Innovators in Mathematics Education

TEXAS INSTRUMENTS

MEI Conference 2013

Exploring the Normal distribution

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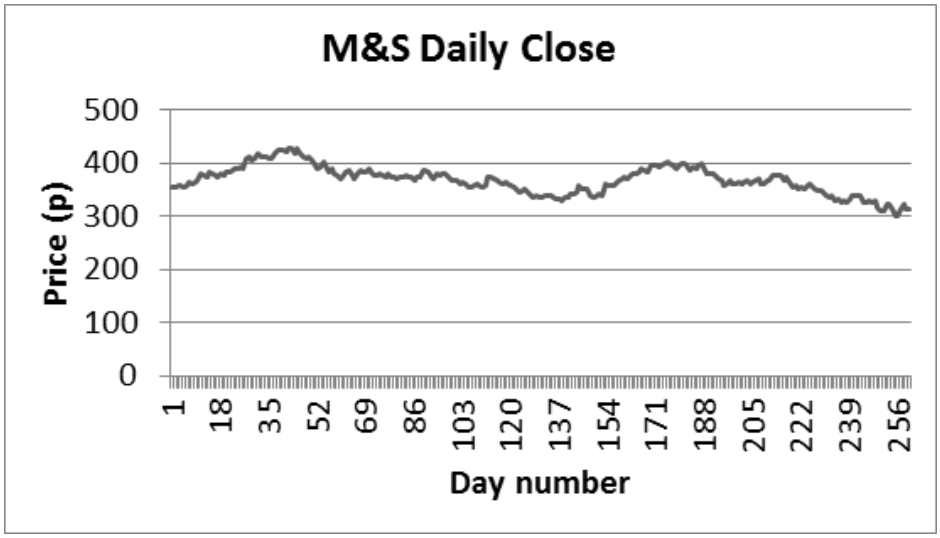
Modelling the Market (Part 1)

The graph below shows the daily closing price of Marks and Spencer shares over about a year.



The problem
Is there a way to model the changes in price of a share?

Some theory

Louis Bachelier

- The father of financial mathematics
- PhD thesis 1900 "Théorie de la Spéculation"
- "L' ésperance mathematique du spéculateur est nul"

Comment on Bachelier's thesis

"The buyer believes in a probable rise, otherwise he would not buy, but if he buys, it is because someone sells to him, and this seller obviously believes in a probable decline. From this results that the market, considered as a whole, takes the mathematical expectation of all operations and of all combinations of operations to be zero."

Henri Poincaré

Deterministic and stochastic models

- Deterministic
  - If the starting condition is known, the future can be predicted with accuracy
- Stochastic
  - If the starting condition is known, possible futures can be assigned probabilities

A random walk

- The idea of a random walk is a simple stochastic model.
- A drunkard is equally likely to take a step forward or a step backward.
- You can model a random walk by tossing a coin and moving forward for heads and back for tails.
- The walk will look different if you repeat it BUT the probability of ending up at any point after a certain number of steps can be worked out.

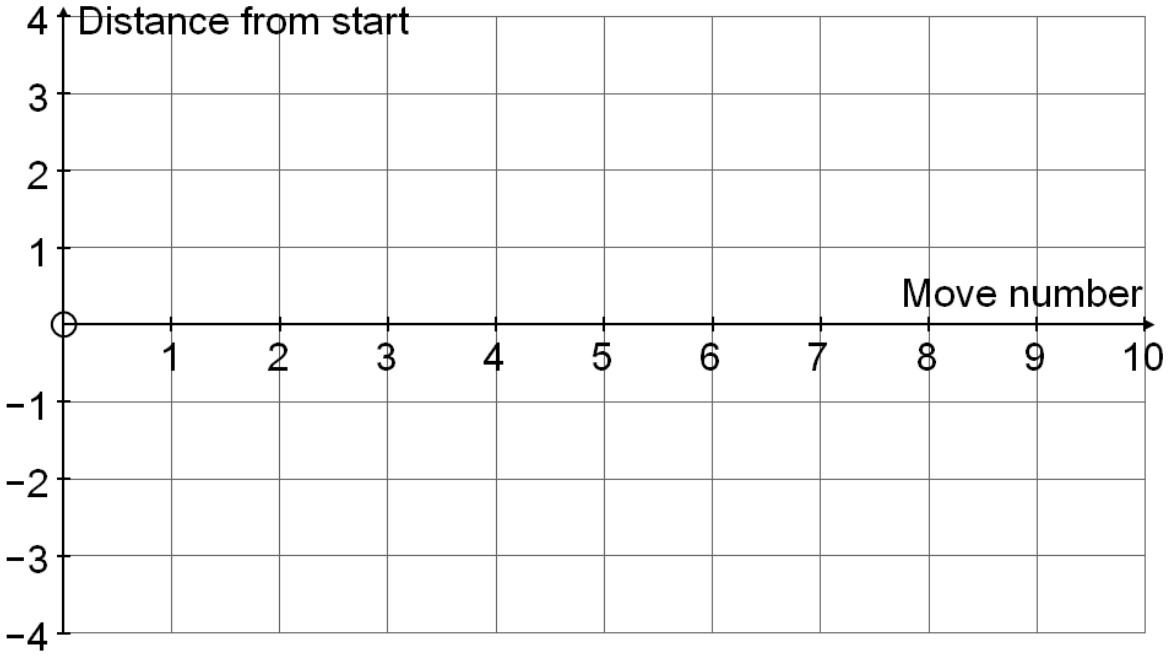
Some of Bachelier's ideas in a nutshell

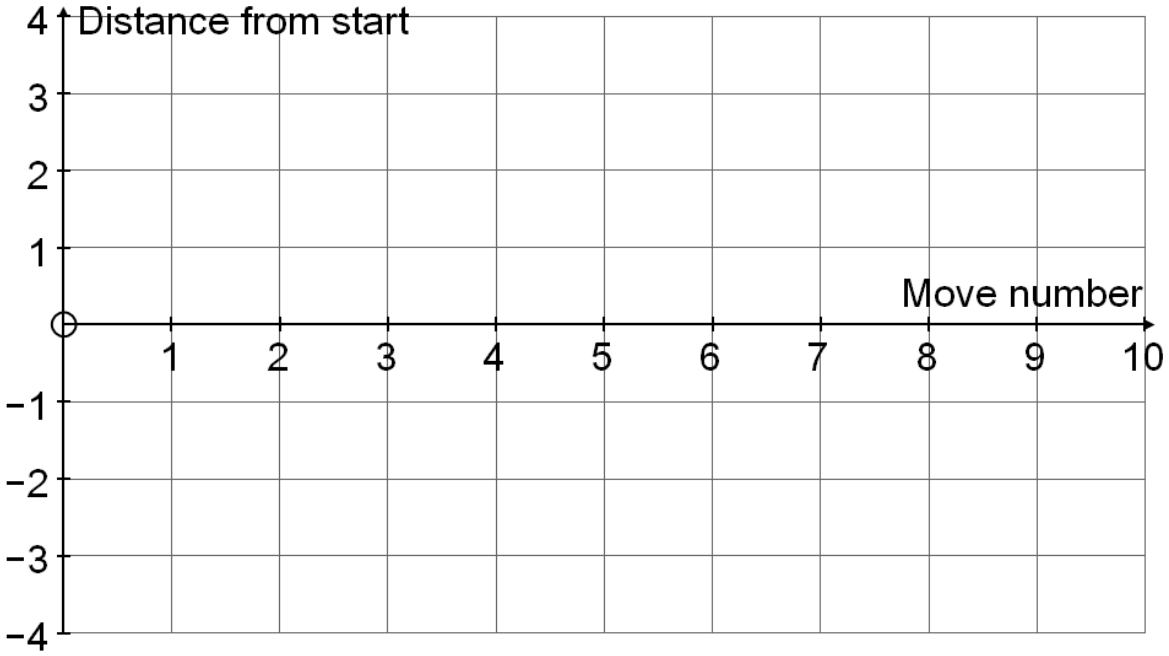
- The previous history of stock market prices does not enable a prediction of whether they will go up or down next.
- Price changes behave like a random walk (over continuous time).

Simulating a random walk

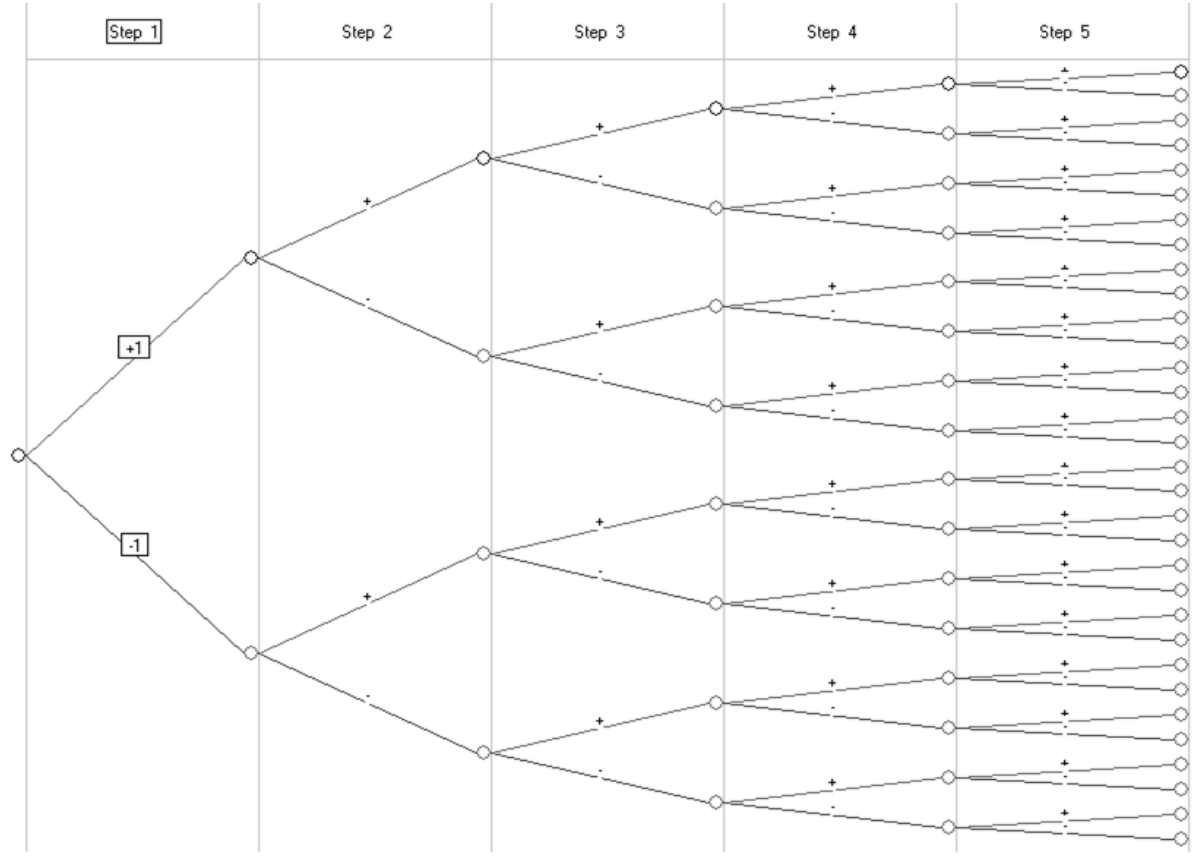
You need a coin.

- Start at zero distance from start at move number zero.
- Toss the coin.
- Go forward one for heads and back one for tails.
- Record the random walk on the grid below.
- Now do another random walk on the second grid.





Probabilities for a 5 move random walk



Distance from start	Probability
+5	
+4	
+3	
+2	
+1	
0	
-1	
-2	
-3	
-4	
-5	

- What does the probability distribution look like for longer random walks?

Samples from Normal distributions with variance 1.6

A	B	C	D	E	F	G
8.124	7.488	4.031	3.339	6.230	8.220	3.833
5.441	4.321	3.904	7.510	5.106	6.470	5.121
6.582	6.555	2.270	7.567	4.969	9.048	2.415
5.886	5.462	5.016	4.996	5.736	9.464	5.296
6.013	7.508	4.269	7.171	7.760	6.347	4.202
6.760	6.262	5.889	6.760	8.284	6.226	4.670
6.535	6.498	1.176	7.411	9.300	5.158	4.218
6.878	5.230	2.823	4.662	5.780	3.177	2.379
5.853	6.319	3.952	6.773	8.295	5.265	1.510
6.367	2.818	2.356	5.284	7.149	5.964	3.693
H	**J**	**K**	**L**	**M**	**N**	**P**
5.045	5.160	4.861	5.507	5.812	2.858	7.292
6.495	7.594	4.314	6.993	8.261	5.088	6.293
6.756	7.714	4.027	6.320	8.650	2.144	6.271
5.394	7.824	1.787	5.920	5.090	3.196	7.790
7.825	4.137	2.062	6.723	6.941	3.135	5.918
3.499	8.013	3.352	7.065	7.132	5.986	7.934
3.908	5.516	3.591	8.666	6.172	4.845	3.943
5.890	7.722	2.023	6.274	5.955	5.134	4.062
4.555	5.112	3.975	7.052	7.030	4.387	7.680
7.025	6.042	3.813	8.682	6.286	3.349	7.447

Q	R
3.203	8.225
6.481	7.049
2.318	5.767
6.585	6.060
2.777	6.003
4.532	4.417
2.645	9.948
4.561	5.547
4.945	6.833
4.850	7.898

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Normal Distribution

UK babies

- How much do you think a typical baby weighs at birth?
- What proportion of babies do you think are born weighing less than 2.5 kg?

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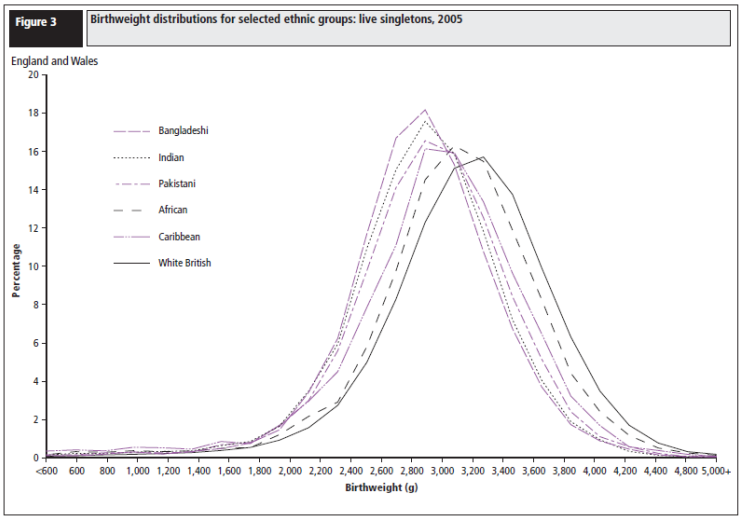

Mean weight

- The graph on the previous slide is from the ONS Health Statistics Quarterly Autumn 2008.
- The mean birth weight was reported as 3352 g with a 95% confidence interval of (3351, 3354).
- Does this mean that 95% of babies had weights between 3351g and 3354 g?

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Below 2.5 kg

- About 6% of babies were below 2.5 kg at birth.

Two papers

- Imagine two exam papers:
  - Paper A has mean score 73 and standard deviation 6
  - Paper B has mean score 78 and standard deviation 15
- Which paper would have more students scoring over 90?
- Which paper would have more students scoring less than 60?
- Which paper would you rather do?

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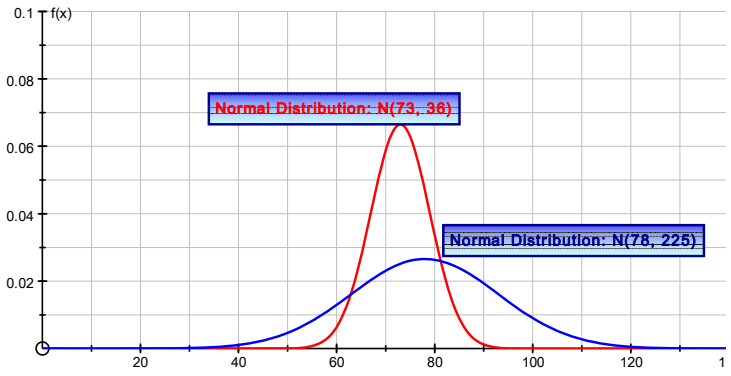
Two papers



Hypothesis testing template

Step	Notes	
Decide null hypothesis	This is often that a population parameter = a particular value. Remember to say what any letters stand for.	
Decide alternative hypothesis	The form of this can depend on what is being investigated	
Decide significance level	Often 5%, but it need not be. You will be told what to use in exam questions.	
Is it a one or two-tail test?	You will be able to tell from the form of the alternative hypothesis.	
Write down key information	Work out the sample mean. If the null hypothesis is true, write down the distribution the sample mean comes from. For a sample of size *n* from $N(\mu, \quad \sigma^2)$ the sample mean, $\overline{X}$, has a normal distribution with mean μ and variance $\frac{\sigma^2}{n}$.	
Calculate test statistic from the sample	You will need to standardise so that you can use Normal tables. (Subtract the mean and divide by the standard deviation).	
Make a decision on the basis of the test statistic	This can be done by using a critical region.	
State the decision in context		