Understanding the assessment of Core Mathematics

MEI Conference 2010

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Agenda

- Importance of the specification
- Understanding a mark scheme
- Common errors made by candidates

Maximizing success: know the Specification

- Read the detail of the spec
- Know what is in the formula book
- Check that other formulae in the spec are learnt by heart
- Look at the questions set in past papers

Maximizing Success : know your way round the Exam

- Read the Principal and Chief Examiner's Reports
- Know where candidates make the most mistakes
- Emphasize the key things to gain marks

The mark scheme

- When the M, A and B marks are used
- What happens when a candidate makes multiple attempts at a question?
- Are errors followed through?
- How accurate do answers have to be?
- Right answer but no working?

Marking actual questions

- Applying the mark scheme
- How we try to ensure that marking is as consistent as possible:
  Standardization meetings. Team Leaders meet before to discuss actual answers.
  Regular checks that everyone is applying the scheme consistently

Points arising from C1 scripts : Graphs

- Sketching graphs.
  Weaker candidates try to plot points which is usually gives an inaccurate impression
- A clear careful sketch is all that is required
- On graph sketches and elsewhere, points are expected to be simplified, e.g. $\frac{4}{2}$ or $\sqrt{1}$ + 2 are penalized
- Many cannot recall the equation of a circle which is centred at the origin. Some think this is a parabola or even a straight line!

Points from C1: Terminology

- We expect to see : Translates ... units in... Direction OR rotate clockwise/anticlockwise through angle ..., OR stretch parallel to...axis by scale factor ..., OR reflect in...
- **SHIFT, MOVE, FLIP, etc, NO.**
- **Instead of STRETCH "... co-ordinate gets twice as high", NO.**
- **TRANSLATES upwards, ...up ...axis, ...in ...axis, ...on ... axis, in positive direction, NO.**
- **Instead of TRANSLATES "increases by ... values", NO.**

Points from C1 scripts : Indices

- Candidates can handle fractional indices well like 1/3 or ½ but are less good at dealing with 2/3 or 4/3 or adding powers e.g $\sqrt{(500)} + \sqrt{(125)}$.
- Rationalizing denominators is usually well done but handling expressions involving roots is quite poorly handled, e.g. lots of $\sqrt{(16 + a^2)} = 4 + a$,
  and simplifying $\sqrt{(9 + k)} = 4$ to $3 + \sqrt{k} = 4$

Points from C1 scripts: Quadratics

- Candidates often do not make substitutions clear e.g.
  Solve $x^4 - 10x^2 + 25 = 0$ is often turned into
  $x^2 - 10x + 5 = 0$ or $x^2 + \sqrt{10}x + 5 = 0$, instead of $(x^2 - 5)(x^2 - 5) = 0$. It is important for the examiner to see $y = x^2$ explicitly, otherwise solving $x^2 - 10x + 5 = 0$ for x and stopping implies candidate believes equation is quadratic in x and not in x^2
- If $y = x^{1/2}$ is substituted then we need to see new quadratic equation in terms of y and not just changed to x, otherwise unclear that any substitution has occurred.

Points from C1 scripts: Links

- Candidates do not see links between topics e.g. often there is a question on finding the discriminant of a quadratic then they are asked for the number of roots of the equation. Many fail to realise the parts are linked, and often further linked into graphs of quadratic function.
- Quadratic equations could expect to occur in questions involving
  (i) "solve".... (ii) simultaneous equations
  (iii) quadratic inequalities
  (iv) sketching a graph of a quadratic
  (v) in questions involving a quadratic function in f(x)
  (vi) a circle and line intersection

Points from C1 scripts: Inequalities

- Linear inequalities such as $1 \le 2x - 5 \le 9$ are badly done since candidates only modify one of the inequalities.
- With quadratic inequalities, such as $x^2 - 3x + 2 = 0$. candidates should be drawing a sketch and **labelling** the acceptable region. If the region is shaded examiners do not know if this shaded region is the acceptable region or the rejection region and it is difficult to award any method mark.
- Often something like $x^2 - 3x + 2 \ge 0$ is rearranged to $x^2 - 3x \ge -2$ or $x^2 \ge 3x - 2$ and then sqrt(...) attempted.

Points from C1 scripts: Differentiation

- Usually well done but candidates often can't convert $3/x^2$, $\sqrt[5]{(x)}$ or $4\sqrt{x}$ to ax^p .
- Whilst maximum/minimum well understood, the idea of increasing and decreasing functions is little known
- There are several methods for establishing whether max/min.
  (i) An accurate sketch with comment, and this may be ideal for cubic or very simple functions.
  (ii) d^2y/dx^2 together with reference to sign, but if the x value of the stationary point is wrong or only one of two stationary points found then the method mark for this procedure is unlikely to be awarded.

Points from C1 scripts : Differentiation 2

- (iii) examine dy/dx or y values at **close** points either side of stationary point **BUT** a sketch of the curve is essential, e.g. Find the type of stationary points for $y = x + 4/x$. If $x = -1$, $dy/dx = -3$. If $x = 3 \Rightarrow dy/dx = 5/9$ so dy/dx changes from negative to positive and we have a minimum at $x = 2$ where $dy/dx = 0$. However, this is incorrect (although appears correct) since one is looking at the gradient on a different branch of the curve.

Points from C2 scripts: formulae

- Know your formulae
  e.g. Cosine rule with sines in it,
- nth term formulae like a + nd,
- A.P. sum formulae with product signs in,
- G.P. Sum formula with n outside the bracket.
- radian formulae for sector area and arc length with degrees used,
- trapezium rule with x values used or integration attempted first to get y values

Formulae problems in C3 and C4

- Use of given list of formulae is often sadly adrift, e.g. As the integral of tan x is shown to be ln(sec x) , the integral of should not be thought to be ln ($\sec^2 x$) or $(\ln \sec x)^2$

Points from C2 scripts: Accuracy

- "exact" means no calculator answer
  e.g. Surds tend to be decimalized
- 3 significant figures is the norm for the final answer:
  many candidates use less while calculating and lose credit for inaccurate results

Points from C2 scripts: showing working

- If answers are given, too few steps are given e.g. A value might be substituted into a factor theorem expression. The candidate knows the answer has to be 0 but shows no working out to prove it
- Answers only. If a particular method is specified in the question, an answer only receives 0

Points from C2 scripts: Terminology again

- Root and factor is badly distinguished
- Confusion over dy, integral signs and +c
- Composition of functions – either used as multiplication or the order of the functions reversed
- Term and coefficient not clear

Important point for C2 : the best method

- Candidates are poor at choosing the best method e.g. Factor theorem, long division or compare coefficients
- N.B. FACT - Candidates who compare coefficients in C2 rarely do it successfully
- Long division has problems – some add lines instead of subtracting them – don't make remainder clear

Points from C2 scripts: graphs

- Sketches require the initial conditions, the basic shape and the end condition like asymptotes
- Critical values needed on axes but not plots or scales
- N.B Copying from calculators is usually disastrous
- Reinforce:

A sketch is not something slapped down on the page carelessly

Common Points in C2 and C1

- Indices
- Surds
- Solving quadratics
- Squaring – square term by term
- Brackets omitted – time and again!!!
  Basic Algebraic errors recur – GCSE, C1, C2

General points about C3 and C4

- In general, the PE's feel that many candidates are not prepared for these modules:
  they can cope with routine questions which they have obviously practised.
  They cannot deal with open ended problems which admit of more than one solution. This is a disadvantage with "stretch and challenge" questions on these modules.
- Many errors made in AS modules are repeated at A2

Points in C4 scripts: Algebraic

- The solving of simple quadratic equations which are not given in the standard form

  $x^2 + 2x = 3$ implies $x(x+2) = 3$ so $x = 3$ or $x = 1$

- In questions involving the division of a polynomial by a linear or quadratic polynomial, 2 methods obviously come into mind: long division and comparing coefficients. Very often the long division method proves to be the easier way but the identity method is often seen and this more often produces errors.

Points in C4 scripts : calculus and vectors

- The differentiation of logarithmic functions is often done incorrectly; candidates needed to differentiate $\ln(9t)$ recently and a very common result was $\frac{1}{9t}$
- In straight-forward integration questions, we do not penalise candidates if they omit the '+c'; but, in practical modelling problems, it is surprising how many candidates do forget the '+c'.
- When vector equations are given in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$, a large minority of candidates are not aware of what the direction vector is.

Key Points

- Know the formulae not in the formula book by heart
- Know the meanings of mathematical terms
- Give answers appropriately
- Emphasize the requirements for graph sketches
- Emphasize the importance of correctly placed brackets again and again
- Know the correct use of the log laws

Thank you for coming

- We hope this day has been useful for you.
- Now please help US to improve. Before you go, do complete the feedback form and hand it in so we can do better next time!
- Many thanks and have a good journey home

Guidance for marking C2

Accuracy

Allow answers to 3sf or better, unless an integer is specified or clearly required.

3sf is sometimes explicitly specified in a question - this is telling candidates that a decimal is required rather than an exact answer eg in logs, and more than 3sf should not be penalised unless stated in mark scheme.

If more than 3sf is given, allow the marks for an answer that rounds to the one in the mark scheme, with no obvious errors.

Extra solutions

Candidates will usually be penalised if an extra, incorrect, solution is given. However, in trigonometry questions only look at solutions in the given range and ignore any others, correct or incorrect.

Solving equations

With simultaneous equations, the method mark is given for eliminating one variable allowing sign errors, addition / subtraction confusion or incorrect order of operations. Any valid method is allowed ie balancing or substitution for two linear equations, substitution only if at least one is non-linear.

Solving quadratic equations

Factorising - candidates must get as far as factorising into two brackets which, on expansion, would give the correct coefficient of x^2 and at least one of the other two coefficients.

Completing the square - candidates must get as far as $(x + p) = \pm \sqrt{q}$, with reasonable attempts at p and q.

Using the formula - candidates need to substitute values into the formula and do at least one further step. Sign slips are allowed on b and $4ac$, but all other aspects of the formula must be seen correct, either algebraic or numerical. If the algebraic formula is quoted then candidates are allowed to make one slip when substituting their values. Condone not dividing by $2a$ as long as it has been seen earlier.