MEI

Innovators in Mathematics Education

Mathematics in Education and Industry

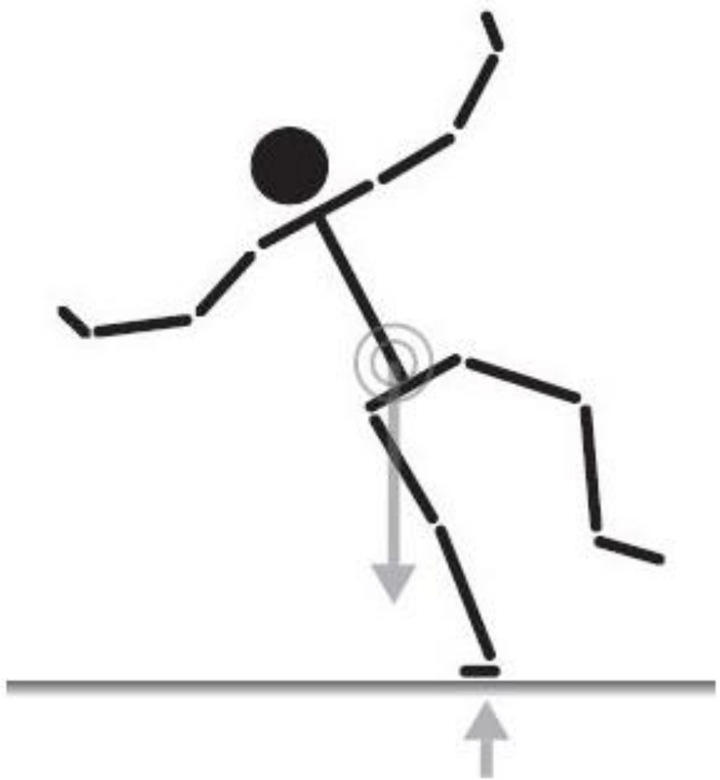
50 years at the forefront of Mathematics Education

Teaching Moments

MEI Conference 2013
Sharon Tripconey

A judo throw

- Why do judo throws work?
- How can you throw someone with a relatively small force?
- Can you explain?



In judo competitors are constantly trying to make their opponents lose their balance in subtle ways.

Every day experiences

- Turning a door knob or handle
- Pushing a door open
- Turning a tap on and off
- Turing the steering wheel on a car

The effects of forces

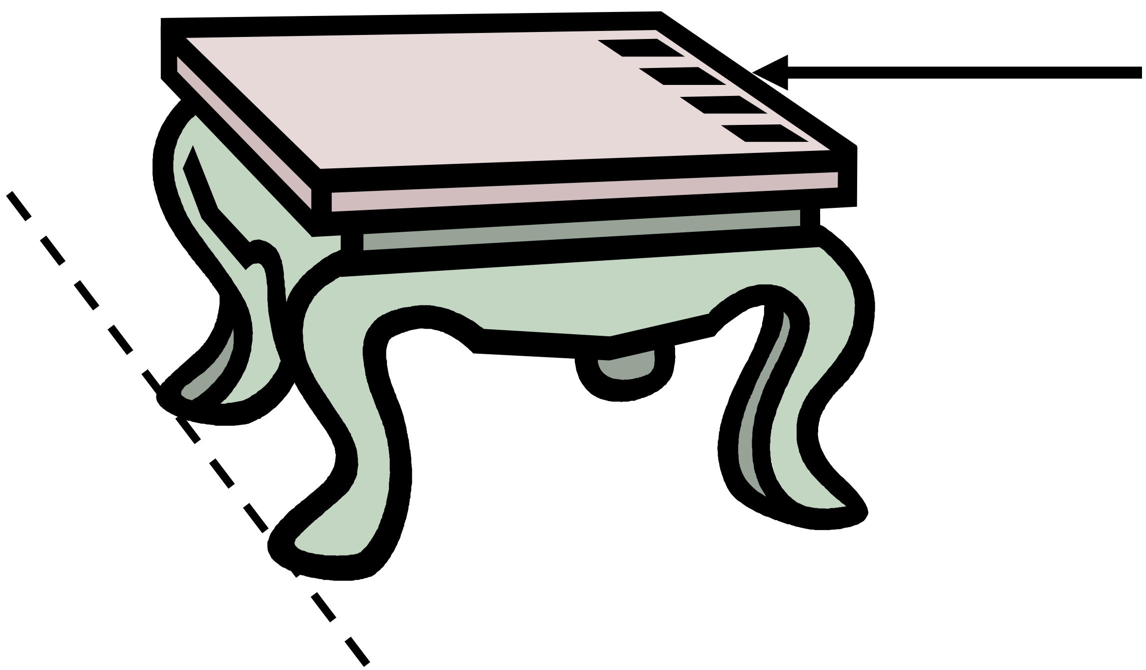
When a system of forces acts on a particle, the particle may be in one of the following states

- static equilibrium
- constant velocity
- accelerating in the direction of the resultant force

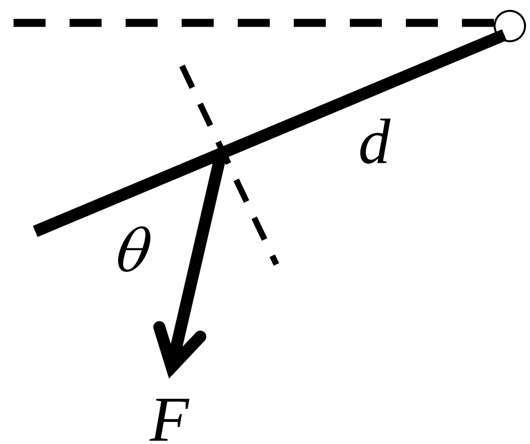
The effects of forces

- There are situations where bodies may be successfully modelled by single particles
  - a book at rest on a slope
  - a book sliding down a slope
  - a car and trailer accelerating along a straight road
- We know that forces applied to bodies may have different effects to those above. Knowing the magnitude and direction of a force is not enough as different lines of action for the force may cause turning effects.

The table could slide or topple

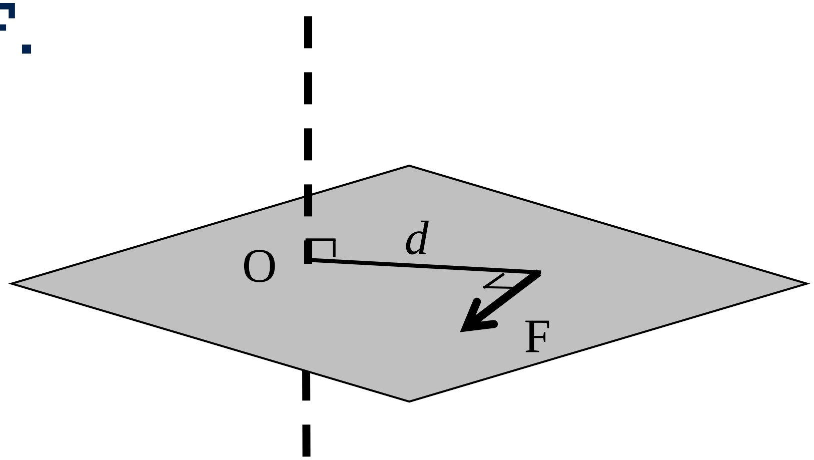


Maximising the turning effect of a force



Moment

The moment of a force F about an axis through O perpendicular to the plane containing O and the line of action of F is Fd, where d is the perpendicular distance from O to the line of action of F.



Moment

- Moment has *sense,* usually described as clockwise or anti-clockwise and is signed positive or negative according to the convention adopted for that problem.
- The SI unit of moment is the N m.

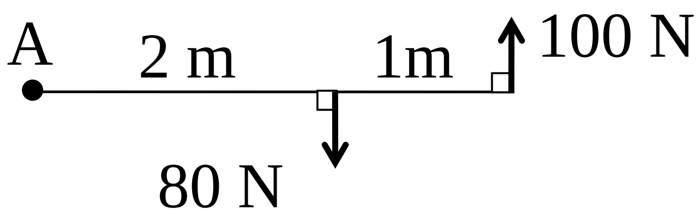
Combining moments

- The total moment of a set of coplanar forces about an axis perpendicular to the plane is the sum of the signed moments of the forces about the axis.

Combining moments

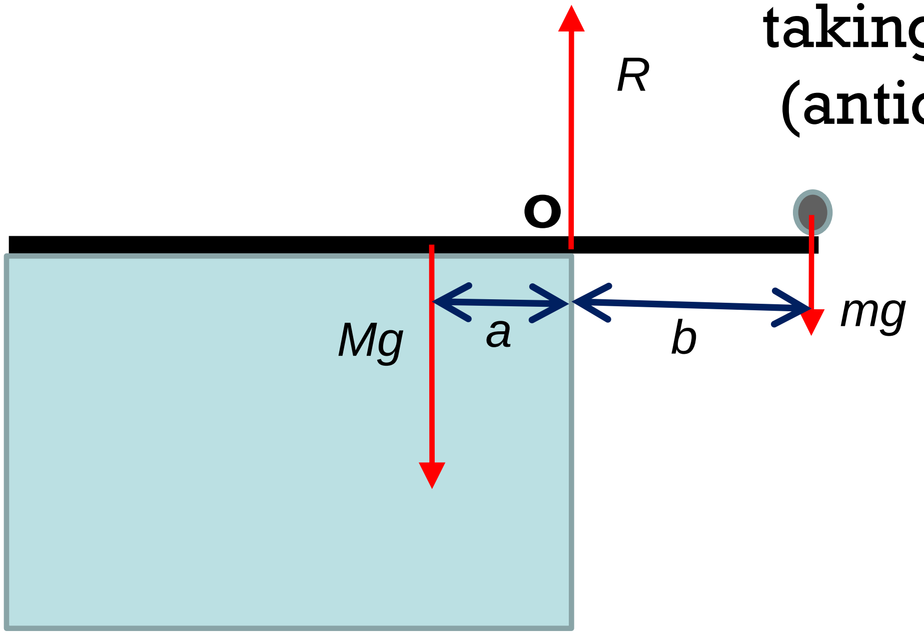
Example

In the following planar system, find the anti-clockwise moment of the two forces about an axis through A perpendicular to the plane.



We have $3 \times 100 - 2 \times 80 = 140 \text{ N m}$

Determining the Mass of a Metre Rule

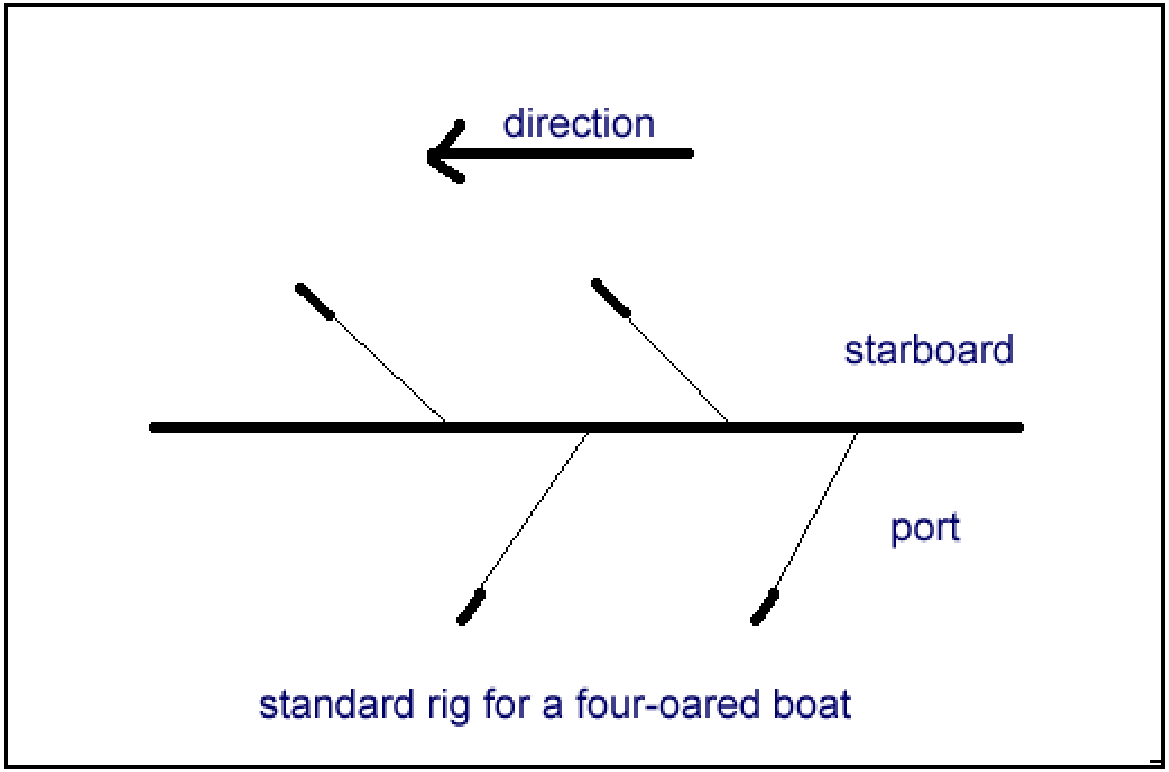


If on the point of toppling, taking moments about O (anticlockwise +ve)

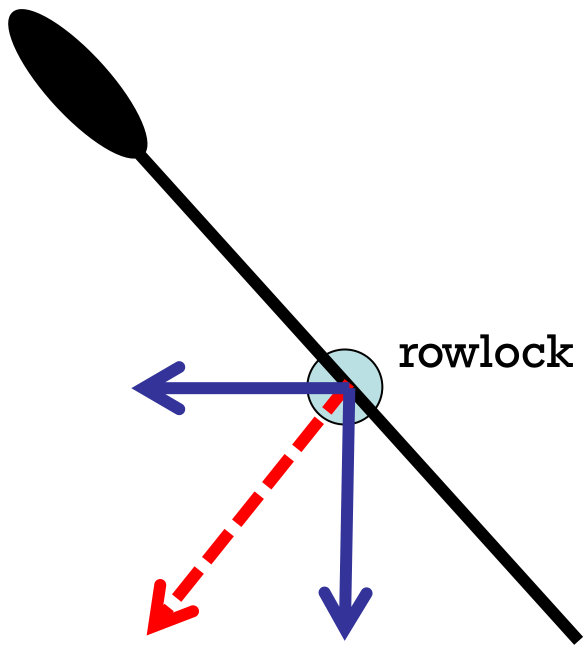
$$Mg \times a - mg \times b = 0$$

$$M = \frac{mb}{a}$$

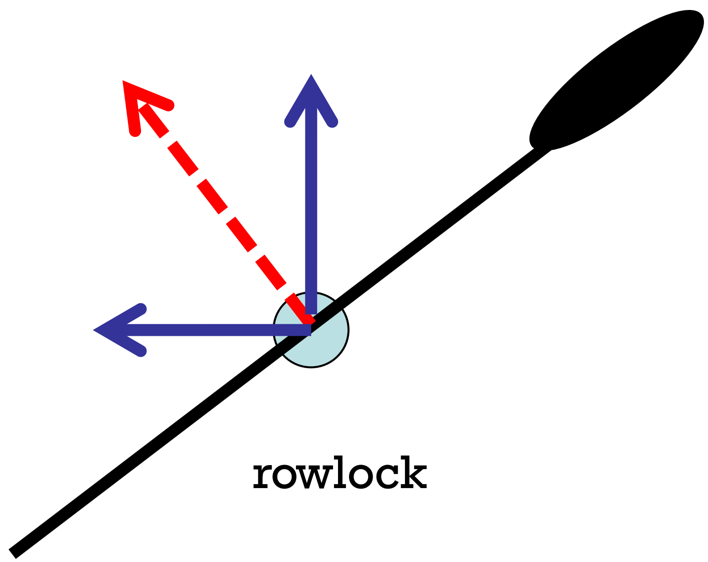
Rowing has its Moments!



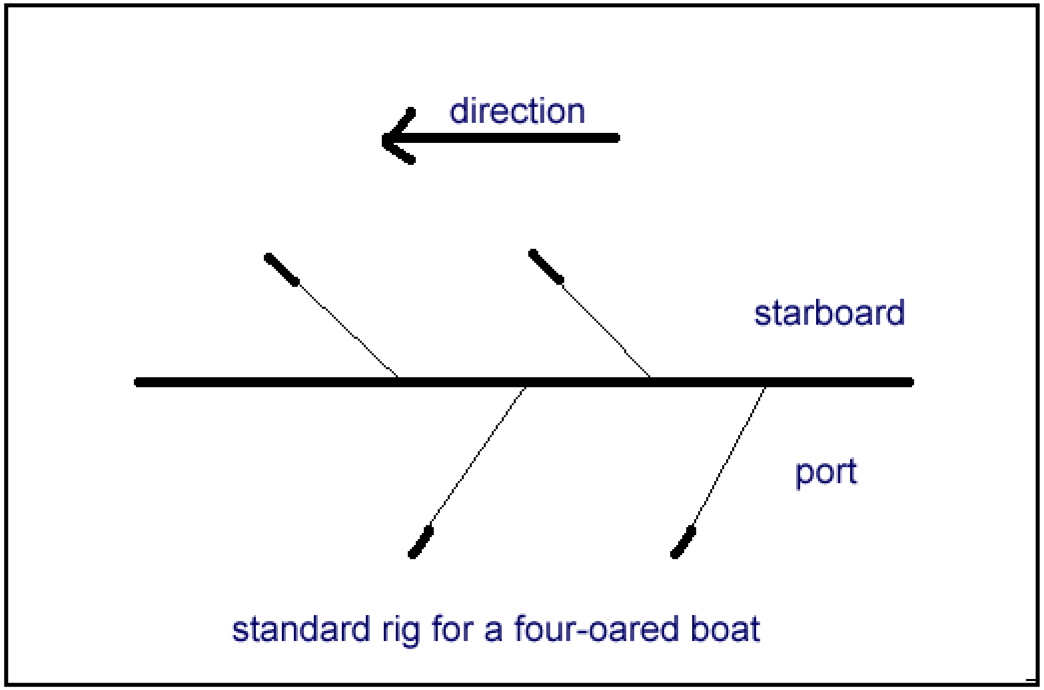
Alternating arrangement of rowers L R L R (rig)



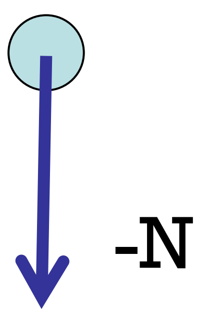
As the rower pulls on the oar (toward them) in the first half of the stroke there is a force on the boat which can be split into two perpendicular components, one in the direction of motion and one directed towards the boat



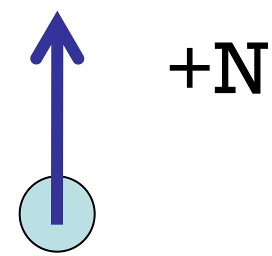
In the second half of the stroke (recovery phase) the force can also be split into two perpendicular components, one in the direction of motion and one directed away from the boat



Component of the force perpendicular to the boat



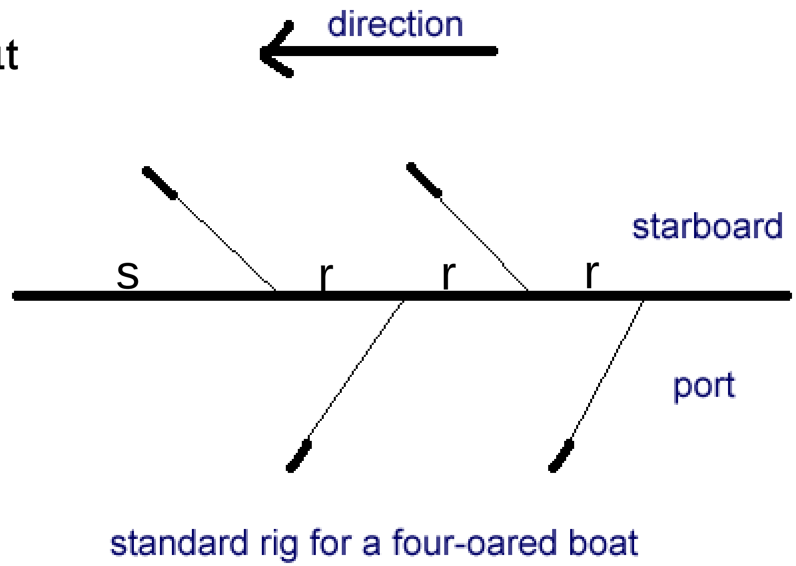
and



(recovery phase)

The Wiggling Boat

Taking moments about the front of the boat (anticlockwise +ve)

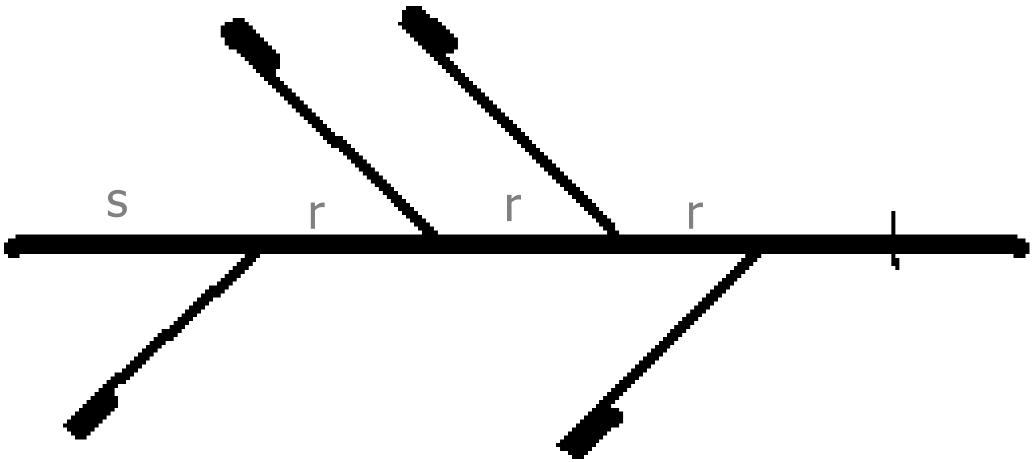


Moment on boat = -Ns + N(s+r) - N(s+2r) + N(s+3r) = +2Nr

Then, half a stroke later…N reverses to –N and..

Moment on boat = -2Nr

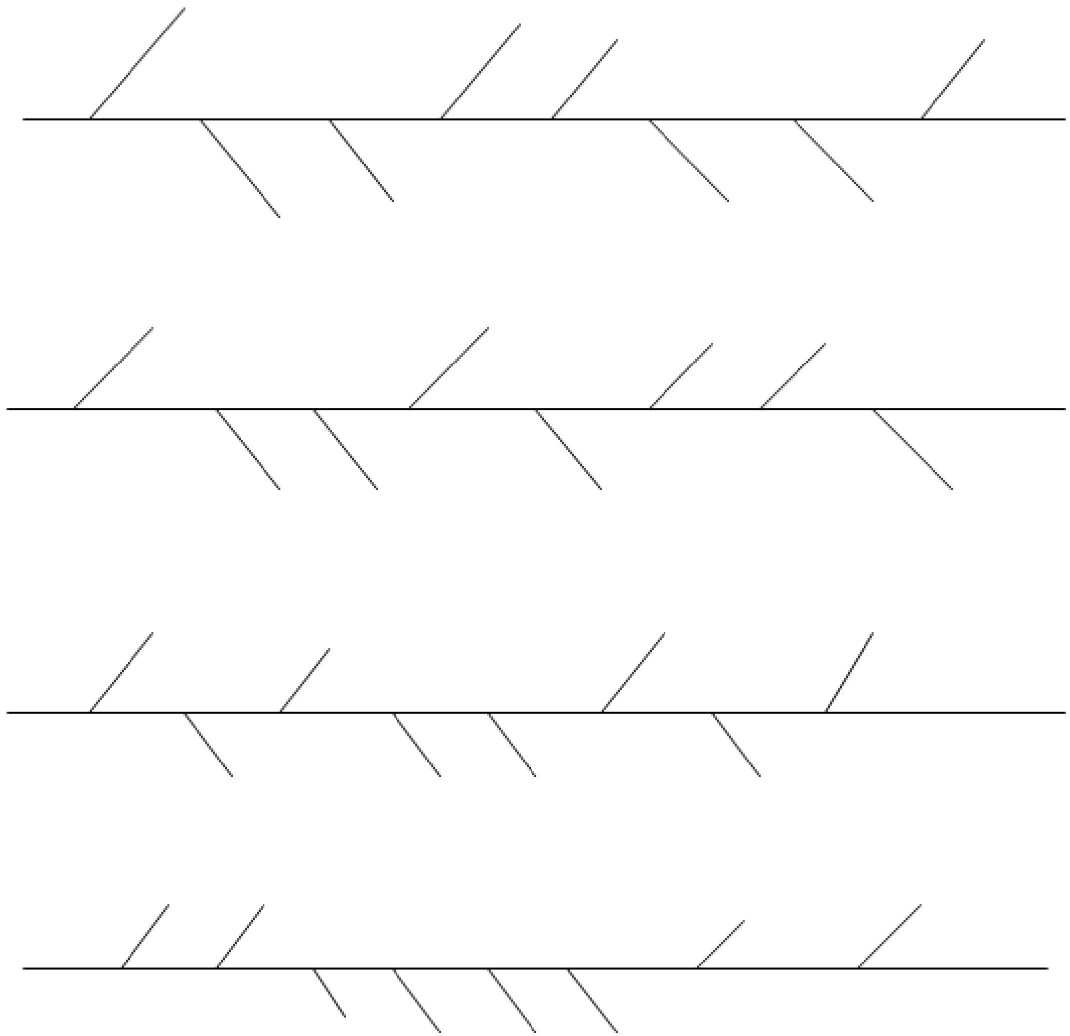
The Italian Rig



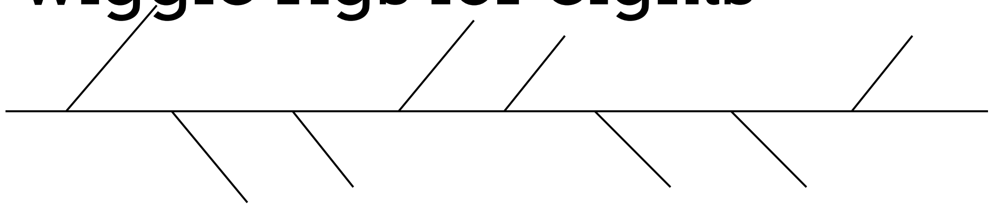
Moment = Ns - N(s+r) - N(s+2r) + N(s+3r) = 0

No wiggle!

Try these eights…

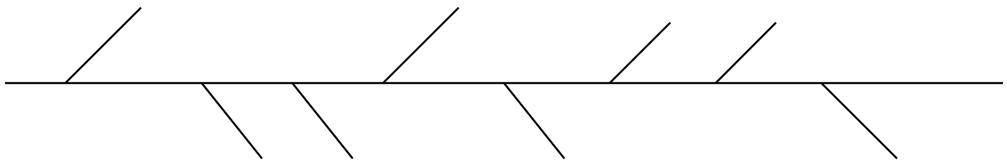


Four no-wiggle rigs for eights



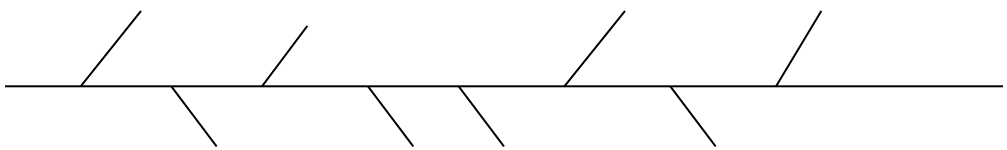
'Italian tandem Rig'

Moment = 1-2-3+4+5-6-7+8 = 0



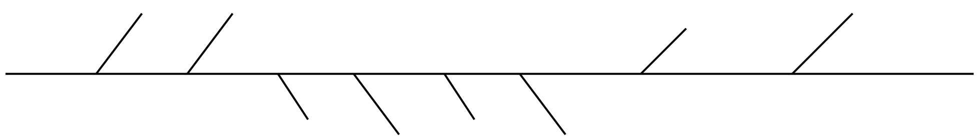
New*

Moment = 1-2-3+4-5+6+7-8 = 0



'German Rig'

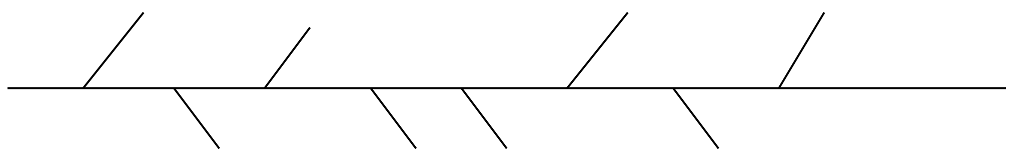
Moment = 1-2+3-4-5+6-7+8 = 0



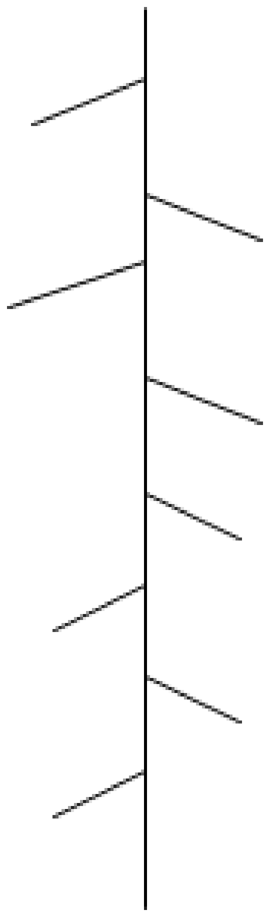
New*

Moment = 1+2-3-4-5-6+7+8 = 0

Canada used the German rig to win 2008 Olympics



1-2+3-4-5+6-7+8 = 0



Oxford used a German Rig winning the 2011 Boat Race

And Cambridge in 2012

And also used by Germany to win at the London Olympics

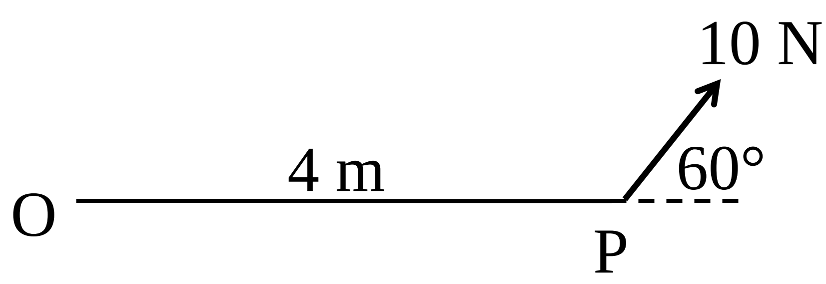
1-2+3-4-5+6-7+8 = 0

More detail in Prof John Barrow’s book, Mathletics published on 20th June 2013 .

Calculating moments

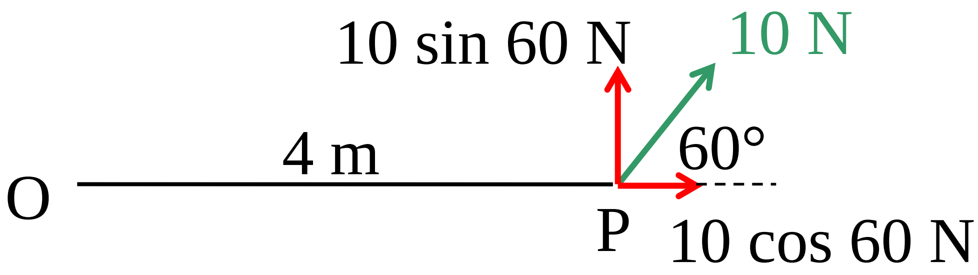
- Finding the perpendicular distance onto the line of action of a force is not always easy. The following approach is often efficient and effective.

Suppose that the force of 10 N acts through P which is 4 m from O and that the force is at an angle of 60° with OP. We want the moment of the force about an axis through O perpendicular to the plane containing the force and OP



Calculating moments

Replace the 10 N force with components parallel to and perpendicular to OP.



The anti clockwise moment about an axis perpendicular to the plane through O is now seen to be $(10\times\sin 60)\times 4+(10\cos 60)\times 0$

so $20\sqrt{3}$ N m

Modelling assumptions

- The body is rigid – it is not deformed by the forces that act on it
- The axis is fixed
- The body is free to rotate about the axis – for instance, there is no frictional or other force impeding rotation about a hinge
- The moment of the whole weight may be found by taking its line of action to be through the centre of mass

Simplifications

For M1 units, the following simplifications are made

- Only static equilibrium situations are considered
- The moments are easy to calculate, mostly with forces that are parallel
- We do not usually mention that the moment is about an axis perpendicular to the plane through a point, say A, we just say the moment about A.

Conditions for static equilibrium

- For the equilibrium of a body it is necessary and sufficient that the resultant of all the external forces is zero and that the total moment of these forces is zero about any axis.
- In the case of coplanar forces it is sufficient for the equilibrium of a body that the resultant of all the external forces is zero and that any point can be found about which these forces have zero moment.

Solving problems

- To meet the conditions for equilibrium we have to resolve in two directions (we choose) to establish a zero resultant force and take moments about one point (we choose) to establish that the forces have zero moment about this point.
- As you might suppose, a wise choice of the directions and point can make the solution easier.
- Not every problem requires two resolutions and moments taken

Solving problems

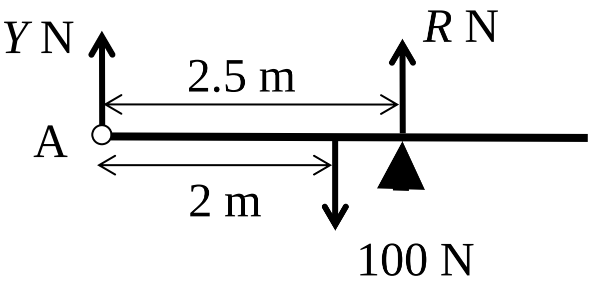
- It is sometimes more efficient to take moments about more than one point instead of doing one of the resolutions

A beam hinged at one end and with a support

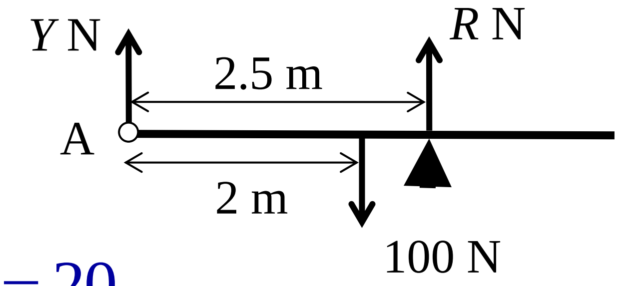
- A uniform heavy beam of weight 100 N and length 4 m is freely hinged at one end A and held in horizontal equilibrium by a smooth support 2.5 m from A. Find the normal reaction of the support on the beam.
- First we need a diagram on which all the forces on the beam are shown.
- Taking anti-clockwise moments about A gives

  $2.5R - 100 \times 2 = 0 \text{ so } R = 80$

  and the reaction is 80 N.



A beam hinged at one end and with a support



- Suppose we want to know the vertical reaction at the hinge
- Either

  Resolve vertically giving $Y + 80 - 100 = 0$ so $Y = 20$

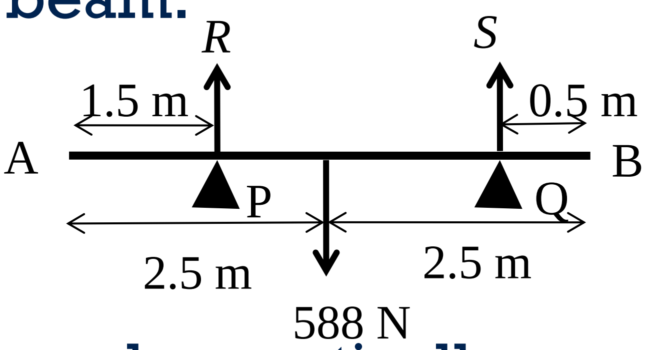
  and the reaction is 20 N
- Or

  Take clockwise moments about the support to give

  $2.5Y - 100 \times 0.5 = 0$ so $Y = 20$ and the reaction is 20 N

A beam with two supports

A uniform heavy beam, AB, of mass 60 kg and length 5 m is held in horizontal equilibrium by smooth supports P and Q. P is 1.5 m from A and Q is 0.5 m from B. Find the normal reactions of the supports on the beam.



- As before we need a diagram.
- Taking c.w. moments about Q

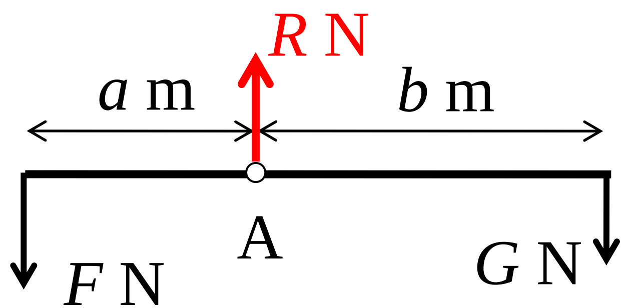
  $3R - 2 \times 588 = 0$ so $R = 392$
- Now either take moments about P or resolve vertically

  Resolving vertically $392 + S - 588 = 0$ so $S = 196$

  The reactions at P and Q are 392 N and 196 N respectively

A light beam hinged not at one end

- The beam is light, freely hinged at A and in horizontal equilibrium with two applied forces as shown on the diagram.



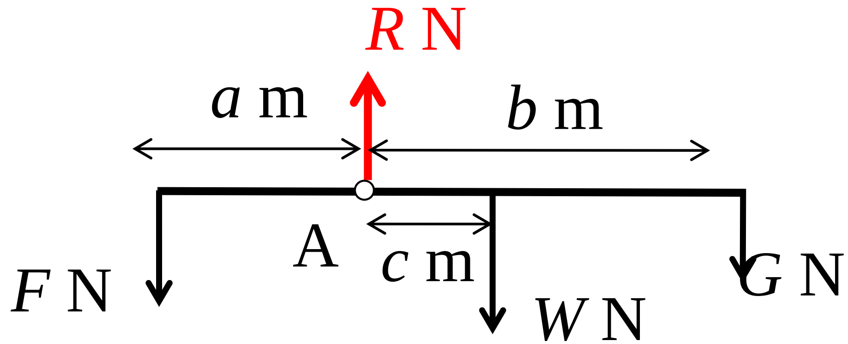
- Show all the forces present so mark in the reaction at the hinge
- Taking anti-clockwise moments about A gives

$$Fa - Gb = 0 \text{ so } Fa = Gb$$

- Given any three of the variables the fourth may be found.

A heavy beam hinged not at one end

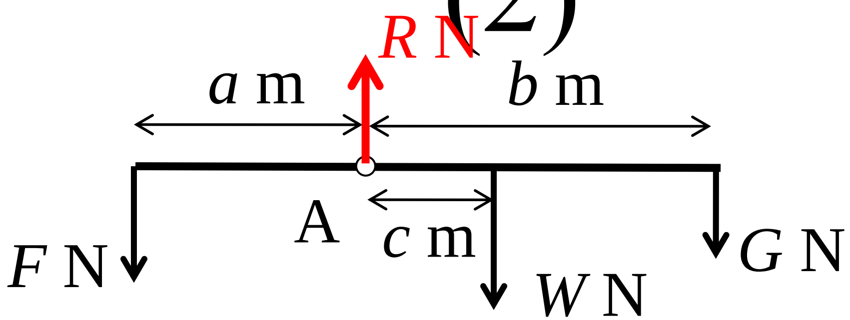
- The beam has weight *W* N, is freely hinged at A and in horizontal equilibrium with two other applied forces, as shown on the diagram. Note that the beam is not necessarily uniform.



- Show all the forces present so mark in the reaction at the hinge
- Taking a.c. moments about A and solving for *G* gives

$$Fa - Wc - Gb = 0 \text{ so } G = \frac{Fa - Wc}{b}$$

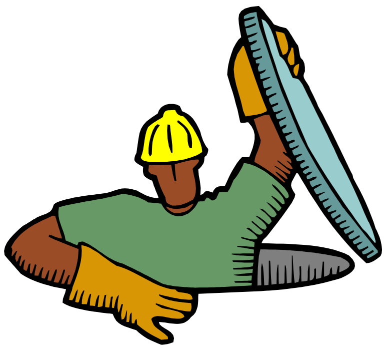
A heavy beam hinged not at one end (2)

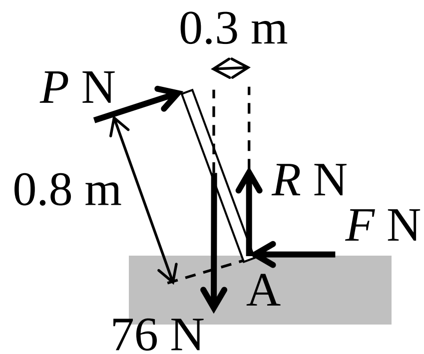


$$G = \frac{Fa - Wc}{b}$$

- We see that *G* may be + ve or – ve (i.e act upwards)
- If $F = 10$, $W = 10$, $a = 2$, $b = 4$ and $c = 1$ then $G = 2.5$
- If $F = 10$, $W = 40$, $a = 2$, $b = 4$ and $c = 1$ then $G = -5$

The man-hole cover





- Suppose the dimension and forces are as shown in the diagram with the man-hole of weight 76 N in equilibrium and *P* perpendicular to the cover.
- Taking c.w moments about A gives
  $P \times 0.8 - 76 \times 0.3 = 0$ so $P = 28.5$
- Note that we could now find *R* and *F*.