Practical Activites in Mechanics

Ted Graham
Centre for Teaching Mathematics
University of Plymouth

Friction Demonstrations

Right way up.

Friction Demonstrations

Button side down!

Friction Demonstrations

Press the hands together gently. With a small reaction force the book can easily be pulled down as there is little friction.

Press the hands together as hard as possible. With a larges reaction force the book cannot easily be pulled down as there is a lot of friction.

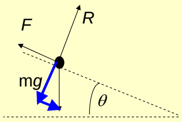
The Friction Law

$$F \leq \mu R$$

A Calculator on a Slope

What is the coefficient of friction between the calculator and the slope?

Solution



Parallel to the slope

$$F = mg\sin\theta$$

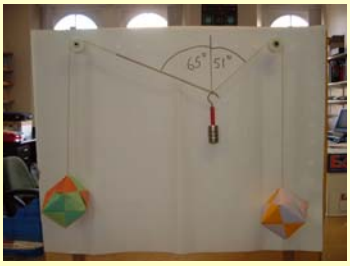
Perpendicular to the slope

$$R = mg\cos\theta$$

Use the friction inequality

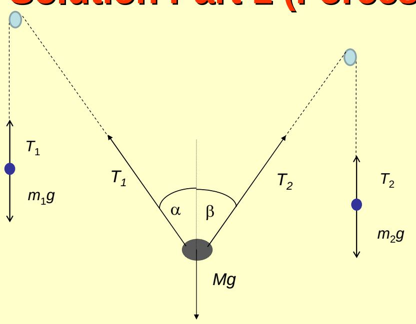
$$F \le \mu R$$
$$mg\sin\theta \le \mu mg\cos\theta$$
$$\mu \ge \frac{mg\sin\theta}{mg\cos\theta}$$
$$\mu \ge \tan\theta$$

Unknown Hanging Masses

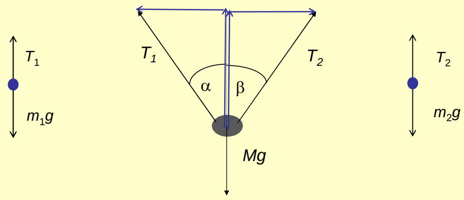


What are the unknown masses.

Solution Part 1 (Forces)



Solution Part 1 (Equations)



$$T_1 = m_1 g$$

$$T_1\cos\alpha + T_2\cos\beta = Mg$$

$$T_2 = m_2 g$$

$$T_1\sin\alpha = T_2\sin\beta$$

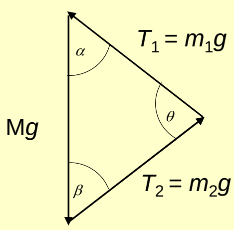
Solution Part 3 (Algebra)

$$T_1\sin\alpha = T_2\sin\beta$$
$$m_1g\sin\alpha = m_2g\sin\beta$$
$$m_1 = \frac{m_2\sin\beta}{\sin\alpha}$$

$$T_1\cos\alpha + T_2\cos\beta = Mg$$
$$m_1\cos\alpha + m_2\cos\beta = M$$
$$\frac{m_2\sin\beta}{\sin\alpha}\cos\alpha + m_2\cos\beta = M$$
$$m_2\left(\frac{\sin\beta}{\sin\alpha}\cos\alpha + \cos\beta\right) = M$$
$$m_2 = \frac{M}{\left(\frac{\sin\beta}{\sin\alpha}\cos\alpha + \cos\beta\right)}$$

$$m_1 = \frac{m_2\sin\beta}{\sin\alpha} = \frac{M\sin\beta}{\sin\beta\cos\alpha + \sin\alpha\cos\beta} = \frac{M\sin\beta}{\sin(\alpha+\beta)}$$

Alternative Solution



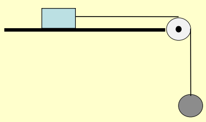
$$\theta = 180 - \alpha - \beta$$

$$\frac{m_1 g}{\sin\beta} = \frac{Mg}{\sin\theta}$$
$$m_1 = \frac{M\sin\beta}{\sin\theta}$$

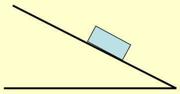
$$\frac{m_2 g}{\sin\alpha} = \frac{Mg}{\sin\theta}$$
$$m_2 = \frac{M\sin\alpha}{\sin\theta}$$

Coefficient and Angle of Friction

First determine the coefficient of friction.



Then predict the greatest angle for which the block will remain at rest on the slope.

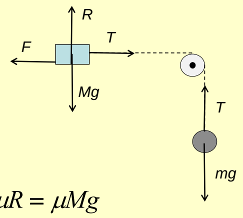


Test you prediction.

Coefficient and Angle of Friction

$T = F$

$R = Mg$



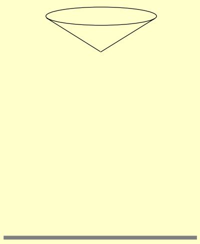
$T = mg$

$T = F$

$mg = \mu R = \mu Mg$

$\mu = \dfrac{m}{M}$

Air Resistance Problem

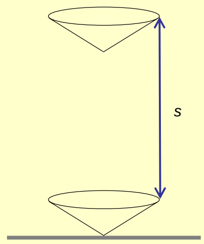


Find the time taken for the cone to fall from a reasonable height to the ground.

Compare with the time to fall under gravity alone.

Determine the average resistance force acting as the object falls

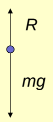
Air Resistance Problem



$$s = ut + \frac{1}{2}at^2$$

$$a = \frac{2s}{t^2}$$

Air Resistance Problem



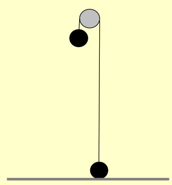
$$\text{Resultant Force} = mg - R$$

$$mg - R = ma$$

$$mg - R = \frac{2ms}{t^2}$$

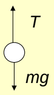
$$R = mg - \frac{2ms}{t^2}$$

Vertical Connected Particles

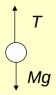


Set up the apparatus as shown in the diagram. Use a difference of 10 grams. Predict the time for the lower mass to rise to level of the pulley. Confirm with an experiment.

Vertical Connected Particles



$$T - mg = ma$$



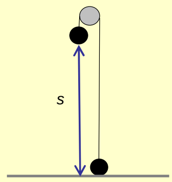
$$Mg - T = Ma$$

$$Mg - mg = Ma + ma$$

$$a = \frac{(M - m)g}{M + m}$$

Version 1.0

Vertical Connected Particles



$$a = \frac{(M - m)g}{M + m}$$

$$s = ut + \frac{1}{2}at^2$$

Version 1.0

Conical Pendulum Practical

Version 1.0

Conical Pendulum Practical

Set up a conical pendulum with the mass close to the floor.

Measure:
- Diameter of circle
- Height of suspension point
- Time for 10 circles

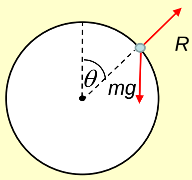
Calculate the angular speed in two different ways and compare.

Version 1.0

A Coin on a Cylinder

Version 1.0

A Coin on a Cylinder



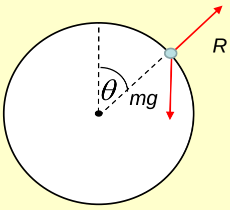
Assume $R = 0$

$$mg\cos\theta = m\frac{v^2}{r}$$

$$\cos\theta = \frac{v^2}{gr}$$

Version 1.0

A Coin on a Cylinder



$$mgr(1-\cos\theta)=\frac{1}{2}mv^2$$

$$v^2=2gr(1-\cos\theta)$$

$$\cos\theta=\frac{v^2}{gr}=2(1-\cos\theta)$$

$$\cos\theta=\frac{2}{3}\quad\Rightarrow\quad\theta=48°$$

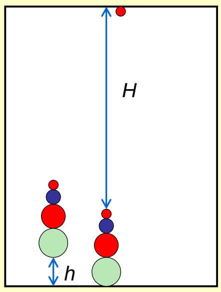
A Coin on a Cylinder

Start the coin part way round the cylinder as shown in the photo.

Make a prediction and test your result.

Energy Lost?

Energy Lost?



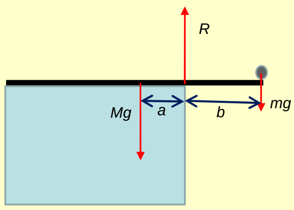
Drop the whole thing so that the small ball just reaches a particular height, for example the ceiling.

Find the energy lost during the bounce.

Determining the Mass of a Metre Rule

Determining the Mass of a Metre Rule

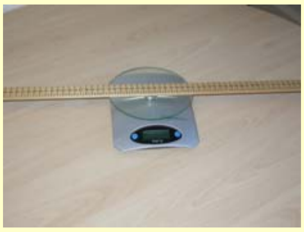
If on the point of toppling:



$$Mg\times a=mg\times b$$

$$M=\frac{mb}{a}$$

Determining the Mass of a Metre Rule



Checking the prediction.

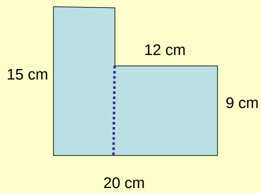
The Suspended Beam

Perpendicular Distances

Perpendicular Distances

Balance a Coke Can

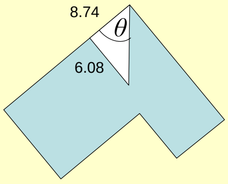
The Centre of Mass of a Lamina



$$\bar{x} = \frac{120\times4+108\times14}{120+108} = \frac{166}{19} = 8.74$$

$$\bar{y} = \frac{120\times7.5+108\times4.5}{120+108} = \frac{231}{38} = 6.08$$

The Centre of Mass of a Lamina



$$\tan\theta = \frac{6.08}{8.74}$$
$$\theta = 34.8°$$

Centre of Mass of a Lamina