MEI
Mathematics in Education and Industry
Innovators in Mathematics Education

MEI Conference 2010

A Journey of Discovery

Roger Porkess

Some lessons from a Journey of Discovery

Mathematics

- **Mathematics is all around us.**
- **In many cultures, mathematics is seen as one of the good things in life.**
- **However, in the UK those who enjoy mathematics can find it hard to share their enthusiasm with other people.**

Students

- **Never underestimate the fear of failure and the value of success.**
- **Never write anybody off.**
- **Be very careful about accelerating students, particularly when it involves placing them out of their age group.**

Curriculum

- **Mathematical modelling, at all levels, is an important life skill.**
- **School mathematics should never be too far from its applications.**
- **It is important for students to want to know the answer, at least some of the time.**
- **Modular courses give more people access to learning.**
- **Good design, incorporating connections and avoiding undue fragmentation, is critical for modular courses.**

Teachers

- **Teachers need a say in what they are required to teach during their professional lives.**
- **Regular CPD is essential if teachers are to fulfil their classroom potential.**

Regulation

- **We have become seriously over-regulated.**
- **Regulation has nothing to do with good learning or teaching.**
- **Mathematics should not be subject to the same rules as other subjects.**
- **Emphasising standards on their own is a recipe for disaster; they must be considered in conjunction with fitness-for-purpose and national needs.**

Roger Porkess
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Question 1 **(A journey of discovery)**

Given that $\cos\theta$ is rational, prove that θ is 0^o or a multiple of either 60^o or 90^o

Question 2 **(Fostering ideas)**

What is the simplest proof that the square root of 2 is irrational ?

Question 3 **(Not in the school syllabus)**

A hunter left his tent.

He walked 5 miles due South and came across the tracks of a bear.

He followed the tracks for 5 miles due East when he came up to the bear and killed it.

He then had 5 miles to walk back to his tent.

What colour was the bear ?

Question 4 **(For discussion)**

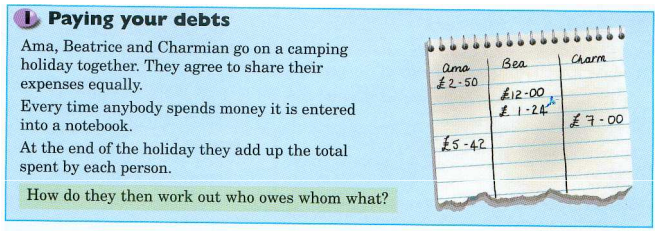
The opening line of the Cambridge course in probability and statistics in 1962 (when these topics were not taught in school)

"Consider the set of n random variables $X_1(x)$, $X_2(x)$, … $X_n(x)$ with probabilities $p_1(x)$, $p_2(x)$, … $p_n(x)$."

Is statistics well taught in universities today ?

Why is this question particularly important ?

Question 5 (Formula One Maths C3)



1 Paying your debts

Ama, Beatrice and Charmian go on a camping holiday together. They agree to share their expenses equally.

Every time anybody spends money it is entered into a notebook.

At the end of the holiday they add up the total spent by each person.

How do they then work out who owes whom what?

Question 6 (Cosmology)

Hubble' Law "Other galaxies are receding from our own at a speed proportional to their distance from it."

Prove that this does not imply that we are at the centre of the universe.

Question 7 (SMP)

As the sun was setting in a clear African sky, it was noticed in a Super VC10 flying north that the outline of the westward windows was projected on the other side of the cabin about 6 inches above the windows on that side.

Estimate roughly the height of the aircraft.

Question 8 (An unsolved problem)

A perfect number is equal to the sum of its factors (including 1 but excluding itself). The first perfect number is 6: 1+2+3=6.

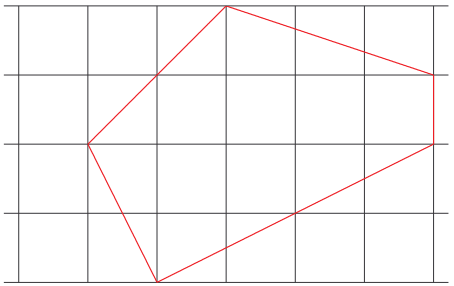
Prove that there is no odd perfect number.

Question 9 (MEI Comprehension paper, January 2003)

Why do planets retrograde ?

Question 10 (Pick's Theorem)

Each vertex of a polygon is a point on a 1 by 1 grid. Find a formula for its area.



Question 11 (Tribute to an OU student)

Prove that Pascal's triangle gives the binomial coefficients

Question 12 (Thinking mathematically)

Which is the greater, e^{π} or π^{e} ?

Question 13 (Mechanics 1, Jan 1992 & textbook)

A bird leaves its nest for a short horizontal flight along a straight line and then returns. Michelle models its distance, s metres, from the nest at time t seconds by

$$s = 25t - \tfrac{5}{2}t^2, \quad 0 \leqslant t \leqslant 10.$$

(i) Find the value of s when $t = 2$.
(ii) Explain the restriction $0 \leqslant t \leqslant 10$.
(iii) Find the velocity of the bird at time t seconds.
(iv) What is the greatest distance of the bird from the nest?
(v) Michelle's teacher tells her that a better model would be

$$s = 10t^2 - 2t^3 + \tfrac{1}{10}t^4.$$

Show that the two models agree about the time of the journey and the greatest distance travelled. Compare their predictions about velocity and suggest why the teacher's model is better.

Question 14 (Constrained trivialisation)

Find a number which is

- **an integer**
- **a perfect cube**
- **positive**
- **a perfect square**
- **less than 100**
- **odd**

Question 15 (Mechanics 1 textbook)

1 The picture shows a boy, Halley, holding onto a post while his two older sisters, Sheuli and Veronica, try to pull him away.
Taking **i** and **j** to be unit vectors in perpendicular horizontal directions the forces, in newtons, exerted by the two girls are:

Sheuli	$24\mathbf{i} + 18\mathbf{j}$
Veronica	$25\mathbf{i} + 60\mathbf{j}$

(i) Calculate the magnitude and direction of the force of each of the girls.
(ii) Use a scale drawing to estimate the magnitude and direction of the resultant of the forces exerted by the two girls.
(iii) Write the resultant in terms of **i** and **j** and so calculate (to 3 significant figures) its magnitude and direction.

Check that your answers agree with those obtained by scale drawing in part (ii).

Question 16 (MEI Comprehension paper, June 2004)

What is the equation of the shape of an egg ?

The sculptor, Sam Plagerson, may be contacted on sam.plagerson@network.rca.ac.uk

Question 17 (Specimen Pure 1 paper for Curriculum 2000, reproduced in "The curious incident of the dog in the night-time"

8 Prove the following result.

"A triangle with sides that can be written in the form $n^2 + 1$, $n^2 - 1$ and $2n$ (where $n > 1$) is right-angled."

Show, by means of a counter example, that the converse is false.

Question 18 (Inspiration at the bar)

When a can of lager is full, its centre of mass is half way up. When it is empty, the centre of mass is also half way up. When some of the beer has been drunk, the centre of mass lies below the half way point.

What is the lowest position of the centre of mass ?

Question 19 (MEI staff photograph)

A group of *n* people is selected at random.

What is the smallest value of *n* such that the probability that at least 2 of them share a birthday is greater than 0.5 ?

Question 20 (Minister fails maths !)

At the 2010 ACME conference, Diana Johnson, a minister in the DCSF said

"With independent predictions suggesting that there will be 3 million science, maths and technology related jobs worldwide by 2017, it is integral that our young people have a solid understanding of mathematics when they leave education."

What did she mean ?

Question 21 (The logistic equation)

Investigate the iteration

$x_{n+1} = kx_n(1 - x_n)$

for different values of k and x_0.