

Binomial and Normal distributions

Teaching Ideas

Which is more likely?

- A. Roll 4 fair dice and get one 6.
- B. Roll 5 fair dice and get no 6s.

By the end of this session, you will be able to work this out.

A gambling problem



In the 17th century, a French nobleman, the Chevalier de Mere, played two different games of chance.

- Rolling at least one 6 in four throws of a single die
- Rolling at least one double 6 in 24 throws of a pair of dice.

The Chevalier's reasoning

On one throw of a die,

$$P(\text{six}) = \frac{1}{6}$$

Average number of 6s in four throws =

$$4 \times \frac{1}{6} = \frac{2}{3}$$

Throwing two dice,

$$P(\text{double six}) = \frac{1}{36}$$

Average number of double 6s in 24 rolls =

$$24 \times \frac{1}{36} = \frac{2}{3}$$

Why did he lose more often on the second game?

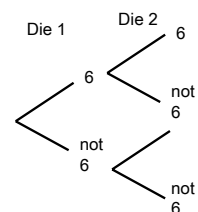
De Mere wrote to his friend Pascal

- Pascal and Fermat solved the problem between them, starting the study of probability
- For four throws of one die, find the probabilities

Number of 6s	0	1	2	3	4
probability					

Working out probabilities

The probabilities can be worked out using a tree diagram but the more dice there are the bigger it gets; this takes more time and makes it easier to go wrong.



Finding a pattern

Number of dice	No 6s	One 6	Two 6s	Three 6s	Four 6s
1	$1 \times \left(\frac{5}{6}\right)$	$1 \times \left(\frac{1}{6}\right)$			
2	$\left(\frac{5}{6}\right)^2$	$2 \times \left(\frac{1}{6}\right) \times \left(\frac{5}{6}\right)$	$\left(\frac{1}{6}\right)^2$		
3	$\left(\frac{5}{6}\right)^3$	$3 \times \left(\frac{1}{6}\right) \times \left(\frac{5}{6}\right)^2$	$3 \times \left(\frac{1}{6}\right)^2 \times \left(\frac{5}{6}\right)$	$\left(\frac{1}{6}\right)^3$	
4	$\left(\frac{5}{6}\right)^4$	$4 \times \left(\frac{1}{6}\right) \times \left(\frac{5}{6}\right)^3$	$6 \times \left(\frac{1}{6}\right)^2 \times \left(\frac{5}{6}\right)^2$	$4 \times \left(\frac{1}{6}\right)^3 \times \left(\frac{5}{6}\right)$	$\left(\frac{1}{6}\right)^4$
5					

In general

$$P(X = r) = {}^n C_r p^r q^{n-r}$$

$$r = 0, 1, 2, \dots, n$$

where p is the probability of success and q is the probability of failure

$X \sim B(n, p)$ shows that the random variable, X , has a binomial distribution with n trials and probability p of success each time.

What is success?

In this case, 6 on a die.

What is failure?

In this case "not 6" on a die.

$p = P(\text{success}) = 1/6$ in this case

$q = P(\text{failure}) = 5/6$ in this case

$r = 0, 1, 2, 3$ $n = 3$ for 3 dice

Back to the first question

$$P(X = r) = {}^n C_r p^r q^{n-r}$$

A. Roll 4 fair dice and get one 6.

$n =$ $p =$ $r =$

B. Roll 5 fair dice and get no 6s.

$n =$ $p =$ $r =$

Where does the Binomial distribution occur?

- You are conducting an experiment or trial n times
 - Eg tossing a fair coin 8 times
- There are 2 outcomes, which we can think of as "success" or "failure"
 - Eg heads and tails
- The probability of "success" is the same each time (symbol p). The probability of "failure" is $1-p$
 - The probability of "success" on any trial is independent of what has happened in previous trials.
- The random variable we are interested in is "the number of successes"

Situation: Toss a biased coin 10 times

Random variable: The number of heads

Situation: Throw a fair die 7 times

Random variable: The number of 4s

Situation: Throw a fair die 12 times

Random variable: The number of even scores

Situation: Throw a fair die

Random variable: The score

Situation: Throw a fair die repeatedly

Random variable: The number of times you have to throw it to get a 6

Situation: Choose 6 students from a class of 30

Random variable: The number of girls chosen

Situation: Choose 6 students from a large school

Random variable: The number of boys chosen

Situation: Open a tube of “smarties” and count the number of each colour

Random variable: The number of orange smarties

Situation: Choose a jury

Random variable: The number of black people on the jury

Situation: Take a 40 question multiple choice test by guessing each answer

Random variable: The number of questions wrong

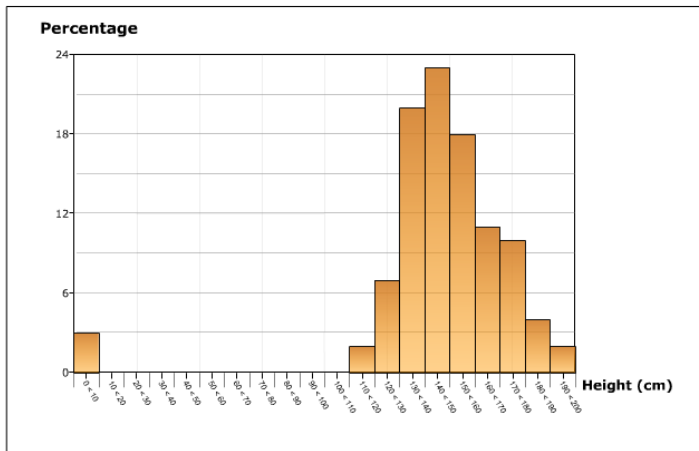
Situation: Each student in the class tosses a coin

Random variable: The number of heads

Situation: The probability of my suitcase getting to the right destination is 0.95.

Random variable: The number of passengers on a flight with “lost” luggage

Histogram of Height (cm)



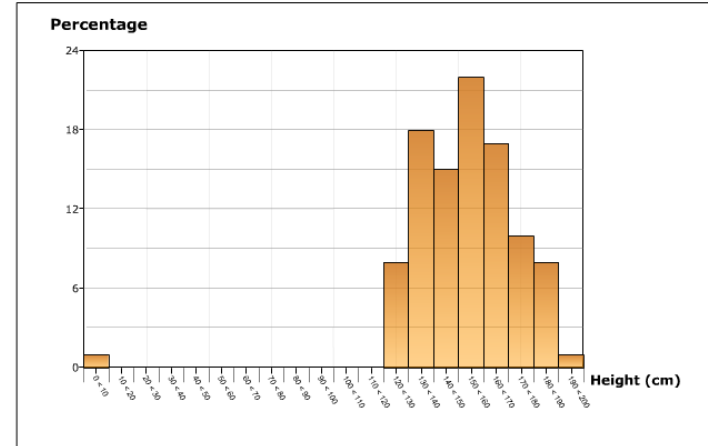
Source UK Secondary 2000-1
Sample A Gender = Male:

These 4 samples are taken from the Census at School database.

What population is the sample from?
Do you think the sample is from a Normal population?
Why?

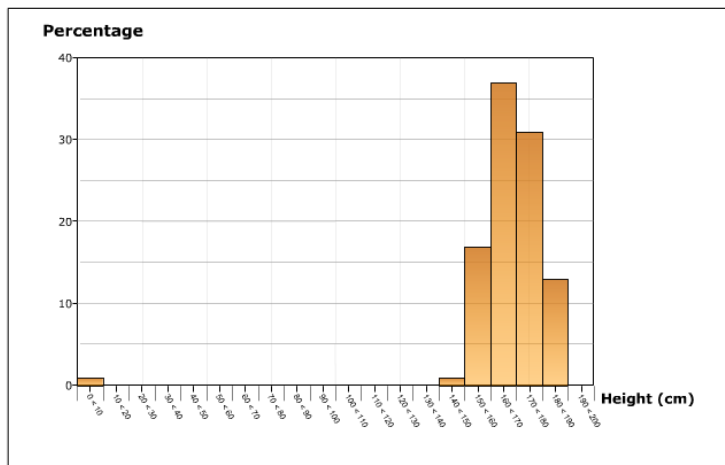
<http://datatool.censusatschool.org.uk/datatool.swf>

Histogram of Height (cm)



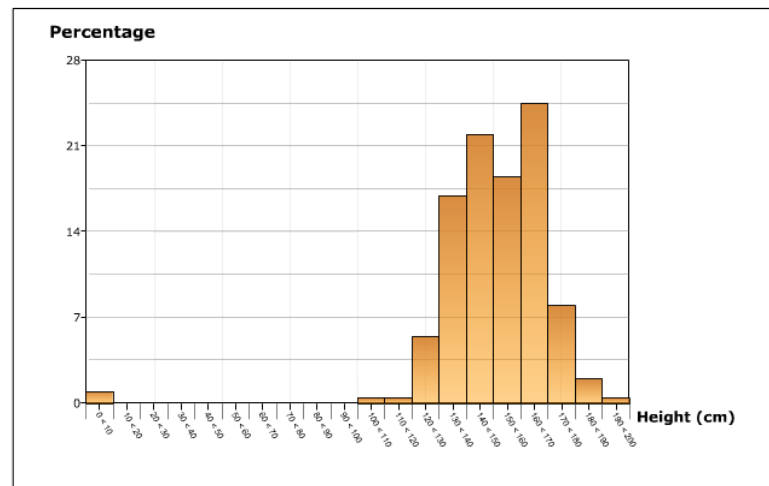
Source UK Secondary 2000-1
Sample A Gender = Male:

Histogram of Height (cm)



Source UK Secondary 2000-1
Sample A Gender = Male, Female: Year Group = 11:

Histogram of Height (cm)



Source UK Secondary 2000-1
Sample A Random sample: No filters used