


MEI Mathematics in Education and Industry
Innovators in Mathematics Education

Ideas for Teaching Moments

Charlie Stripp
Stephen Lee



MEI Mathematics in Education and Industry
Innovators in Mathematics Education

In summary

Fd

MEI Mathematics in Education and Industry
Innovators in Mathematics Education

Abstract

- This session explores ways to teach moments, including the use of contexts and simple experiments to bring the topic to life!

MEI Mathematics in Education and Industry
Innovators in Mathematics Education

Introduction

- Moments is a relatively straightforward topic
- It is an application which many students find somewhat easy, yet one with which a slip can easily be made in more complex questions
- Clear presentation, diagrams and statements of 'directions' are key

MEI Mathematics in Education and Industry
Innovators in Mathematics Education

Moments in the syllabus

- Moments is in Edexcel M1, all other boards M2 (and beyond)
- OCR (M2)

Equilibrium of a Rigid Body

Candidates should be able to:

- calculate the moment of a force about a point in two dimensional situations only (understanding of the vector nature of moments is not required);
- use the principle that, under the action of coplanar forces, a rigid body is in equilibrium if and only if (i) the vector sum of the forces is zero, and (ii) the sum of the moments of the forces about any point is zero;
- solve problems involving the equilibrium of a single rigid body under the action of coplanar forces, including those involving tipping or sliding (problems set will not involve complicated trigonometry).

MEI Mathematics in Education and Industry
Innovators in Mathematics Education

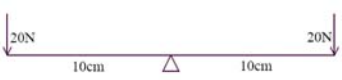
AQA (M2) (M4 bottom box)

<p>Moments and Centres of Mass</p> <p>Finding the moment of a force about a given point.</p> <p>Determining the forces acting on a rigid body when in equilibrium.</p>	<p>Knowledge that when a rigid body is in equilibrium, the resultant force and the resultant moment are both zero.</p> <p>This will include situations where all the forces are parallel, as on a horizontal beam or where the forces act in two dimensions, as on a ladder leaning against a wall.</p>
<p>Moments</p> <p>Couples</p> <p>Reduction of systems of coplanar forces.</p> <p>Conditions for sliding and tipping.</p>	<p>Rotations Moment of a Force = $r \times F$</p> <p>Understanding of the concept of a couple.</p> <p>Reduction to a single force, a single couple or to a couple and a force acting at a point. The line of action of a resultant force may be required.</p> <p>Determining how equilibrium will be broken in situations, such as a force applied to a solid on a horizontal surface or on an inclined plane with an increasing slope. Derivation of inequalities that must be satisfied for equilibrium.</p>


MEI Mathematics in Education and Industry
Innovators in Mathematics Education

Introduction to the ideas

Consider a light rod as shown. Is this system in equilibrium? What force is exerted by the fulcrum on the rod?



Now move the fulcrum as shown. Is this system in equilibrium? The forces are the same.

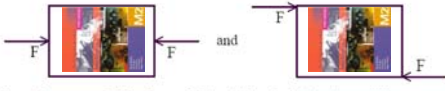


Clearly not: why not? The turning effect of the right-hand force is greater. What force would we need to apply to the left-hand end to balance it, and what would now be the force exerted by the fulcrum on the rod? Discuss.

MEI Mathematics in Education and Industry
Innovators in Mathematics Education

Introduction to the ideas

Our Mechanics 1 particle model is now inadequate, because we have to consider the effect of forces which are acting through different points. For example, for this tray, the particle model makes no distinction between



We need to use a **rigid body model**, in which a body has size and shape (which recognizes the effect of turning forces) but does not deform.

MEI Mathematics in Education and Industry
Innovators in Mathematics Education

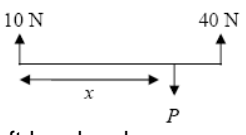
Introduction to the ideas (cont)

- The moment of the force, F , about an axis through A , perpendicular to the plane containing A and F , is Fd .
- Many problems can be solved by using a combination of resolving forces and taking moments.
- Always draw a diagram** – if you try to work without a diagram, you are very likely to make mistakes with signs, or to miss out forces.
- Remember to include reaction forces at a support or hinge in the force diagram.** These have no effect when you take moments about the support or hinge, but you need to take them into account when you resolve forces or take moments about a different point.

MEI Mathematics in Education and Industry
Innovators in Mathematics Education

Basic example

- What is the distance, x , required for a light horizontal rod, of length 1.5 m to be resting in equilibrium?

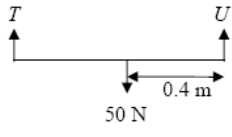


- Resolving vertically:
 $10 + 40 - P = 0,$
 $P = 50 \text{ N}$
- Taking moments about left-hand end:
 $40 \times 1.5 - Px = 0$
 $50x = 60$
 $x = 6/5 = 1.2 \text{ m}$

MEI Mathematics in Education and Industry
Innovators in Mathematics Education

Basic example

- What are the forces T , U required for a light horizontal rod, of length 1 m to be resting in equilibrium?



- Taking moments about right-hand end:
 $50 \times 0.4 - T \times 1 = 0$
 $T = 20 \text{ N}$
- Resolving vertically:
 $T + U - 50 = 0,$
 $U = 50 - 20$
 $= 30 \text{ N}$

MEI Mathematics in Education and Industry
Innovators in Mathematics Education

Alternatively... (or in addition)

Use lots of contexts!

MEI Mathematics in Education and Industry
Innovators in Mathematics Education

Contexts

See-saw

(Image restricted by copyright - search Google images for suitable image)

Standard

MEI Mathematics in Education and Industry
Innovators in Mathematics Education

Contexts

See-saw

(Image restricted by copyright - search Google images for suitable image)

Standard with one person either side

MEI Mathematics in Education and Industry
Innovators in Mathematics Education

Contexts

See-saw

(Image restricted by copyright - search Google images for suitable image)

Standard with one person either side and one in the 'middle'

MEI Mathematics in Education and Industry
Innovators in Mathematics Education

Contexts

See-saw

(Image restricted by copyright - search Google images for suitable image)

Standard with one heavy person and one light

MEI Mathematics in Education and Industry
Innovators in Mathematics Education

Contexts

See-saw

2 v 1

(Image restricted by copyright - search Google images for suitable image)

MEI Mathematics in Education and Industry
Innovators in Mathematics Education

Contexts

See-saw Bookcase!


Moments used in marketing!

(Image restricted by copyright - search Google images for suitable image)

MEI Mathematics in Education and Industry
Innovators in Mathematics Education

Video action

Doing the calculations 'instinctively'



MEI Mathematics in Education and Industry
Innovators in Mathematics Education

Contexts

Cranes




This was actually outside the conference building!

MEI Mathematics in Education and Industry
Innovators in Mathematics Education

Contexts

Cranes




MEI Mathematics in Education and Industry
Innovators in Mathematics Education

Contexts

(A much more complex example)


What is happening in this video clip?



MEI Mathematics in Education and Industry
Innovators in Mathematics Education

Investigation

Is Mr Bump safe under this construction?



Where is the mathematics? Can you recreate this, with rulers, books, bricks?

MEI Mathematics in Education and Industry
Innovators in Mathematics Education

Starter exercise

Here is a man painting a ledge

Discussion point:

Where is the mathematics?

(Image restricted by copyright - search Google images for suitable image)

Starter exercise

Comments may be made on how much paint is needed etc.

Should draw out the health and safety aspects of how the ladder is staying in place!

(Image restricted by copyright - search Google images for suitable image)

Concept

Couples

A couple is represented by two equal parallel forces not in the same line that are in opposite directions.

This system has a zero resultant force and a non-zero moment about any point.

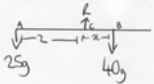
Any good examples?

Contexts



Example

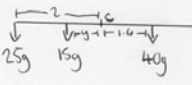
- Edexcel M1 Jan 2006 Qu 3

3 a)  EQUILIBRIUM.

Moments about point C: $25g(2) = 40g(x)$

$$\Rightarrow x = \frac{50g}{40g} = \frac{5}{4} = 1.25 \text{ m.}$$

b) If the beam has uniform weight then its mass/weight can be taken to act through the midpoint C (which was important for the previous question)

c)  Still in EQUILIBRIUM

Moments about point C:

$$40g(1) - 15g(1) - 25g(1) = 0$$

$$\Rightarrow 50g - 50g = 15g(y)$$

$$\frac{6g}{15g} = y = \frac{6}{15} = \frac{2}{5} \text{ m}$$

(or 0.4m)

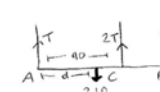
MEI Mathematics in Education and Industry
Innovators in Mathematics Education

Example

- Edexcel M1 June 2006 Qu 5

MEI Mathematics in Education and Industry
Innovators in Mathematics Education

Edx June 06 solution

5.  (w) Resulting vertically
 $T + 2T - 210 = 0$
 $3T = 210$
 $T = 70\text{ N}$

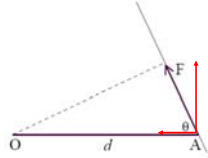
MEI Mathematics in Education and Industry
Innovators in Mathematics Education

Forces at an angle

Consider a rod OA, of length d , and a force acting at A at an angle θ with OA.

Moment of F about O is $F \times$ the perpendicular distance from O to the line of action of F
 $= F d \sin \theta$

$= d \times F \sin \theta$
 $= d \times$ component of F perpendicular to OA
 (the other component of F passes through O and has no moment about O)



MEI Mathematics in Education and Industry
Innovators in Mathematics Education

Interesting point of detail

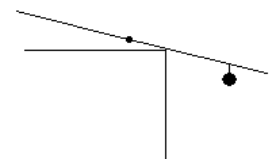
What is the correct force diagram for a ladder (sitting on rough ground) resting against:

- a smooth wall
- a window ledge
- a peg

MEI Mathematics in Education and Industry
Innovators in Mathematics Education

Investigation

Measuring the mass of a metre ruler



MEI Mathematics in Education and Industry
Innovators in Mathematics Education

More difficult example

- [Example](#) exam question that has 'forces at an angle'

MEI Mathematics in Education and Industry
Innovators in Mathematics Education

3 (i)

Taking moments about B $\Sigma +ve$

$$200 \times 0.6 - R_A \times 0.6 = 0$$

$$R_A = 150 \text{ N}$$

Resolving $\uparrow +ve$ $R_B + R_A - 200 = 0$

$$R_B = 200 - R_A = 50 \text{ N}$$

MEI Mathematics in Education and Industry
Innovators in Mathematics Education

(ii)

Taking moments about D $\Sigma +ve$

$$200 \times 1.2 - R_C \times 0.8 = 0$$

$$R_C = 300 \text{ N}$$

MEI Mathematics in Education and Industry
Innovators in Mathematics Education

3 (ii)

Taking moments about P $\Sigma +ve$

$$0.4 \times 200 \cos \alpha - 0.8 \times R_A = 0$$

$$R_A = 96 \text{ N}$$

Resolving perpendicular to plank $\Sigma +ve$

$$R_P - R_A - 200 \cos \alpha = 0$$

$$R_P = R_A + 200 \cos \alpha = 288 \text{ N}$$

Loss sin alpha but given

MEI Mathematics in Education and Industry
Innovators in Mathematics Education

Contexts

Rucksack - how should you pack it and carry it?

(Image restricted by copyright - search Google images for suitable image)

What would you discuss?

MEI Mathematics in Education and Industry
Innovators in Mathematics Education

Contexts

Exercise

Side plank exercise

(Image restricted by copyright - search Google images for suitable image)

MEI Mathematics in Education and Industry
Innovators in Mathematics Education

Experiments/investigations

- Mechanics lends itself well to undertaking practical and computer experiments/investigations and contextual examples
- Take the opportunity wherever possible, e.g. those cited earlier in the talk, those cited in earlier sessions

Key Points

- Equilibrium is a key idea
- Resolving and taking moments are standard practice
- Be more careful when considering 'forces at an angle'