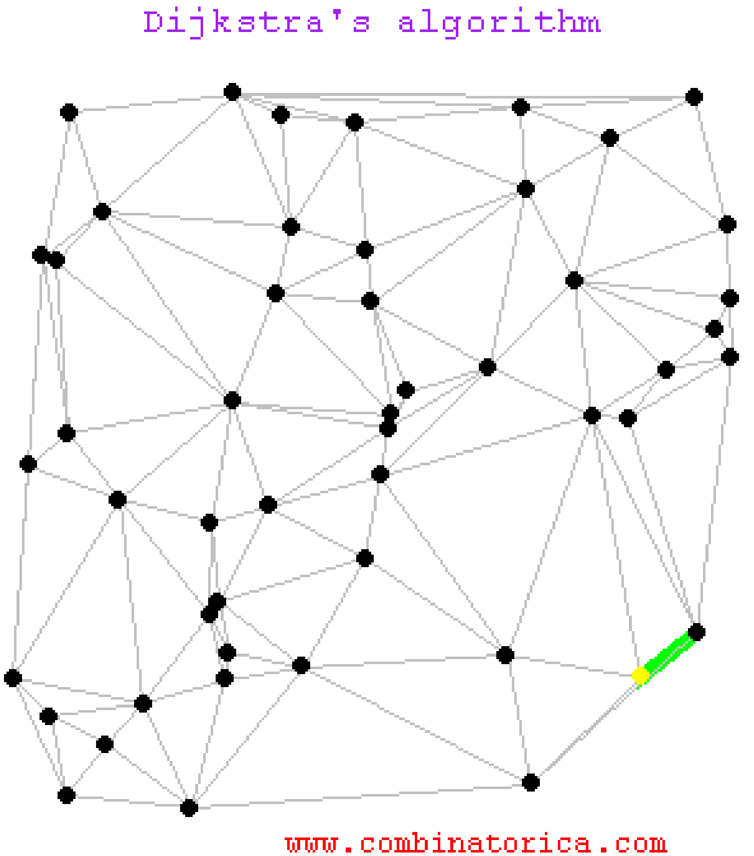
These algorithms are covered in D1/D2.

Minimum Connector & Shortest Paths

Minimum Connector



Shortest Path



http://www.unf.edu/~wkloster/foundations/DijkstraApplet/DijkstraApplet.htm

Linear Programming (LP)

Although the mathematical methods required for LP were known by Fourier (1768-1830) it was first used as a mathematical model by the allies during WW2 – and it was kept secret until 1947.

Dantzig published the simplex method in 1947.

A typical LP Problem

Given a set of constraints…

…you must optimize ‘Utility’

As the name suggests, the constraints and the utility function are always Linear!

MacDiet (a LP with 30 variables, 19 constraints)

The LP is based on data for 22 MacDonald's foods.

Diet		1	2	3	4
Cost	£	1.30	4.83	5.23	5.99
Big Mac	.99	1.3	1		
Cheeseburger	.91				1
MacChicken sandwich	.99				1
Scambled eggs & muffin	.86			2	
French fries	.57		2	3	2
Tomato ketchup portion	.00	70.5	4	6	4
Banana milkshake	.90		3	2	2
Hot chocolate	.55				1
Orange juice	.60				1
Calories		2400	2308	2303	2303
Protein		55.0	59.8	66.2	63.8
Fat		41.4	76.2	76.3	75.4
Saturated fat		16.5	33.8	29.3	27.1
Carbohydrate		450.0	343.0	336.0	340.4
Sugars		326.2	204.4	148.2	190.1
Sodium		1.2	2.0	1.7	2.5
Fibre		4.9	15.3	22.0	19.1

Diet 1. At least 2300 calories of which no more than 30% from fat. At least 55g protein and no more than 3g sodium.

Diet 2. Require an integer solution. No more than twice as many tomato ketcup portions as french fries.

Diet 3. No more than 2 banana milkshakes.

Diet 4. No more than 2 french fries and less than 64g protein.

Typical D1 LP Question

Jane is baking Cookies and Donuts for the school fair.

A Cookie needs 30g of flour and 1 egg, and makes 10p profit.

A Donut needs 20g of flour and 2 eggs, and makes 15p profit.

Jane has 360g of flour and 20 eggs to hand.

Other ingredients are readily available.

How many of each should Jane bake to maximise her profit, and how much profit can she make?

Formulation

x=number of cookies: y=number of donuts

<u>Constraints</u>

Flour: $30x+20y \leq 360$

Eggs: $x+2y \leq 20$

<u>Utility</u>

Profit: $P=10x+15y$

http://www.meidistance.co.uk/flash/d1/linpro.html

Linear Programming (maximising)

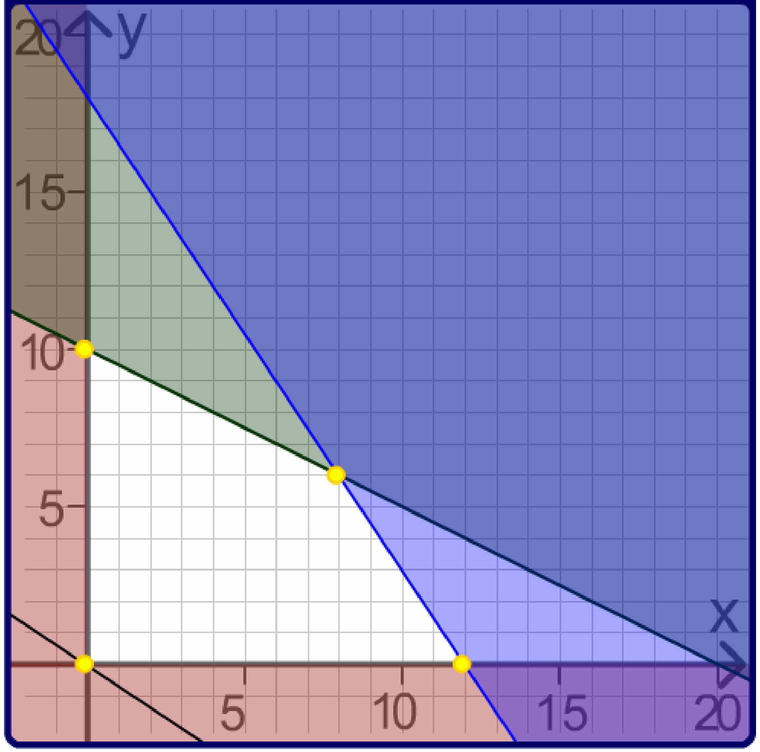
Objective function

$P = 10x + 15y$

Constraints

$3x + 2y \leq 36$ $\quad x + 2y \leq 20$ $\quad x \geq 0,\ y \geq 0$

Graphs



Vertices

$x = 8,\ y = 6$

$P = 10 \times 8 + 15 \times 6$

$= 170$

Maximum value of P (so far)

$x = 8,\ y = 6$

$P = 170$

Objective function line

$10x + 15y = 0$

help

reset

Critical Path Analysis (CPA)

CPA, is a mathematically based algorithm for scheduling a set of project activities.

It was developed in the 1950s in a joint venture between DuPont Corporation and Remington Rand Corporation for managing plant maintenance projects.

Modelling CPA

A model of the project is constructed that includes the following:

- A list of all activities required to complete the project
- The time (duration) that each activity will take to completion, and
- The dependencies between the activities.

Real World

In the real world, for large projects, there is commercially available software which allows the input of activities, precedence and durations, which will then produce the schedule of events and plot a cascade diagram.

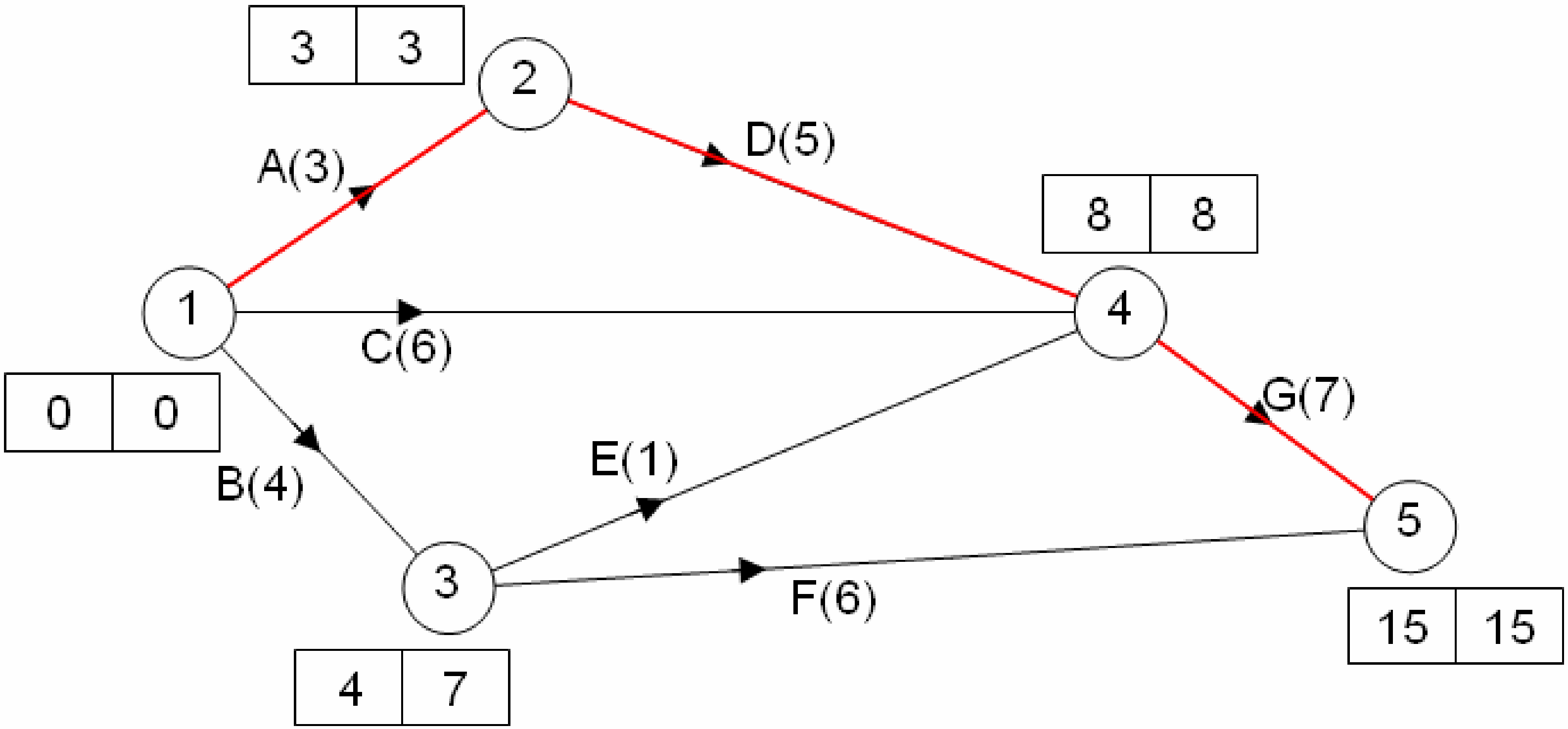
One such program is Microsoft Project.

Precedence Table

http://www.meidistance.co.uk/pdf/d1/mei/c/1/Activity%20networks%20example%201.ppt

Task	Duration (hours)	Immediate predecessors
A	3	-
B	4	-
C	6	-
D	5	A
E	1	B
F	6	B
G	7	C, D, E

Activity Network

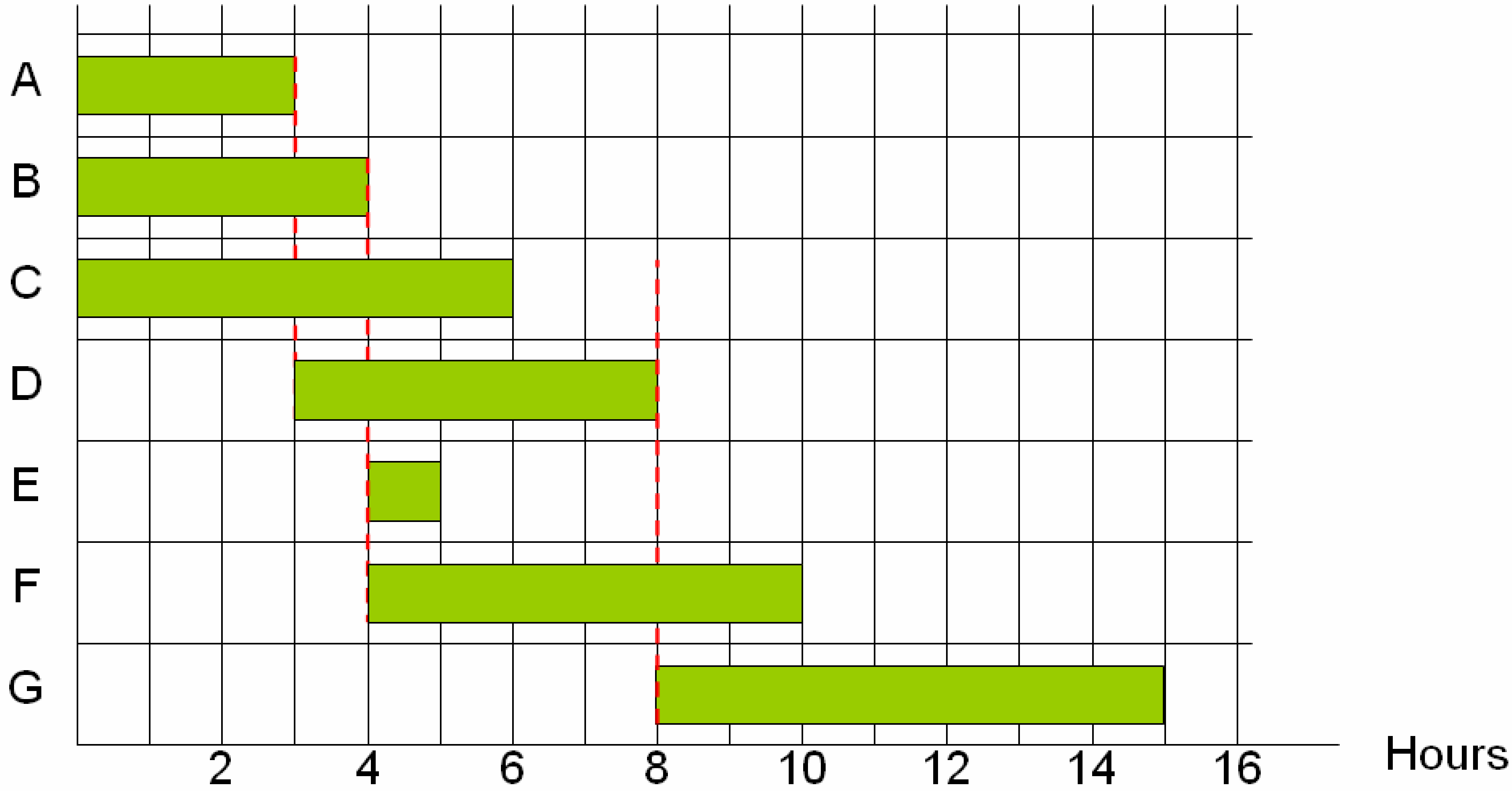


The completed network shows that the project can be completed in 15 hours.

The critical activities are A, D and G.

Cascade Diagram

http://www.meidistance.co.uk/pdf/d1/mei/c/2/Cascade%20diagrams%20example%201.ppt



Do students like D1?

YES:

- Little algebra required
- Accessible straight from GCSE
- Most marks are method marks
- Links with subjects like Business and Computing
- Full online resources with interactivities.

Do students like D1?

NO:

- memorise some standard algorithms.
- Students take shortcuts and do not score full marks
- Doesn’t reinforce many skills learnt in Core
- Exam questions tend to be wordy.

Learn More...

An introduction to Decision Mathematics 1

An in-depth introduction to the mathematics in this module from a teaching and learning perspective to the point where they feel confident to teach it in the knowledge that further support is available after the course.

More details:

http://www.mei.org.uk/cpd/alevel.shtml