MEI Conference 2013

Using GeoGebra in A level Core

www.mei.org.uk/?page=icttasks#geogebra

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Generating a dynamic line in the form $y - y_1 = m(x - x_1)$

Adding a point (x_1, y_1), a slider, m, for the gradient and the line

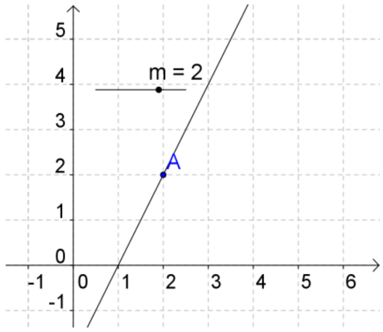
1 Add a **New Point**, A, (2nd menu).

2 In the input bar type **x_1=x(A)** and press enter.

3 In the input bar type **y_1=y(A)** and press enter.

4 Add a slider (11th menu) and name it **m**.

5 In the input bar type **y-y_1=m*(x-x_1)** and press enter.

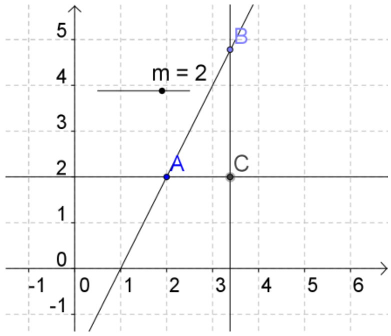


Adding a general point and creating the third point in a gradient triangle

6 Add a **New Point**, B, (2nd menu) on the line (the cursor should change as you hover over the line).

7 Use **Perpendicular Line** (4th menu) to construct perpendiculars to the *y*-axis through A and the *x*-axis through B.

8 Use **Intersect Two Objects** (2nd menu) to find the intersection of the perpendicular lines, C.



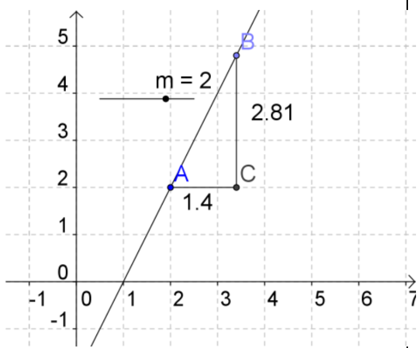
Creating the gradient triangle

9 Hide the perpendicular lines: click on the blue circles next to the lines in the Algebra pane.

10 Use **Segment between Two Points** to add segments AC and BC.

11 Right-click on each of the segments and select **Object Properties** then enable Label > Value.

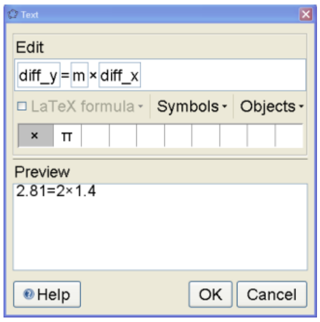
12 Right-click on the segments and rename them as **diff_x** and **diff_y**.



Adding the dynamic text

13 Use **Insert Text** (10th menu) to add a text-box.
Enter **diff_y=m×diff_x**.
diff_x, **diff_y** and **m** should be selected from Objects.
× can be found in Symbols.

14 Use **Insert Text** (10th menu) to add a text-box.
Enter **y-y_1=m(x-x_1)**.
x_1, **y_1** and **m** should be selected from Objects.
Enable the LaTeX formula.



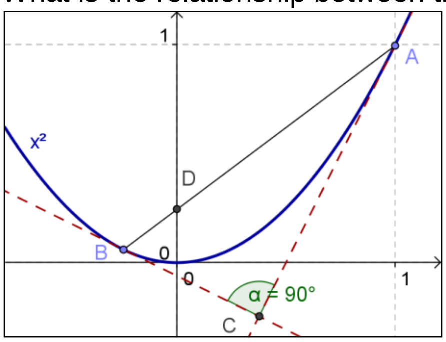
ICT for AS Core Mathematics – Differentiation/Integration

1. Plot a function and find the gradient of the chord between two points
   a. What happens to the gradient of the chord as the chord reduces in length?
   b. How is this related to the gradient of the tangent?
   c. Could you predict what the gradient of the chord would be if it was infinitely short?

2. Explore the gradient at a point on a curve
   a. Plot a curve: e.g. $\mathbf{f(x) = x^2}$
   b. Measure the gradient of the tangent to the curve at a point:
      i. Add a point on the curve
      ii. Find the tangent to the curve at the point
      iii. Measure the slope of the tangent
   c. In a spreadsheet record the value of the gradient for different x-values
   d. Find a formula, in terms of x, that fits the gradient values
   e. Try changing the initial equation of the curve to a different function:
   f. e.g. $\mathbf{f(x) = x^2}$, $\mathbf{f(x)=x^3}$, $\mathbf{f(x)=x^4}$, $\mathbf{f(x)=5x^2}$, $\mathbf{f(x)=x^2+3x}$

3. Gradient functions and stationary points
   a. Plot a function and its gradient function
      i. Plot a curve e.g. $\mathbf{f(x)=x^3-3x^2-2x}$ (a cubic function works well for this task)
      ii. Plot the gradient function $\mathbf{g(x)=f'(x)}$
   b. Add a point to the original curve and display the tangent to the curve at this point
   c. Describe the gradient of the tangent in terms of the gradient function
   d. When is the tangent horizontal?
   e. When is the gradient of the tangent at a maximum/minimum?

4. Tangents and Normals
   a. Draw the curve $\mathbf{f(x) = x^2}$
   b. Add a point on the curve **A**
   c. Display the tangent to the curve at this point
   d. What are the coordinates of **B**, the point where the tangent is perpendicular to the tangent at **A**?
   e. Find the point of intersection of the two tangents
   f. Check the angle between the two tangents
   g. Add the line segment **AB** and find where this intersects the y-axis
   i. Find the coordinates of the point of intersection of the tangents.
   h. Repeat this for different positions of the tangents that are perpendicular. What is the relationship between the points of intersection?



Innovators in Mathematics Education

5. Stationary points
    a. Find a cubic with two distinct stationary points and plot it, e.g. **$f(x) = x^3 + 3x^2 - 5x - 3$**.
    b. Find the turning stationary points: using the function **TurningPoint[f]**
    c. Find the inflection point: using the function **InflectionPoint[f]**
    d. Repeat this for different cubics and observe a relationship between the points
        i. Prove this relationship using differentiation.

6. Introduction to integration (finding areas under lines)
    a. Add sliders named for **m** and **c**
    b. Draw a line **y=m*x+c** (starting with +ve **m** and **c** is easiest)
    c. Find the area under the line up to an x-value:
        i. Add a point **A** at the origin and a point B on the x-axis
        ii. Find the perpendicular to the x-axis through the point **B**
        iii. Find the intersection of the y-axis and the line **y=m*x+c**
        iv. Find the intersection of the perpendicular and the line **y=m*x+c**
        v. Add a perpendicular to the y-axis through the y-intersection
        vi. Find the intersection of the two perpendiculars to the axes
        vii. Use the polygon tool to draw the triangle and rectangle created (and also give their areas as **poly1** and **poly2**)
        viii. Add the areas to get a total area
    d. Can you generalise this? What is the area under **y=m*x+c** up to **$x=x_1$**?
    e. What is the relationship between this and differentiation?
    f. Does it work if either m or c are negative?
    g. Can you extend this to the area between $x = x_1$ and $x = x_2$?

7. Introduction to integration (finding areas under curves)
    a. Plot a function and its integral
        i. Plot a curve e.g. **$f(x) = x^2$**
        ii. Add a point **A** at the origin and a point B on the x-axis
        iii. Set **a=x(A)** and **b=x(B)**
        iv. Find the integral of the curve between the points: **Area=Integral[f,a,b]**
        v. Plot a point **C** which shows the area for a given value of b: **(b,Area)**
        vi. Switch Trace On for this point and vary the point **B**
    b. What function would pass through these points?
    c. Try some different functions for f. How is the Area function related to the original function?

8. Integration (reverse of differentiation)
    a. Add sliders **a** and **b**
    b. Plot **g(x)=a*x+b**
    c. Find a function **f(x)** that would have g**(x)** as its gradient function:
        i. Plot a curve e.g. **f(x)=x²**
        ii. Add a point **A** on **f(x)**
        iii. Draw the tangent to the curve at **A**
        iv. Measure the slope of the tangent (this should be automatically recorded as **m**).
        v. Plot the point **(x(A),m)**
        vi. If **g(x)** is the gradient function for **f(x)** then the new point should move along **g(x)** as **A** is dragged along **f(x).**
    d. Repeat c. for different values of **a** and **b**
    e. Add a third slider **c**
    f. Change the function **g(x)** to **g(x) =a*x²+b*x+c**
    g. Find a function **f(x)** that would have g**(x)** as its gradient function.

9. Finding general cases of cubics
    a. Add sliders named **a**, **b c** and **d**
    b. Plot **f(x)=a*x³+b*x²+c*x+d**
    c. Find values of **a**, **b c** and **d** so that the curve has:
        i. 2 stationary points
        ii. 1 stationary point
        iii. 0 stationary points
    d. What are the conditions on a cubic that determine whether it has 0, 1, 2 stationary points?

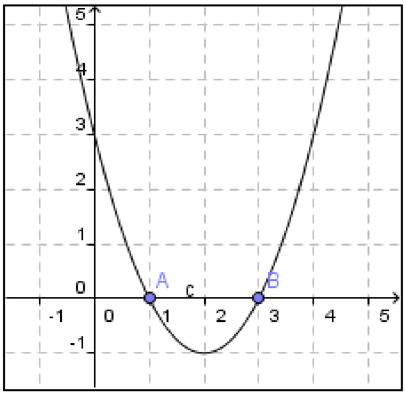
10. Trapezium rule
    a. Open a new Graphs page and plot a function that you can't integrate e.g. f(x)=sinx, f(x)=x*cosx, f(x)=2^x, f(x)=√(1-x^(2))
    b. Enable the Spreadsheet view (View > Spreadsheet) and use this to estimate the area under the curve between 0 and 1 by the trapezium rule with a strip size of 0.2:
        i. In cells A1:A6 enter 0, 0.2, 0.4, 0.6, 0.8, 1
        ii. In cell B1 enter **f(A1)** and fill down to B6
        iii. In cells C1 and C6 enter **1**, in cells C2:C6 enter **2**
        iv. In cell D1 enter **B1*C1** and fill down to D6
        v. In cell D7 enter **sum[D1:D6]**
        vi. In cell D8 enter **0.2** (the strip size)
        vii. In cell D9 enter **.5*D8*D7**
    c. Compare the value of the area to the integral on the graphs page: **Integral[f,0,1]** (you may wish to increase the rounding to 3 or 4 decimal places).
       What is the percentage error in the trapezium rule estimate?
    d. How could you find an improved trapezium rule estimate?

Constructing objects in Geogebra

Testing students' understanding of ideas and reinforcing generalisation

Example

0. Create a two points A and B on the x axis. Construct a quadratic graph that passes through A and B.



A solution (there are others)

Use the New Point button to add points A and B fixed to the *x*-axis.
In the Input bar define two variables:
a = x(A) and **b = x(B)**.
Define a new curve on the Input bar:
y = (x–a)(x–b)

Ideas for AS Core Mathematics

1. Create two points A and B. Construct a third point C which lies on the line perpendicular to AB passing through A and is twice as far away from A as B is.

2. Create points A, B and C fixed to the *x*-axis and D fixed to the *y*-axis. Construct a cubic that passes through A, B, C and D.

3. Create a triangle with one point on the origin and one point on the x-axis. Construct circles centred on each vertex such that all three circles touch each other.

4. Create a graph of a quadratic equation that can be moved by dragging the vertex.
- **a.** Construct the tangent to the curve with gradient 2 (that works for the vertex in any position).
- **b.** Construct the tangent to the curve with gradient *b* (that works for the vertex in any position).

5. Draw the graph of a straight line through the origin (NB this must be defined as a function, e.g. f(*x*)= *x* or f(*x*)=2*x*). Add a point A on the positive *x*-axis.
- **a.** Construct a point B such that the integral of f(*x*) between A and B is 8.

  The Geogebra function for $\int_a^b \mathrm{f}(x)\mathrm{d}x$ is: **Integral[f, a, b]**
- **b.** Construct a point B such that the integral of f(*x*) between A and B is *d*.
- **c.** Construct a point B such that the integral of f(*x*)=*mx* between A and B is *d* for any value of *m* or *d*.

6. Construct a triangle with sides *a* and *b* and angle A that demonstrates the ambiguous case of the Sine rule.

7. Draw the graph of $y = a^x$ and add the point A on the curve. Construct a point B based on A that you can use with Trace function to obtain the shape of $y = \log_a x$.

8. (A challenge!)
Create two points A and B. Construct a cubic that has stationary points at A and B. (Hint – the midpoint of A and B may help).