

## M1 and M2: some common misconceptions and errors

It is important to recognise that in every mechanics unit in every session there are many very well-prepared candidates whose performance shows evidence of expert instruction and their own diligence (including making a real effort to do their best on the day). There are many very good examples to be seen of accurate analysis, efficient methods, clear diagrams, careful accurate algebra, lucid explanations etc.

There are some persistent areas of general weakness such as the production of diagrams too poor to be useful (which is found widely in both M1 and M2). One common area of weakness has been in the formulation of explanations and so it has been particularly pleasing to see so many good explanations from candidates in the recent M1 papers.

The following notes concentrate on the main weaknesses seen both in examination responses and in class and try to classify them broadly into common *misconceptions* and common *errors* (made despite a reasonably sound grasp of the concept). There are also some suggestions about approaches that can help to establish sound understanding and good technique.

It is sometimes difficult to tell whether a student has gone wrong because of a slip or because of a misconception. However, the misconceptions in the following list are fairly widely held and when seeing them in a student's work one should be suspicious of there being more wrong than a simple slip.

### Misconceptions

#### Vectors

Many students do not manipulate vectors with any confidence and do not have a clear concept of a vector equation.

*Use of column vectors instead of the  $a\mathbf{i} + b\mathbf{j}$  form can help.*

#### Kinematics

Taking the direction of the motion of a particle from its position vector  $\mathbf{r}$  instead of its velocity vector  $\mathbf{v}$ .

Not realizing that for a particle to be (instantaneously) at rest it is necessary for  $\mathbf{v} = \mathbf{0}$ .

Using constant acceleration formulae inappropriately and also using *distance = speed  $\times$  time* when the acceleration is not zero.

Confusion of  $x$ - $y$  and  $t$ - $y$  graphs so that the direction of motion of a particle is described in terms of the shape of its  $t$ - $y$  graph.

## Forces

False normal reactions are drawn (e.g. at the point of contact of a string with a block),

There are two common incorrect uses of friction results.

- Many students learn Coulomb's law as  $F = \mu R$  and replace all occurrences of  $F$  with  $\mu R$ . Having said that friction is limiting it is, of course, impossible to find its value if it is not.

*Students do best with  $F \leq \mu R$  if they can deal with the inequality; writing  $F_{\max} = \mu R$  does also seem to be effective.*

- Many students believe that in all circumstances of an object sliding down a slope inclined at  $\lambda$ , they may use  $\mu = \tan \lambda$ .

In M2, giving a wrong line of action to a normal reaction (see below).

## Dynamics

Not dealing properly with connected particles. In particular, not being able to find the force in a coupling and not being able to deal with systems involving pulleys where a connecting string has sections in different directions.

## Projectiles

Various wrong ideas often based on false assumptions of linearity; for instance, that half the maximum height is achieved in half the time taken to reach the full height.

## Things often not done well (but not necessarily because of a misconception)

### General

The use of clear complete diagrams for all mechanics topics (produced even if not instructed to draw them) so that positive directions (and an origin) are properly defined, thus reducing the occurrence of sign errors etc.

*The need for good diagrams will be emphasized in the notes below. Without them, strong students make unnecessary errors and weaker ones often sink in a mire of misunderstanding. In examinations, there seems to have been a recent increase in the number of candidates who, apparently, annotate diagrams given on the question paper, leaving the examiner without this essential key to the solution.*

Recognizing all the forces present and when their sense is determined by the situation (e.g. tension in strings, friction acting in a direction opposed to the motion).

Recognizing that the situation being modelled has changed (e.g. removal of one force may change the value of another).

Correct resolution in *sensible* directions, especially the ones given in a question.

Using vectors appropriately.

### **Interpretation**

Students are expected to be able to explain a result using a concise and sound mathematical argument with the correct use of mathematical language. They are also expected to understand the modelling process and be able to relate results obtained from the mathematical model to the situation being modelled. Students frequently reveal an almost total lack of understanding of the significance of their results. This is sometimes compounded by a lack of command of technical language so that terms like force, work, energy etc are often used as elegant variations in a piece of prose.

*Students should be encouraged to use diagrams and equations to aid their explanations.*

### **Algebra, trigonometry and calculus**

The algebraic demands of the mathematical models are not always negligible. Some scenarios require models where at least one of the quantities involved is a variable. Students whose simple algebra and trigonometry are not sound are likely to produce a confused mess very early in such questions and are not really ready for them.

*Students should be able to deal efficiently and accurately with simultaneous equations in two variables and with quadratic equations.*

### **Showing a stated result is true**

Many examination questions ask the students to establish a given result in order to ensure that they have the correct expression for further use. The argument must be made **very** clear involving statements such as *resolving vertically, applying Newton's second law down the plane* etc. Methods tend to be especially sketchy when students are asked to formulate an equation of motion.

### **Statics** (also see moments below)

Diagrams are often not clear enough to allow subsequent resolution, false reaction forces are introduced and new diagrams are not produced when the situation being modelled is changed.

Many students seem to be able to resolve forces in a given direction but not be able to establish an equation given that there is equilibrium. Many students cannot draw a polygon whose sides represent the forces acting in a given situation; in particular, the triangle of forces is not often considered as a method of solution of a three force problem.

## Dynamics

Generally, Newton's second law is applied accurately to single particles or bodies but connected particle problems reveal many misunderstandings. Many students are unable to separate the component parts when there are several particles present and then do not seem to be able to relate common sense to their analysis and are content to have one part of a system in equilibrium and a connected part accelerating.

In some situations, students take friction in a direction to oppose acceleration instead of motion. For example, suppose a sledge is being pulled along horizontal ground by a string and then the string breaks; the frictional force continues to oppose the forward motion even though the sledge is decelerating.

## Kinematics

Whether or not vector notion is being used, the constant acceleration formulae are applied even when the acceleration is manifestly not constant. When vector forms are required many students struggle with them.

When formulae are quoted there is frequent confusion with the current value of ' $u$ ', ' $t$ ' etc, so in a question which divides the motion into two parts the value taken for ' $u$ ' in the second part is likely to be that used in the first. The absence of a defined origin and of the direction being taken as positive often lead to confusion and sign errors.

*Students need techniques such as the introduction of question specific variables to help them avoid confusion with the notation.*

*Students should always draw diagrams to establish the origin and the positive direction.*

## Projectiles

Many problems seem to stem from students learning formulae for horizontal range, maximum height or even the trajectory and then applying them inappropriately. It is worth noting that examination questions are usually structured to suggest an efficient method and no assumption is made that the student has committed to memory, say, the general trajectory formulae.

Many students, when using the ' $uvas$ ' formulae, allow ' $u$ ', ' $t$ ', ' $v$ ' etc to stand for different measures in the same question; this invariably leads to confusion.

*As with general kinematics the students need techniques to help them avoid confusion with the notation. I would use  $x$  for the horizontal position,  $\dot{x}$  for the general horizontal component of velocity,  $\dot{x}_0$  for the initial horizontal component of velocity and similarly for the vertical direction in terms of  $y$ . Some teachers use  $x, v_x, u_x$  etc to achieve the same distinction.*

Many students do not recognize situations where the result  $v^2 = u^2 + 2as$  applied to the vertical motion gives an immediate answer to the question.

The absence of a diagram indicating the direction taken as positive frequently leads to sign errors, often followed by fudging. Many solutions have no indication of method, often making it impossible to tell which part of the motion is being considered.

### **Friction**

The incorrect uses of  $F = \mu R$  and  $\mu = \tan \lambda$  already mentioned as possible misconceptions.

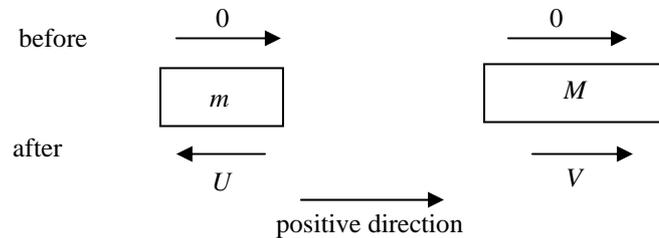
### **Impulse, linear momentum and impact**

Many students seem not properly to understand that impulse and momentum are vector quantities. Situations may involve more than one dimension, in which case they may be represented by directed line segments. However, even if the motion is in one dimension, clear diagrams are required to establish the directions involved and to establish a positive sense. In particular, students' calculations of impulse as  $m\mathbf{v} - m\mathbf{u}$  often involve sign errors.

Many students do not properly understand the application of conservation of linear momentum to collisions. In collisions, by N3L, the forces and hence the impulses in the collisions are of equal magnitude but opposite direction on the two bodies. When no external force acts on a system of particles or bodies that collide, the *total linear momentum of the system is conserved* as the forces involved in the collisions are internal and give zero net impulse. However, the linear momentum of each body involved in a collision *will change* as the force of the collision was external to that body.

Care must be used by students when applying PCLM when relative motion is involved. As an example, suppose that you are firmly seated on a sledge that is on smooth ice and initially at rest. You now throw a ball horizontally away from the sledge. The PCLM tells us that by the time you have the ball moving relative to you, the sledge and you must be moving in the opposite direction to conserve linear momentum; the speed of the ball relative to you will not be the speed of the ball relative to the ground. A particularly important application of this idea is in ballistics; if you fire a gun with a barrel inclined to the horizontal, the recoil will cause it to be moving backwards while the shell is still in it. The true angle of elevation of the shell as it leaves the barrel is *not* the angle of the barrel; this makes ballistics interesting!

*Example* A woman is sitting firmly on a sledge on smooth, horizontal ice. The sledge is at rest. The combined mass of the woman and the sledge is  $M + m$ . She throws a ball of mass  $m$  horizontally from the sledge with a speed of  $u$  relative to her. What is the speed of the sledge when the ball has been thrown?



PCLM gives  $\rightarrow MV - mU = 0$

We also have  $U + V = u$ ,

so  $MV - m(u - V) = 0$  and  $V = \frac{mu}{M + m}$ .

In collision problems, students seem to make fewest sign errors when they have a diagram with a positive sense defined, the sense of each known velocity marked, all unknowns given positive sense and Newton's

experimental law used in the form  $\frac{v_A - v_B}{u_A - u_B} = -e$ .

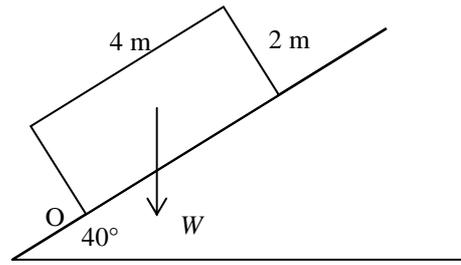
### Moments and static equilibrium

A clear diagram is again vital as missing out forces often completely undermines a solution. A common student error is to omit the forces acting at a pivot point (this could be a hinge or a point of contact with another body). This sometimes encourages students to suppose that a problem may be solved entirely by resolution; more commonly, the student may well start off correctly because moments were taken about the pivot (and the answer confirmed if given in the question) but then go wrong when moments are taken about another point or when resolution is used.

Some students incorrectly apply the analysis of pin-jointed light frameworks to heavy beams.

In some situations it is not easy to find the perpendicular distance from the axis or a point onto the line of action of the force. In such cases it may be helpful to express the force as the sum of two forces in more convenient directions and then find the total moment of those forces about the axis or point.

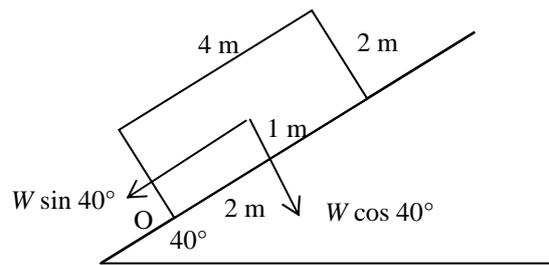
*Example* The situation shown in the diagram on the right is found in many problems. The weight  $W$  acts through the centre of a uniform rectangular body at rest on a slope at  $40^\circ$  to the horizontal. The dimensions of the rectangle are 4 m by 2 m. Calculate the moment of the weight about the point O. Note that, for clarity, no other forces have been shown.



The solution is not too hard if the perpendicular distance from O onto the line of action of  $W$  is attempted but the following is much easier.

Replace  $W$  with its resolved parts parallel to and perpendicular to the plane.

The clockwise moment of  $W$  about O is now clearly  $W \cos 40 \times 2 - W \sin 40 \times 1$ .



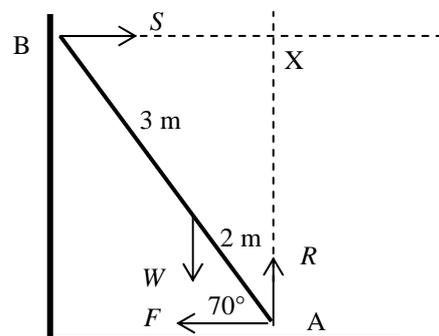
If a body is in equilibrium with several forces acting, the normal reaction will not necessarily have a line of action through the centre of mass and it should not be marked through this point in a diagram. The position of this line of action may be determined by taking moments.

Many students do not seem to remember that three forces acting on a body in equilibrium must be concurrent. This result also gives the possibility of some neat solutions to otherwise quite testing problems. The result is often used in conjunction with a triangle of forces.

The ability to choose the point about which moments are taken can lead to some neat solutions.

*Example* The diagram shows a ladder in equilibrium resting on a smooth vertical wall and on rough horizontal ground. Suppose we want to find the frictional force,  $F$ , acting on the foot of the ladder.

We could take moments about A and resolve horizontally and vertically but a neater method is available.



X is the intersection of the lines of action of  $R$  and  $S$ , neither of which we need to know to find  $F$ .

Taking clockwise moments about X gives  $F \times 5 \sin 70 - W \times 2 \cos 70 = 0$  and the result follows.

## Students' responses to M1 and M2 questions

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### General

- There are many very well-prepared candidates whose performance indicate
  - expert instruction
  - diligence (including making a real effort to do their best on the day).

### General

- There are many very good examples to be seen of
  - accurate analysis,
  - neat methods,
  - clear diagrams,
  - careful accurate algebra,
  - lucid explanations etc.

### General

- There are some persistent areas of general weakness such as the production of diagrams too poor to be useful (which is found widely in both M1 and M2).
- One common area of weakness has been in the formulation of explanations and so it has been particularly pleasing to see so many good explanations from candidates in the recent M1 papers.

### General

- Apart from technical problems with the content of the units, the work of students may suffer from
  - poor arithmetic and working to too few figures
  - the inability to carry out simple algebraic manipulation efficiently and accurately
  - poor notation
  - poor presentation
- There are levels of accuracy and presentation that are
  - enough for the student to work through a question
  - enough to communicate to others
  - enough to *show*

### General

- Sound concept formation depends on the student's experience. This experience may be enhanced by
  - experimentation
  - the use of simulations
  - practice in the correct use of technical (mathematical) language
  - awareness of the modelling cycle and the significance of results in terms of the real-world situation

## General

- Good diagrams are fundamental to problem solving in most mechanics topics – poor diagrams are commonly the sources of errors.
  - Kinematics (including projectiles) requires an origin and the direction taken to be positive
  - Linear momentum and impact require the direction taken to be positive
  - All forces need directions and labels in any topic

## Where are your principles?

- There can be a lack of an attempt to find out what **principle** might apply in a situation
  - formulae may be applied apparently randomly
  - numbers given in a question may be put into a calculator in systematic searches for a given answer
  - there may be little sense of progression
- Even with poor presentation, there is often indication of some knowledge of the principles
- **An attempt to apply a principle is the best strategy**

## Errors and misconceptions

- Classify weaknesses broadly into common
  - *misconceptions*
  - *errors* (made despite a reasonably sound grasp of the concept).
- Some misconceptions may be disguised as errors
- Particularly pernicious are the misconceptions that are false simplifications that take the place of proper analysis such as
  - the force in tow-bar under deceleration must be a thrust
  - for any body on a rough slope in any motion  $\mu = \tan \lambda$

## Some misconceptions: vectors

- Students often make mistakes with vector equations
  - equating vectors to scalars
  - not giving the correct signs

e.g. A mass of 20 kg is subject to two forces,  $\mathbf{F}$  N and  $12 \mathbf{j}$  N. Write down its equation of motion.

Students think of the  $12 \mathbf{j}$  N as a resistance and, using Newton's second law write

$$\mathbf{F} - 12 \mathbf{j} = 20 \mathbf{a} \text{ instead of } \mathbf{F} + 12 \mathbf{j} = 20 \mathbf{a}$$

This is particularly likely if the force is *called* a resistance

## Some misconceptions: vectors

- Using column vectors helps to avoid loss of vector form

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} - \begin{pmatrix} -2 \\ 5 \end{pmatrix} \text{ instead of } (x\mathbf{i} + y\mathbf{j}) = 2\mathbf{j} - (-2\mathbf{i} + 5\mathbf{j})$$

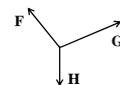
## Some misconceptions: vectors

- In statics applications, information about the direction of vectors can lead to confusion.

*Example*

Told that the forces  $\mathbf{F}$ ,  $\mathbf{G}$  and  $\mathbf{H}$  are in equilibrium, most students will write down  $\mathbf{F} + \mathbf{G} + \mathbf{H} = \mathbf{0}$ .

Given the diagram, most will write  $\mathbf{F} + \mathbf{G} = \mathbf{H}$ .



### Some misconceptions: kinematics

- Taking the direction of motion to be that of the position vector  $\mathbf{r}$  instead of the velocity vector  $\mathbf{v}$
- Not using  $\mathbf{v} = \mathbf{0}$  as the condition for a particle to be instantaneously at rest
- Using constant acceleration and even zero acceleration formulae inappropriately
- Confusing  $x$ - $y$  and  $t$ - $y$  curves

### Some misconceptions: forces

- It is not true that in all circumstances,
  - a friction force may be written as  $\mu R$
  - on a slope at  $\lambda$  to the horizontal, the coefficient of friction may be given by  $\mu = \tan \lambda$
- It is not true that the line of action of the normal reaction of a surface on a body must pass through the centre of mass of the body
- Newton's third law is not well understood
  - 'action' and 'reaction' wrongly both shown acting on a body
  - false normal reactions
  - wrong arguments in lift problems

### Some misconceptions: dynamics

- Wrongly using N2L as  $F - mg = ma$
- Not dealing properly with connected particles in situations such as
  - trucks in a train
  - connected particles not moving in the same line
- Belief that, under deceleration, the force in a coupling is always a thrust

### Some misconceptions: projectiles

- False assumptions of linearity
- Wrongly believing that when a particle is fired at a fixed speed from an origin and there are two trajectories passing through a given point, that the particle
  - is rising on one trajectory and falling on the other,
  - was projected at an angle less than  $45^\circ$  in one case and more than  $45^\circ$  in the other.

### Some common errors: general

- Poor diagrams leading to sign errors and inconsistency
- Failure to recognise all the (relevant) forces present and know when their sense is determined by the problem
- Failure to recognise that the situation being modelled has changed and so some forces already calculated have changed values
- Failure to resolve correctly in *sensible* directions
- Using vectors inappropriately

### Some common errors: statics

- Poor diagrams are at the root of many problems
  - There are forces missing
  - There are 'phantom' reaction forces
  - Fresh diagrams are not drawn when the model is changed
  - Labelling is poor – forces with same magnitude should have the same label; forces that may not have the same magnitude must have different labels
  - Confusion between the length of a string and the magnitude of the force in it.
  - Lack of familiarity with the use of triangles of forces.

### Some common errors: kinematics

- Poor interpretation of graphs
- Inappropriate use of constant (or zero) acceleration formulae
- Loss of vector forms
- Sign problems because of poor diagrams or no diagrams
- Confusion of the meaning of  $s$ ,  $u$ ,  $v$ ,  $a$  and  $t$  when used repeatedly in a problem
  - encourage students to use working variables

### Some common errors: dynamics

- Problems arising from missing forces and poor diagrams
- Poor techniques for dealing with connected particles
- Taking the direction of the friction force to oppose the acceleration instead of the direction of motion

#### *Example*

Suppose a sledge is being pulled along horizontal ground by a string and then the string breaks; the frictional force continues to oppose the forward motion even though the sledge is decelerating.

### Some common errors: projectiles

- Inappropriate use of standard formulae
- Confusion of the meaning of  $u$ ,  $v$ ,  $s$ ,  $a$  and  $t$  in the horizontal and vertical directions and in different parts of the solution
- Not recognising when it is not best to work via  $t$  as a parameter
- Absence of a diagram leading to sign errors
- Lack of indicated method leading to confusion about what part of the motion is being considered

### Some common errors: linear momentum

- Explanations involving the principle of conservation of linear momentum
- Problems involving relative motion
- Sign errors because of lack of diagrams to give positive sense. Impulse calculations and Newton's law of impact suffer often suffer. Students seem to do better with

$$\frac{v_A - v_B}{u_A - u_B} = -e \quad \text{than} \quad \frac{v_A - v_B}{u_B - u_A} = e$$

### Some common errors: moments

- Forces omitted (often at a pivot) so their moments are not included
- Poor choice of points about which to take moments
- Wrong line of action of a normal reaction
- Not recognising that three forces in equilibrium must be concurrent
- Not being able (or remember how) to find the moment of a force using its components.