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- We offer continuing professional development courses, provide specialist tuition for students and work with employers to enhance mathematical skills in the workplace
- We also pioneer the development of innovative teaching and learning resources

Mathematical problem solving in students with Autistic Spectrum Disorder (ASD)

13:15 - 14:15 Thursday 28 June

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An overview

An exploration of mathematical problem solving in students with Autistic Spectrum Disorder (ASD) describing a case study of mathematical problem solving in an individual with autistic spectrum disorder in an English secondary school.

Motivations

- Teaching in a school with a specialist unit for those with ASD *and* teaching a very interesting student with ASD.
- Deepen my own understanding of ASD.
- Develop professionally as a teacher and better understand an area of mathematics education.

Introduction

- Around 70% of 120,000 school aged children in England with ASD attend mainstream schools.
- Mathematics can be thought of as having two different aspects: skills and problem solving.
- Research into those with ASD has predominantly focused on skills.

What is Autistic Spectrum Disorder?

- It used to have several different labels, but APA 2013 introduced the omnibus category. Asperger's syndrome and autism are now diagnosed as ASD

- ASD is a neurodevelopmental disorder, characterised by persistent social communication and interaction deficits.

- It includes deficits in social reciprocity, nonverbal communication and skills developing, maintaining, and understanding relationships.

- Other neurodevelopmental disorders often co-occur with ASD; for example, an individual with ASD may also have an intellectual disability.

What is Autistic Spectrum Disorder?

- Reported frequencies of ASD have approached 1% of the population and it is frequently associated with intellectual impairment.
- It is a spectrum condition so manifestations differ. It is not degenerative.
- Learning and compensation continues throughout life, so symptoms may change and can be masked by compensation mechanisms.
- Approximately 80 per cent are male and 20 per cent female. In clinical samples, females tended to show accompanying intellectual disability, suggesting females may go undiagnosed.

What is Autistic Spectrum Disorder?

ASD is characterised by the failure of false-belief tests (a test which assesses if an individual can account for another's belief which they know to be false.)

Sally-Anne Test

Sally places a marble into a basket and leaves the scene.

Anne enters the scene, moving the marble into the box.

When Sally returns "Where will Sally look for her marble?"

If the subject points to the basket (previous location), they pass by appreciating the doll's now false-belief.

However, if the subject points to the box (current location), they fail by not accounting for the doll's belief.

What is Autistic Spectrum Disorder?

ASD is characterised by the failure of false-belief tests (a test which assesses if an individual can account for another's belief which they know to be false.)

Smarties Test

Subjects were shown a Smarties tube and asked, 'What's in here?'.

It was then revealed that in fact the tube contained a pencil.

They were then asked about the belief of the next subject to participate in the experiment, 'They haven't seen this box. When they come in, I'll show them this box just like this and ask: "What's in here?"'.

The subject was then asked the prediction test ('What will they say?').

It went further than Sally-Anne, as the ASD group were asked checking questions after the prediction test; reality check question ('Is that what's really in the box?') and own-response check question ('Remember when I asked you what was in the box, what did you say?').

What is Autistic Spectrum Disorder (ASD)?

Those with ASD are more likely to:

- Be worse at identifying and correcting errors

- Think that they have got a question correct
- AND claim it was an intentional mistake when they are told that they have got something wrong

- Persevere with previously successful strategies despite receiving feedback indicating it was inappropriate in the new situation

How does ASD impact ability in mathematics?

- There is widely-held belief that those with ASD are mathematically gifted, despite evidence to the contrary.
- Mathematical-giftedness does exist in some with ASD.
- The prevalence of mathematical learning disabilities in those with ASD is much higher than the 5–7 per cent found in the general population.
- Chiang and Lin (2007) found in their literature review of mathematical ability in students with ASD, that the majority had average mathematical abilities.

How does ASD impact ability in mathematics?

- Children with ASD have weaknesses in the areas of writing including graphomotor and organizational skills, attention, complex processing across domains including problem solving, numerical operations, listening comprehension and reading comprehension.

- Some suggest that students with ASD may begin to struggle as the mathematics content becomes more abstract, cognitively complex, emphasising problem solving and mathematical reasoning.

- **Academic strengths/weaknesses of those with ASD is heterogeneous in nature.**

- It is important to compare the relative performance of individuals rather than focusing on group means

Research to date of mathematics in those with ASD

- Intervention studies to date have predominantly been inappropriate for students with ASD in a mainstream school.
- Many interventions featured one to one instruction arrangements, which may not feasible in a mainstream school.
- Previous studies had not addressed the challenges faced by high functioning students and those without other intellectual difficulties.

Research to date of mathematics in those with ASD

- There were very few, if any studies focussing on teaching mathematical problem solving in students with ASD.

- Future intervention studies in mathematics should focus on higher order skills and problem solving.

- **Most studies have focused on the skills aspect of mathematics, perhaps as it allows researchers to easily obtain quantitative measures for statistical analysis.**

What is problem solving? (in mathematics)

- There is no real consensus over the definition of problem solving and it remains an important question to answer

- Broadly speaking there is consensus on what is not problem solving

- Standard textbook “word problems” are not necessarily considered problem solving:
  e.g. *Jon buys 30 apples at 15 pence each. How much does Jon spend on apples?*

What is problem solving? (in mathematics)

“Mathematics has always had two surface aspects, often called skills and problem solving… Skills are seen as a set of well-defined procedures for transforming numbers, symbols or shapes; the role of the ideal performer is that of an automaton, and the role of the student that of learning to be a reliable one… Problem solving, in contrast, involves tackling tasks that are significantly different from those one has learned ‘by heart’; a major part of the challenge lies in deciding how to tackle the problem, and which bits of one’s toolkit of mathematical skills will help”

Burkhardt, H., & Bell, A. (2007). Problem solving in the United Kingdom

A framework for analysis of problem solving

- Metacognition was chosen as many people have suggested that deficits in executive function are present in those with ASD.

- Metacognition seemed a sensible viewpoint to use to analyse both mathematical problem solving (due to the work of Schoenfeld amongst others) and was relevant to the deficits of those with ASD.

Metacognition and problem solving in mathematics

- Broadly speaking "metacognition is defined as the knowledge and control one has over one's thinking and learning activities"
- Metacognition is viewed as central to problem solving in mathematics and can be divided into three distinct categories
- Self-awareness. How accurately can you describe your own thinking?
- Self-regulation. How well can you track what you are doing? For example, when problem solving how well do you use your observations to influence your actions in problem solving?

(• Beliefs and intuition. What ideas do you have about mathematics and how does this influence the way in which you do mathematics?)

Metacognition and problem solving in mathematics

Novice problem solver (Two students who recently studied the topic)

“The students had spent 20 minutes on a wild goose chase. They had ample opportunity to stop during that time and ask themselves ‘Is this getting us anywhere? Should we try something else’ but they didn't. And as long as they didn't they were guaranteed not to solve the problem. This is an all too typical example of the disastrous consequence of an absence of self-regulation.”

This kind of attempt was described as “read the problem, make a decision to do something, and then pursue it come hell or high water”

Metacognition and problem solving in mathematics

<u>Expert problem solver (mathematician who was rusty at geometry)</u>

"(What is) interesting about his solution is that as he work(ed) his way through the problem, he considered quite a few different approaches to it, many of which were just plain wrong. In fact, he generated enough potential wild geese to keep dozens of problem solvers busy. Yet he never went off the deep end the students had, because he was as ruthless about testing and rejecting ideas as he was ingenious in generating them."

Metacognition and problem solving in mathematics

Comparing the problem solving attempts of the students and mathematician, there were clear differences between approaches:

- The mathematician spent most their time thinking rather than doing. On several occasions they asked themselves a monitoring question such as ‘how am I doing?’ and decided on what to do as a result. As such, it is clear self-regulation occurred throughout the mathematician’s problem solving attempt.
- During the second part of the mathematician’s problem solving attempt, the mathematician began a ‘wild goose chase’. The key difference between the mathematician and the student’s problem solving attempt is that this wild goose chase was quickly ceased and the mathematician changed their approach after only a short period of time.

Metacognition and problem solving in mathematics

Schoenfeld believed that problem solving is often a function of luck, as such developing metacognitive skills does not guarantee success, but gives students an opportunity to be successful at solving a non-standard problem.

Metacognition and problem solving in mathematics

Schoenfeld sought to improve metacognition in mathematical problem solving

Based on their experiences from their problem solving classes, suggested that the following can improve metacognitive processes:

(1) Using video tapes (watching other students problem solving and being critical of their approach)
(2) Teacher as a role model for metacognitive behaviour
(3) Class discussion of a problem with the teacher serving as ‘control’
(4) Problem solving in small groups

Metacognition and problem solving in mathematics

- Schoenfeld indicated that their problem solving classes produced marked shifts in problem solving behaviour.
- Before classes, more than half of problem solving sessions analysed were of the kind “read the problem, make a decision to do something, and then pursue it come hell or high water”
- After their classes fewer than 20 per cent of solution attempts were of this kind.

Research questions

- Can a student with ASD adapt their strategy when problem solving in mathematics?
  (can this be achieved using feedback and without using feedback?)

- Can a student with ASD identify and correct errors whilst problem solving in mathematics?
  (can this be achieved using feedback and without using feedback?)

- Can students with ASD take into account more than one perspective when problem solving in mathematics.

Methodology

- A male student in Year 10 with ASD.
- Student was bilingual and was “high-ability” in mathematics.
- Student had previously been taught by myself.
- In a mainstream school with a specialist unit and the student had a TA with them permanently to act as a scribe and for extreme behaviour.

Methodology

- Student was video recorded completing 2 different problem solving task taken from *Problems with Numbers and Patterns* by Shell Centre
(Skelton Tower and The Climbing Game)

- Student was allowed to attempt to solve the problem. Prompt questions and clarification was allowed: “What are you doing?”, “Why are you doing it?” and “How does that help you?”

- The problem solving attempt was then discussed

- Videos of both were then coded and analysed using metacognition

Problems with Patterns and Numbers

http://www.mathshell.com/publications/tss/ppn/ppn_teacher.pdf

Skeleton Tower on page 18

The Climbing Game on page 12

Research findings – Self-awareness

- The subject demonstrated self-awareness of his mathematical habits during both tasks.
- In skeleton tower, he identified that he tended to not update his work whilst working through a problem and discussed a feature of his ASD stating he did not like checking his work in case he spotted a mistake.
- He also demonstrated good self-awareness while performing the addition of three numbers stating he was “stupid with addition”, which led to him taking more time on it to minimise the risk of an error.
- In the climbing game, he identified that when he solves problems he tends to think of similar problems.

Research findings – Self-regulation

- Poor self-regulation was evident in both tasks but on a small number of occasions good self-regulation was observed.
- The subject independently devised strategies in both tasks that would have allowed to him to successfully solve the problems successfully.
- However, errors occurred in both tasks which went unidentified during his problem solving attempt but were then identified during the discussion.
- The subject did not take the time to carefully read both given problems.
- There was no evidence of monitoring questions such as ‘How am I doing?’ and at no point during either problem solving attempts did he stop to reflect on the strategies that he was using.

Research findings – Adapting strategy

- Findings from both problem solving sessions suggest the subject could adapt strategy when solving problem solving both with and without feedback.
- In the problem solving attempt for skeleton tower he did not explicitly adapt strategy. However, in the discussion it was revealed that he had dismissed his initial strategy. Suggesting he may have adapted strategy without feedback.
- During the discussion for skeleton tower, despite initially making an error, he used his initial strategy to determine a correct formula for the number of cubes in a tower in terms of the height of the tower.
- The subject went further and after prompting he verified that both the formulae he had derived were equivalent using a third strategy.
- In the problem solving attempt and discussion for the climbing game his strategy evolved. Suggesting he can adapt strategy without feedback.

Research findings - Error identification and correction

- In both problem solving attempts there were few incidents of error identification and correction.
- During the problem solving attempt for skeleton tower no errors were identified. However, the subject attempted to verify the triangular number formula suggesting he was mindful of errors.
- In the problem solving attempt for the climbing game, the only error that was identified and corrected without feedback was an initial error in colour coding, which suggested the subject can identify and correct errors without feedback.

Research findings - Error identification and correction

- Error identification and correction predominantly occurred in the discussion of the problem solving attempt.
- In the discussion of the skeleton tower task, the subject identified and corrected all the errors made during the problem solving attempt.
- In the discussion of the climbing game the subject identified and corrected all, except one, of his errors.
- These findings highlighted that identifying errors is to some degree a function of luck (and possibly aided by social interaction)

Research findings - Accounting for another perspective

- No observations related to accounting for more than one perspective in skeleton tower. All observations occurred during the climbing game.
- During his early strategy of thinking of the about the problem in terms of a simpler game the subject demonstrated that he could take into account more than one player’s perspective.
- Then throughout his strategy of working back from the finish dot towards the start dot the evidence suggested that he could account more than one player’s perspective in various positions.
- Both these episodes suggested that Alexander could take into account of more than one perspective when problem solving in mathematics.

Discussion – the subject

- Evidence from the problem solving sessions seems to suggest the subject can demonstrate behaviours required to be successful at mathematical problem solving.
- With discussion, they could successfully answer both problem solving tasks.
- However, the evidence also seems to show that the subject had poor self-regulation in mathematical problem solving.

Discussion – questions arising

- Would better self-regulation improve their ability to independently solve a given mathematical problem and what interventions would improve their self-regulation in mathematical problem solving?
- Could interventions suggested by Schoenfeld (1987) be appropriate in a mainstream secondary school?
- If so, given the social deficits in those with ASD could they improve their metacognition and mathematical problem solving?

Conclusion – for researchers into SEND/ASD

- Most studies of mathematics in students with ASD have focused on the skills aspect of mathematics
- Perhaps as it allows researchers to easily obtain quantitative measures for statistical analysis
- OR perhaps because there is no clear definition of what is meant by problem solving
- Unfortunately, this reinforces the idea that mathematical skills are mathematics.
  This is not the case.

Conclusion – for teachers

- Teachers of mathematics should avoid the misconception that mathematical giftedness is pervasive in those with ASD.
- They should understand that ASD is heterogeneous in nature and should consider an individual's strengths and weaknesses rather than generalising those with ASD.
- The findings of this study are broadly positive as the subject in this study showed behaviours that are thought to be important in successful mathematical problem solving.
- This suggests those with ASD may be able to be successful problem solvers in mathematics.

Conclusion – for specialist units for ASD

- They should be aware that mathematics is wider than just skills and that problem solving is central to doing mathematics.
- Several studies have indicated that problem solving in mathematics may present unique challenges for their those with ASD.
- The findings of this study are broadly positive as the subject in this study showed behaviours that are thought to be important in successful mathematical problem solving.
- This suggests those with ASD may be able to be successful problem solvers in mathematics.

Any questions?