Algebra and functions

- expanding and factorising
- surds
- rationalising
- laws of indices
- dividing a polynomial
- factor theorem
- remainder theorem

Sine and cosine rule

- using both to find missing sides and angles
- finding two solutions for missing angles
- area of a triangle =½ ab sinθ

Quadratic functions

- graphs of quadratic functions
- solving equations by factorising
- completing the square
- use of the formulae
- sketching graphs of quadratic formulae

Exponentials and logarithms

- function $y=a^x$
- writing expressions as logs
- calculating using logs to base 10
- laws of logs
- solving equations of the form $a^x=b$
- changing base of logs

Equations and inequalities

- simultaneous equations by elimination
- simultaneous equations by substitution
- linear and quadratic simultaneous equations
- linear equalities
- quadratic unequalities

Binomial Expansion

- Pascal's triangle
- Combinations and factorial rule
- using $\binom{n}{r}$ in the binomial; expansion
- expanding $(a+bx)^n$

Sketching curves

- cubic functions
- reciprocal graphs and functions
- intersection points of graphs to solve equations
- transformations f(x+a), f(x-a), f(ax), af(x)

Radian measure and graphs of trig functions

- radians to measure angles
- length of an arc
- area of a sector
- area of a segment
- graphs of sine, cos and tan
- values of functions in four quadrants
- surd values for functions
- transformations of sin, tan and cos curves

Coordinate geometry

- straight line in form ax+by+c=0
- gradient of a straight line
- $y-y_1=m(x-x_1)$
- equation of a straight line
- parallel and perpendicular
- midpoint of a line
- distance between two points on a line
- equation of a circle

Differentiation

- increasing and decreasing functions
- stationary points, max, min and inflexion
- using turning points to solve problems
- derivative of f(x) as the gradient of the tangent top the graph f(x)=y
- formula for gradient of ax^n
- expanding or simplifying functions to make them easier to differentitate
- second order derivatives
- rate of change at a particular point
- equation of tangent and or normal at a point

Sequences and series

- nth term
- recurrence
- arithmetic sequences
- arithmetic series
- sun to n of an arithmetic series
- using Σ notation
- geometric sequences
- geometric series
- sum of a geometric series
- sum to infinity of geometric series

Trig Identities

- simple trig identities
- solving simple trig equations
- solving equations in the form sin(nθ+a), cos(nθ+a) and tan(nθ+a) = k
- solving quadratic trig equations

Integration

- simple definite integration
- area under a curve
- area under a curve negative values
- area between a line and a curve
- the trapezium rule
- integrating x^n
- integrating simple expressions
- using the integral sign
- simplifying expressions before integrating
- finding the constant of integration

Parametric equations • Using parametric equations • Conversion to Cartesian • Finding the area under a curve given by parametric equations	**Functions** • mapping diagrams • range, mapping diagrams and graphs • composite functions • Inverse functions
The binomial expansion • Expanding $(a+bx)^n$ • Use of partial fractions	**Exponential and log functions** • $y=a^x$ • $y=e^x$ • Using e^x and the inverse of the exponential function $\log_e x$
Differentiation • Parametric differentiation • Implicit relations • a^x • Rates of change • Simple differential equations	**Numerical methods** • finding approximate roots of f(x) = 0 graphically • Iterative and algebraic methods to find approximate roots of f(x)=0
Vectors • Diagrams • Unit vectors • 2D & 3D • Scalar product • Vector equation of a straight line • Angle between two lines	**Transforming graphs of functions** • Modulus function $\|f(x)\|$ and $f(\|x\|)$ • Solving equations involving a modulus • Applying combinations of transformations to curves
Integration • Standard functions • Reverse chain rule • Trgi identities • Partial fractions • Substitution • By parts • Numerical • Areas and volumes • Differential equations	**Trigonometry** • functions of secant, cosecant and cotangent • graphs of secant, cosecant and cotangent • Simplifying expressions, proving identities and solving equations using sec, cosec and cot • using inverse trig functions and their graphs

Partial fractions	Algebraic fractions
Further trig identities • Addition trig formulae • double angle formulae • solving equations and proving identities using double angle • using $a\cos\theta+b\sin\theta$ • factor formula	Differentiation • chain rule • product rule • quotient rule • exponential function • logarithmic function • differentiating sinx, cos x, tanx • further trig functions

Functions challenge

The function $f(x) = 2x + 4$ can be written using a function machine like this:

You can reverse this by undoing the operations like this. This is called finding the inverse.

We say the inverse function is $$f^{-1}(x) = \frac{x-4}{2}$$

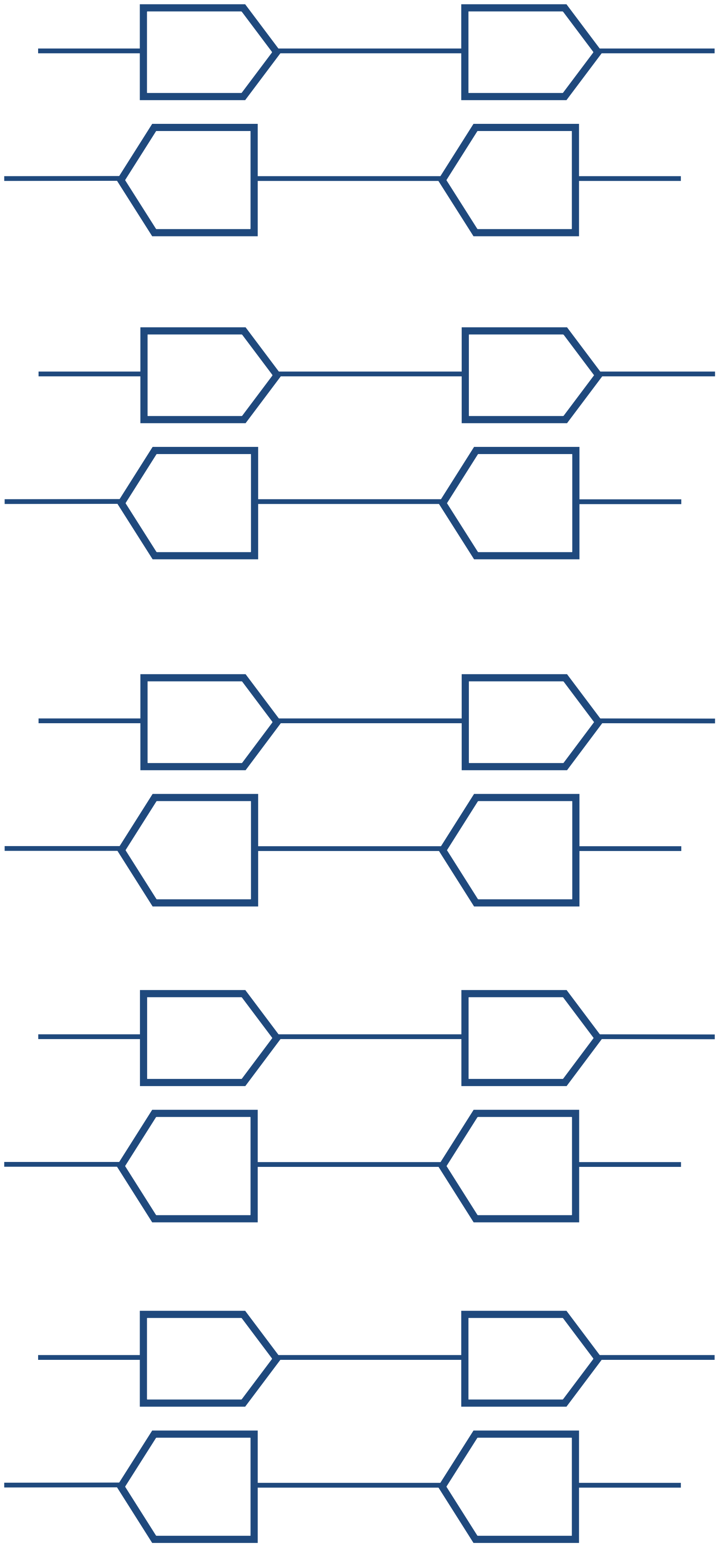
Can you find the inverses of these functions? Use the function machines if you want.

1. $f(x) = 3x+5$
2. $f(x) = 4x+7$
3. $f(x) = \frac{x}{2} + 1$
4. $f(x) = \frac{x+2}{3}$
5. $f(x) = \frac{2}{3}x + 3$
6. $f(x) = 3 – 2x$
7. $f(x) = x^2$
8. $f(x) = \sin x$

Draw a graph of a function and its inverse – on the same pair of axes – what do you notice?

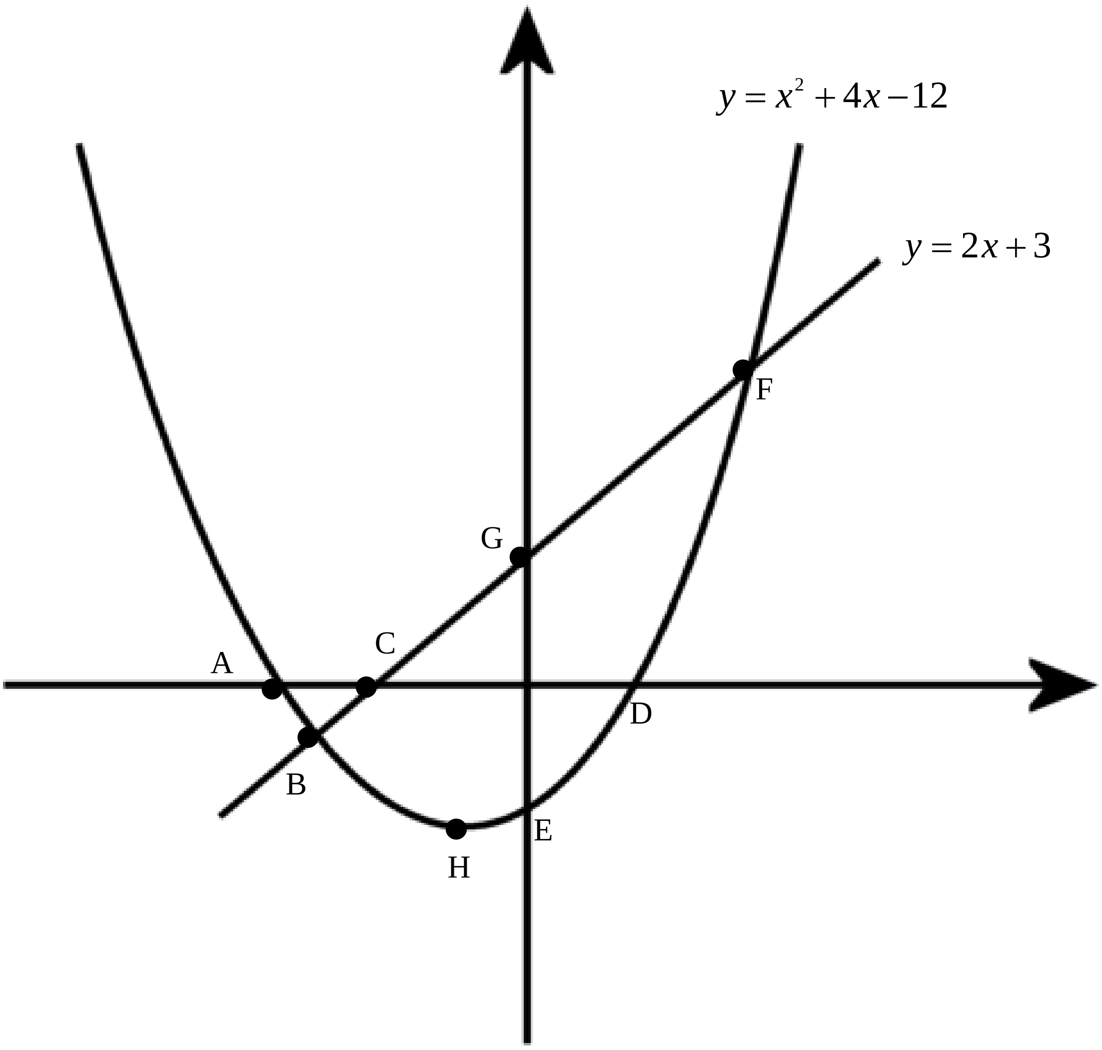
What is different about numbers 7 and 8 – why do you think this happens?

Function machines



Quadratic Challenge I

Can you work out the coordinates of all the points named with letters below?

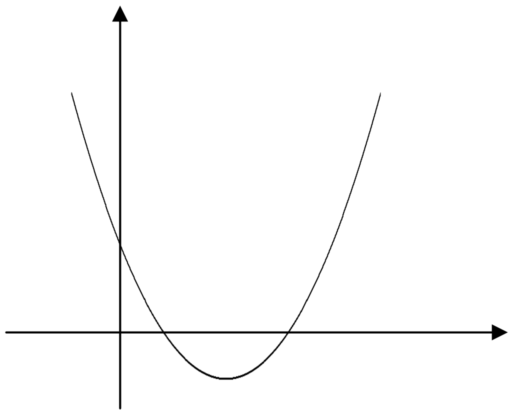


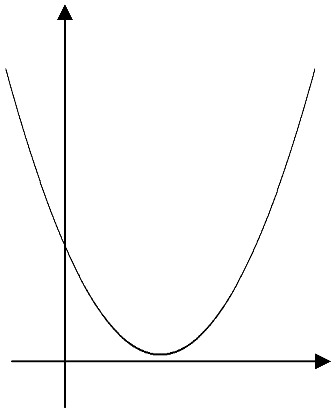
Quadratic Challenge II

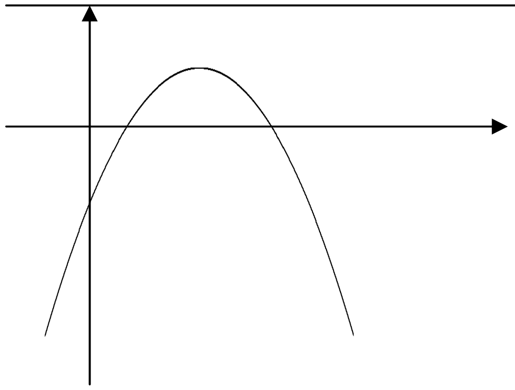
Cut out all the cards. Can you match all the cards?

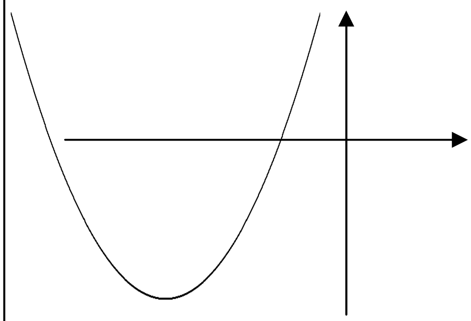
$y=x^2+6x-16$	$y=x^2-8x+16$
$y=8-x^2+2x$	$y=6x-x^2-8$
$y=x^2-10x+16$	$y=x^2+6x+8$
$y=x^2-6x-16$	$y=(x-8)(x+2)$
$y=(x+4)(x+2)$	$y=(x+2)(4-x)$
$y=(x-4)(2-x)$	$y=(x-8)(x-2)$
$y=(x-4)(x-4)$	$y=(x+8)(x-2)$
$y=(x+3)^2-25$	$y=(x-4)^2$
$y=(x-5)^2-9$	$y=-(x-3)^2+1$
$y=-(x-1)^2+9$	$y=(x+3)^2-1$

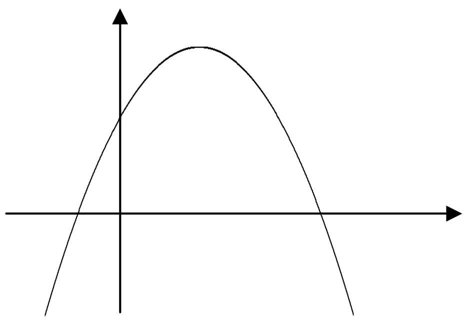
$y=(x-3)^2-25$	Minimum at (3, -25)
Minimum at (-3, -1)	Maximum at (1, 9)
Maximum at (3, 1)	Minimum at (5, -9)
Minimum at (4, 0)	Minimum at (-3, -25)
$x=0,\ y=-16$	$x=0,\ y=16$
$x=0,\ y=16$	$x=0,\ y=-8$
$x=0,\ y=8$	$x=0,\ y=8$
$x=0,\ y=-16$	$y=0,\ x=8 \text{ or } -2$
$y=0,\ x=-4 \text{ or } -2$	$y=0,\ x=-2 \text{ or } 4$
$y=0,\ x=4 \text{ or } 2$	$y=0,\ x=8 \text{ or } 2$
$y=0,\ x=4$	$y=0,\ x=-8 \text{ or } 2$

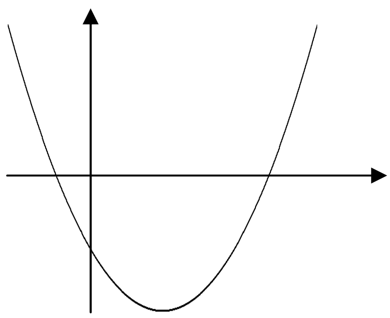


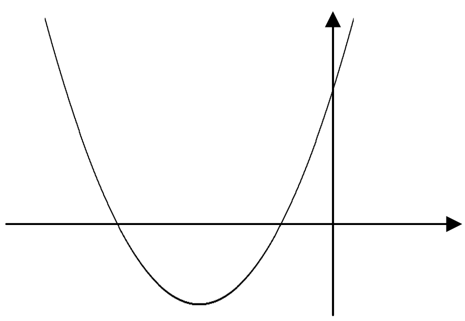












Simultaneous Equations Challenge

You can now solve pairs of simultaneous equations. What happens if there are more than two? Like these:

$$3x+4y+z=3$$
$$x+y+z=2$$
$$2x+y-z=2$$

The solution to this problem is on worksheet A but it is in the wrong order. Can you sort it out so that it is in the right order? Then try worksheet B which uses a different method.

Which method seems to work best for you?

Now try these – can you solve them? Use one of the two methods that you explored on sheets A and B.

$$x-y+zx=10$$
$$3x+y+2z=34$$
$$-5x+2y-z=-14$$

$$x+y+2z=11$$
$$2x-3y-z=-9$$
$$2x-y+3z=7$$

Type the equations above into a graph plotting programme like Autograph or Omnigraph.

- What can you see?
- Is it what you expected?
- Why is there only one solution to the set of equations?
- Can you think of a situation where there might be a different number of solutions?
- Can you describe a situation when there are no solutions – what would it look like?

Worksheet A

$3x+4y+z=3$ – equation 1

$x+y+z=2$ – equation 2

$2x+y-z=2$ – equation 3

Solve to get

x=2 and y=-1

$$\begin{aligned} &2x+y-z=2 \\ +\ &x+y+z=2 \\ \rightarrow\ &3x+2y=4 \end{aligned}$$

Take away equation 2 from equation 1

You now have a pair of simultaneous equations and you know how to solve these.

$2x+3y=1$ & $3x+2y=4$

Add equation 3 to equation 2

Solution is x=2, y=-1 and z=1

3x2 + 4x-1 + z = 3

6 + -4 + z = 3

z = 1

Pick any of the three equations – for example equation 1 : $3x+4y+z=3$

Substitute in the values of x and y

$$\begin{aligned} &3x+4y+z=3 \\ -\ &x+y+z=2 \\ \rightarrow\ &2x+3y=1 \end{aligned}$$

Worksheet B

$3x+4y+z=3$ – equation 1

$x+y+z=2$ – equation 2

$2x+y-z=2$ – equation 3

Now substitute into equation 2

$x+y+z=2$ – equation 2

$x+y+(3-3x-4y)=2$

$-2x-3y+3=2$

$-2x-3y=-1$

We know

$z = 3 - 3x - 4y$

Put in values of $x=2$ and $y=-1$ and get $z=1$

$3x+4y+z=3$ – equation 1

$z = 3 - 3x - 4y$

Solution is $x=2$, $y=-1$ and $z=1$

Solve the two new equations you have made

$-2x-3y=-1$ & $x+y=1$ to get

$x=2$ and $y= -1$

Now substitute into equation $2x+y-z=2$ – equation 3

$2x+y-(3-3x-4y)=2$

$5x+5y-3=2$

$5x+5y=5$

$x+y=1$

Take equation 1 and rearrange it to make z the subject of the formula