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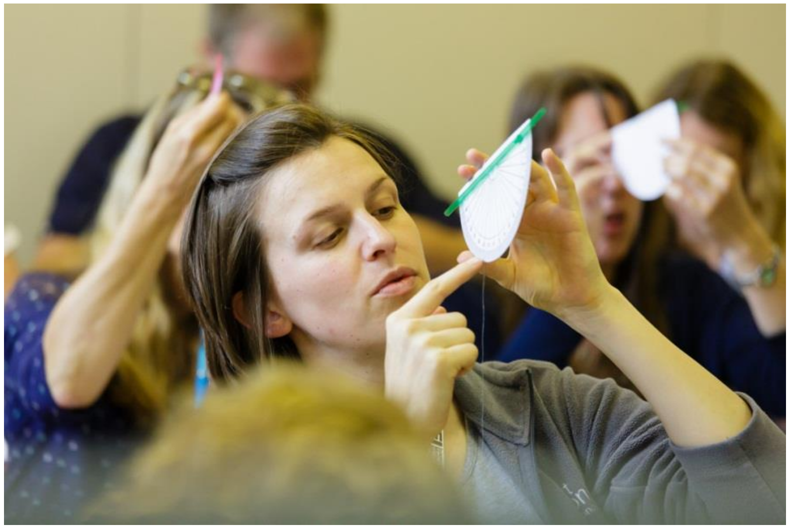

Mechanics in A level Further Mathematics

Work & Energy

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Topics using energy

- Work, energy & power
  - Definitions, conservation of momentum, work-energy principle
- Elastic springs & strings
  - Hooke’s law & elastic potential energy
- Circular motion
  - Vertical circular motion
- Momentum & restitution
  - Loss of energy in collisions

Specifications references

	AQA	Edexcel	OCR (A)	OCR(B) MEI
Work, energy & power	AS level	FM1 AS	AS level	Mech a
Elastic springs & strings	AS level	FM1 AS	A level	Mech b
Circular motion	A level	FM2	AS level	Mech b
Momentum & restitution	AS level	FM1	AS level	Mech a

Work

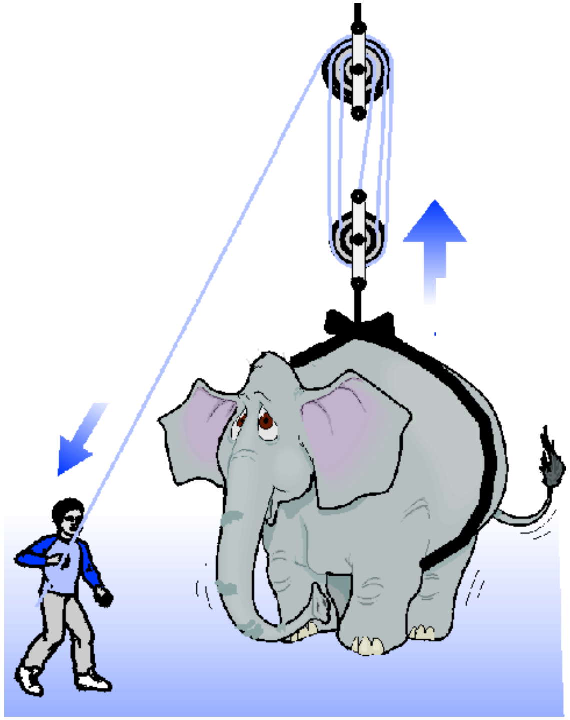
I like to start with work

If you lift an elephant from the floor, you have done some work.

You apply a force to overcome gravity. The higher you try to lift the elephant, the more work you do.

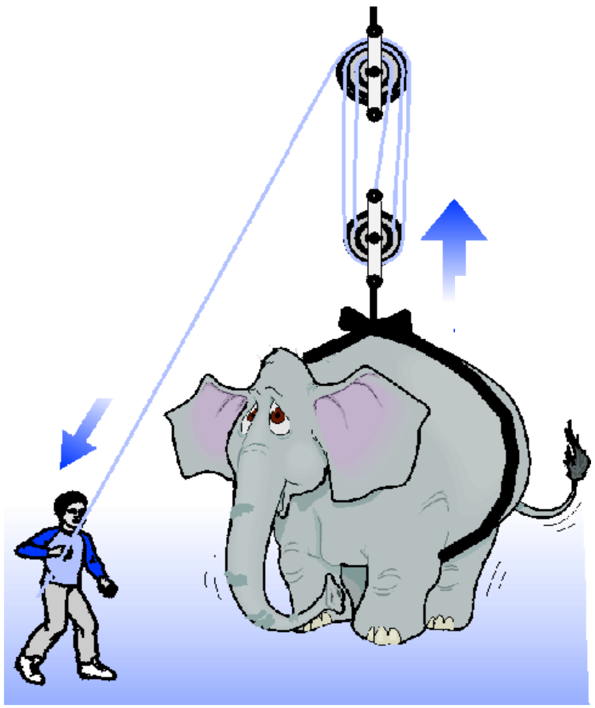
Work = Force X Distance

Unit is the **JOULE (**a Newton-metre)



named after English physicist James Prescott ***Joule*** *(1818–1889)*

Work

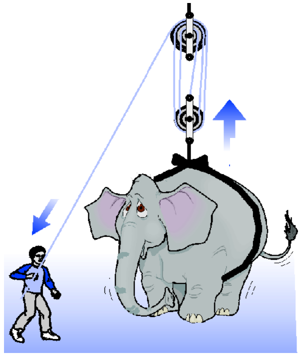


Important points

- Work is done by a force
- A force only does work on an object when it has a component in the direction of motion of the object
- Work is a vector

Potential Energy

And back to the elephant…



- If we lift an object from the floor into the air, it has the potential to do work for us.
- This ability to do work is called POTENTIAL ENERGY
- A lifted object has GRAVITATIONAL POTENTIAL ENERGY

Energy definitions

- Kinetic energy $KE = \frac{1}{2}mv^2$
- Gravitational potential energy $GPE = mgh$
- Elastic potential energy $EPE = \frac{\lambda x^2}{2l}$

$$\left(EPE = \frac{1}{2}kx^2\right)$$

Energy definitions

$$KE = \frac{1}{2}mv^2 \qquad GPE = mgh \qquad EPE = \frac{\lambda x^2}{2l}$$

$m =$ mass

$g =$ acceleration due to gravity

$v =$ speed

$h =$ height (above reference point)

$\lambda =$ modulus of elasticity of spring/string

$x =$ extension of spring/string

$l =$ natural length of spring/string (upstretched length)

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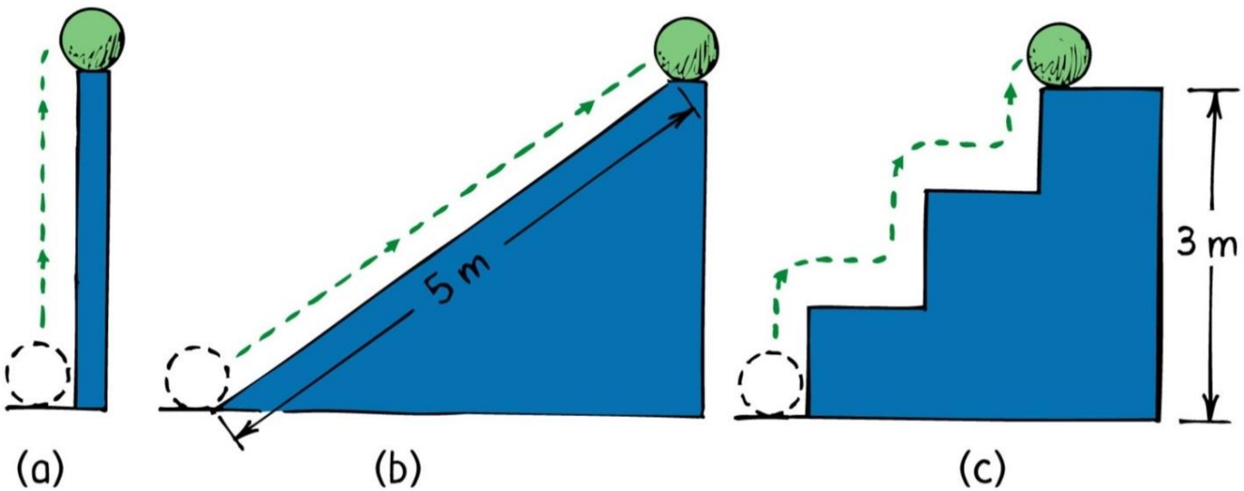
Gravitational Potential Energy

Which ball has the greatest gravitational potential energy?



Mechanical energy and work

The total energy of an object can only be changed when it is acted on by an external force.

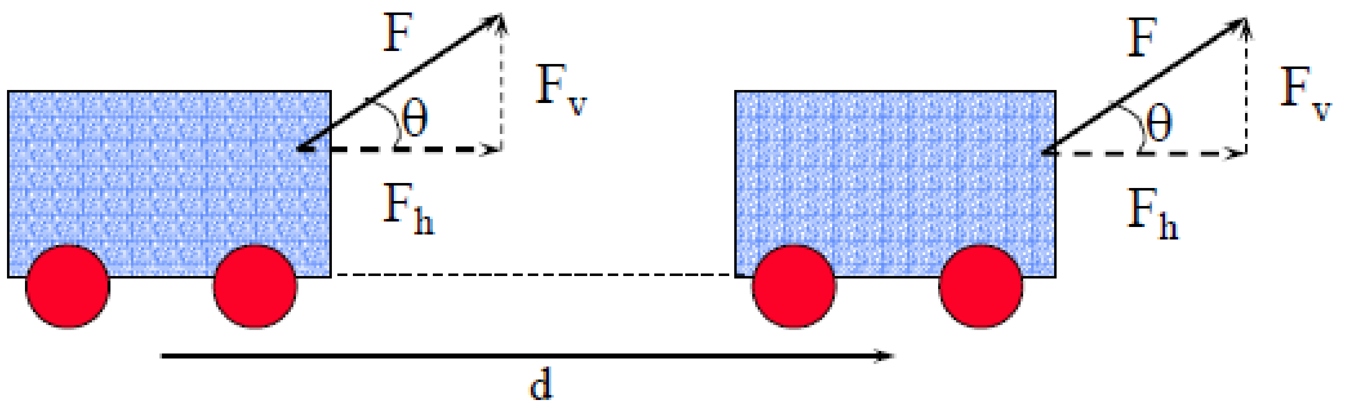
When a force is applied to an object which moves in the direction of its line of action, the force is said to do work.

For a constant force this is defined as:

The work done by a constant force = force X distance moved in the direction of the force

Warning

When the force is not parallel to the direction of motion, we must take the component of the for that is in the direction of motion to calculate work done



$$F_h = F\cos\theta$$
$$F_v = F\sin\theta$$
$$W_k = F_h d = F(\cos\theta)d$$

If F = 100 N and θ = 30 degrees,

Compute Fv and Fh

If d = 5 m, compute W

Key principles

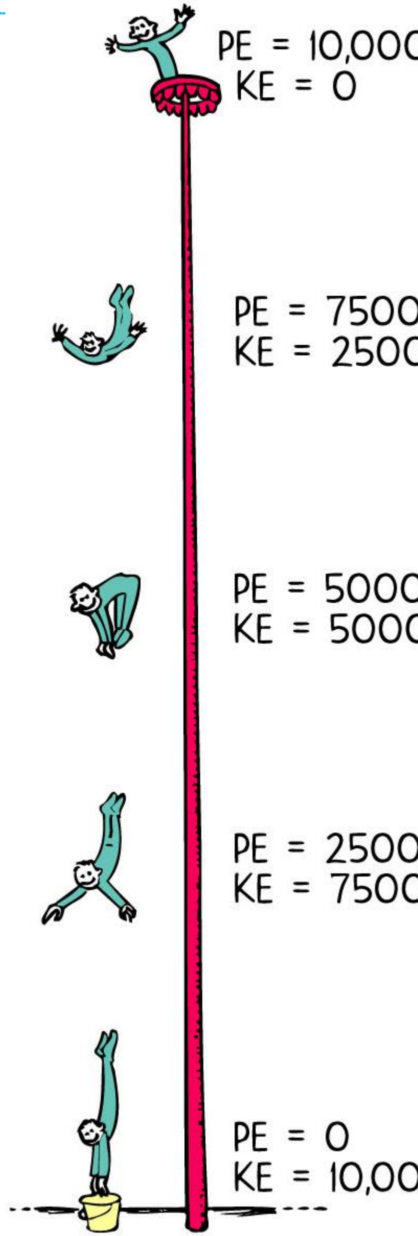
- Conservation of energy

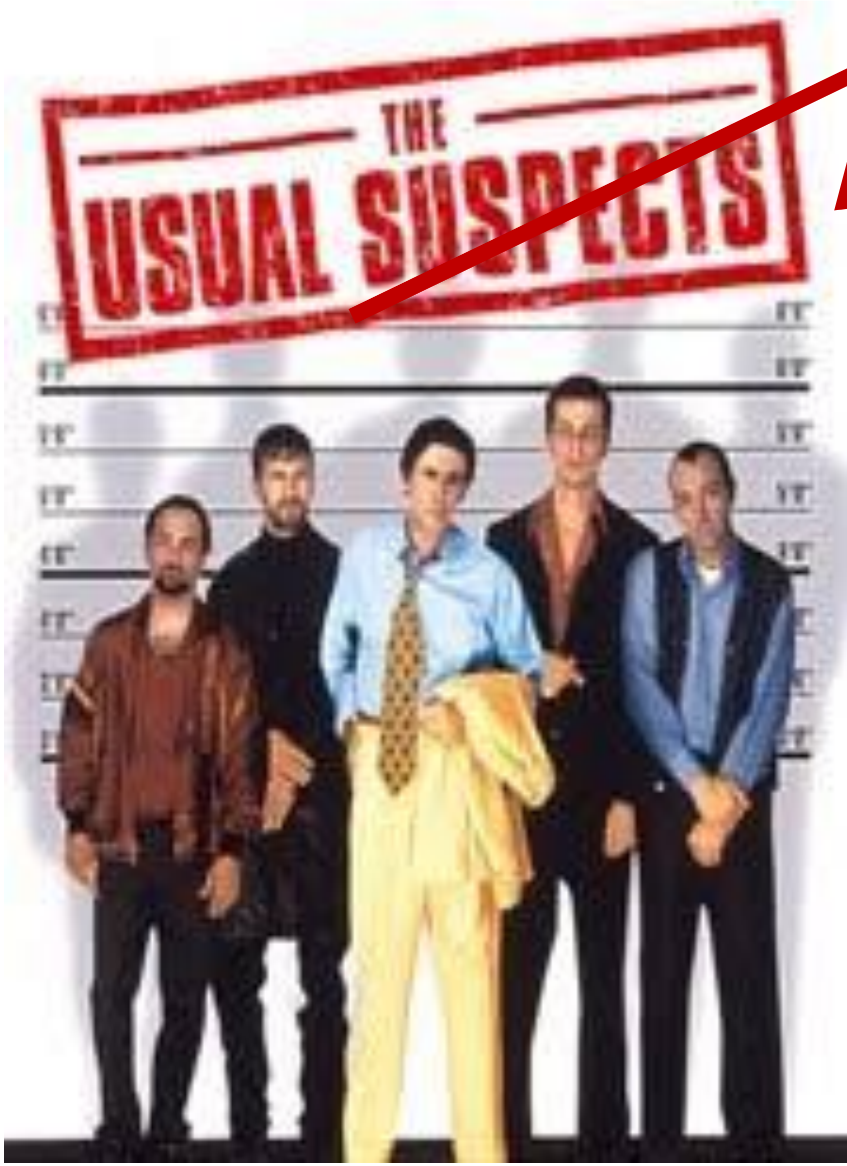
In the absence of any external forces doing work, the total energy of a system remains constant.

- Work-energy principle

Total work done by external forces = change in mechanical energy.

NB: Mechanical energy is KE+GPE, but we extend this principle to include EPE.





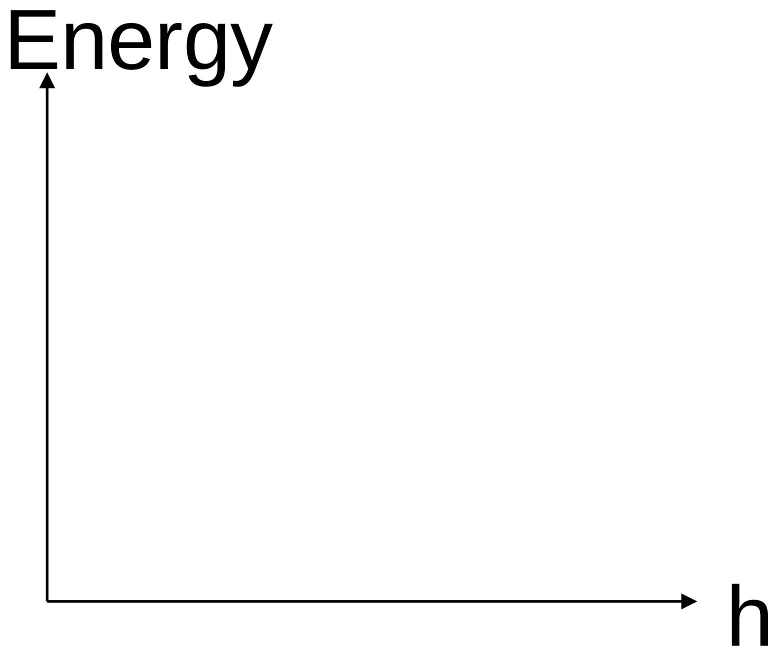
ASSUMPTIONS

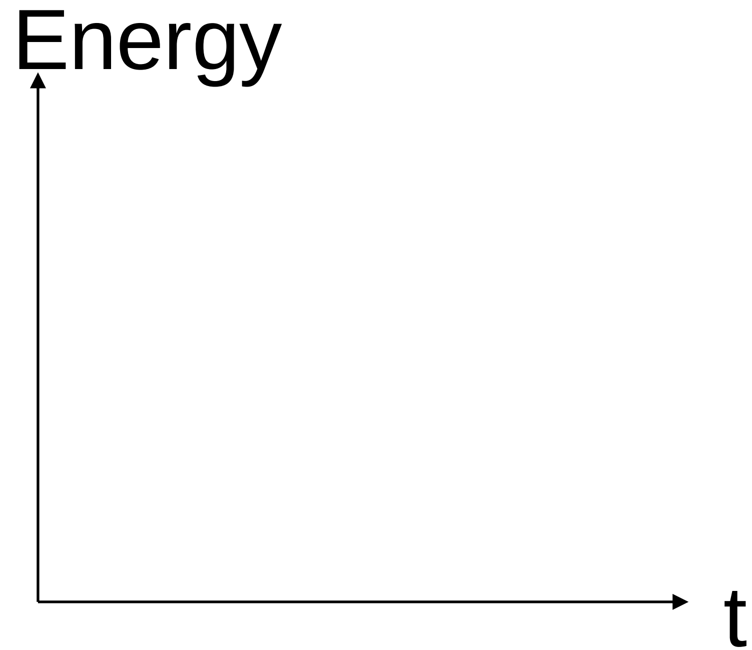
Dropping a ball

- Draw graphs of total energy (GPE + KE) against *h* on the same axes

- Draw graphs of total energy (GPE + KE) against *t* on the same axes

Dropping a ball





What assumptions are we making?

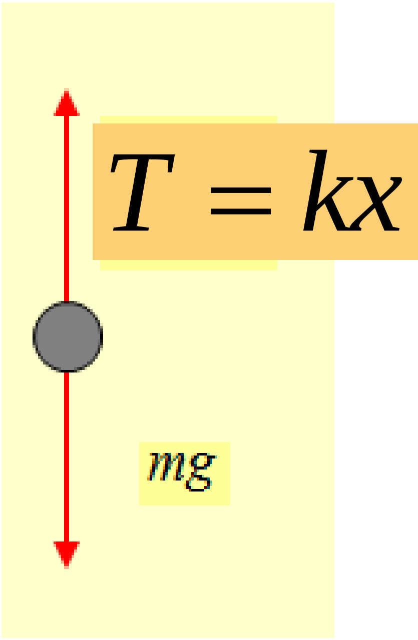
Bagpuss goes bungee jumping

Bungee jumper

- Measure the natural length of your elastic string/spring
- Hang a mass from the elastic so that it is in equilibrium
- Measure the elastic string and determine the modulus of elasticity or stiffness of the elastic.
- Now use energy to predict the maximum length of the elastic string if the mass is released from rest at the same level as the fixed end of the string.

Bungee jumper

- The diagram shows the forces acting the mass, when it is in equilibrium.
- When the forces are in equilibrium:



$$T = mg$$

$$kx = mg$$

$$k = \frac{mg}{x}$$

Note:
k is the stiffness of the string.
The modulus of elasticity, λ, is related to k by

$$k = \frac{\lambda}{l}$$

Bungee jumper

The GPE lost during the downward motion is equal to the EPE gained as the string stretches to its maximum length.

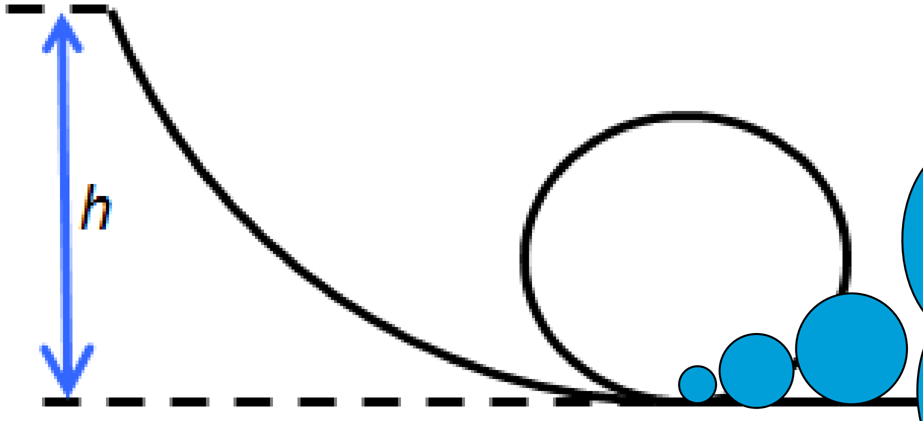
If the mass falls a distance *x*, after it has become taut, the maximum extension of the elastic is *x* and distance fallen is *x* + *l*

Using: EPE Gained = GPE Lost

$$g(x+l)=\frac{kx^2}{2}$$

$$\frac{kx^2}{2}-mgx-mgl=0$$

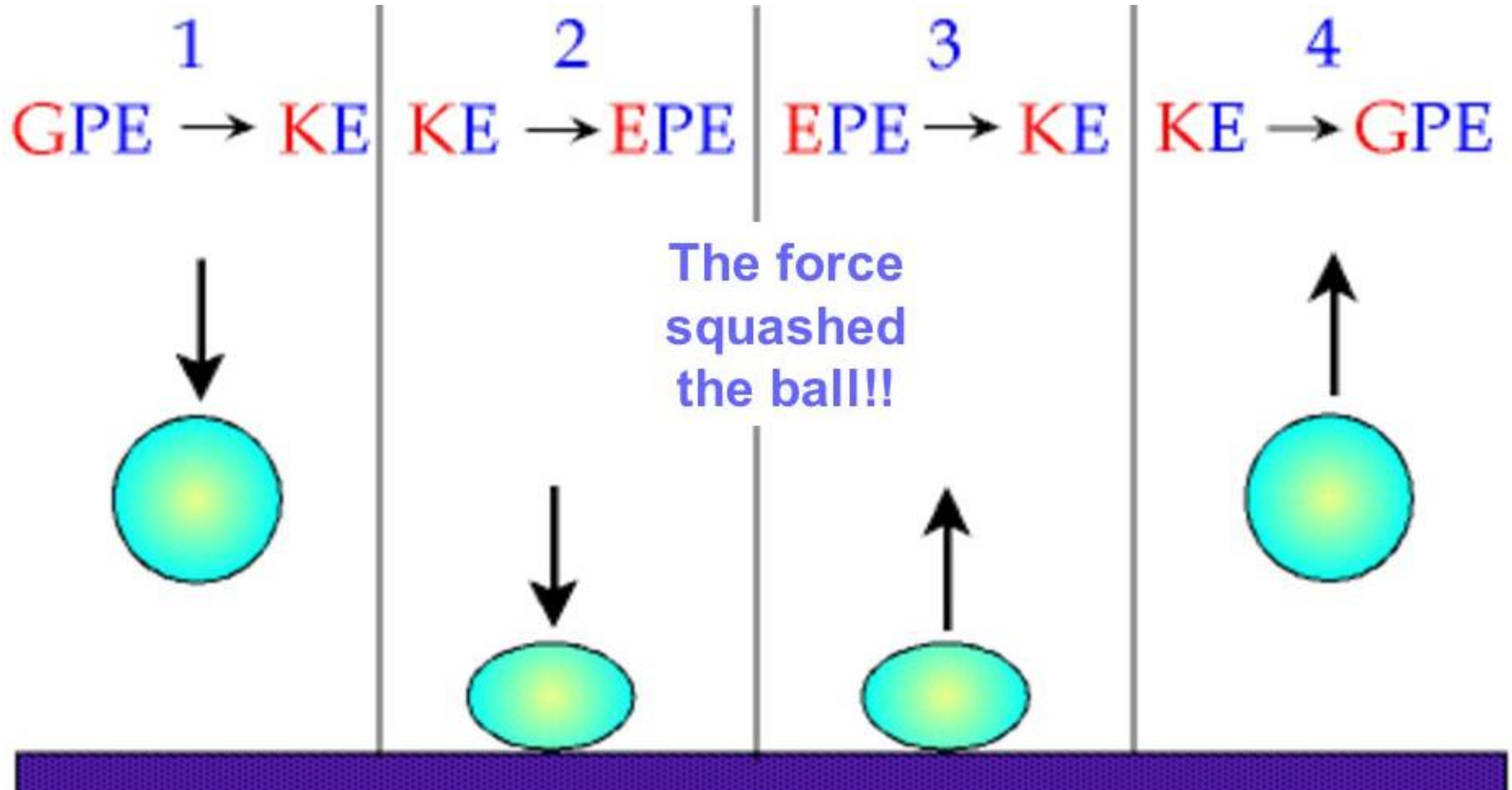
Loop-the-loop



For the particle to make a full loop the speed at the bottom of the circle $> \sqrt{5gr}$

- What happens when the particle is released?
- What is the release height, h, needed for the particle to make a full loop?

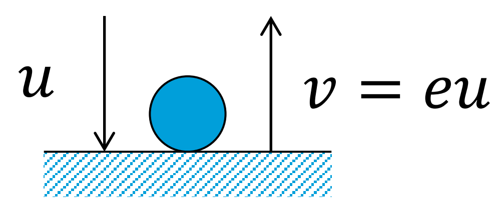
Bouncing balls



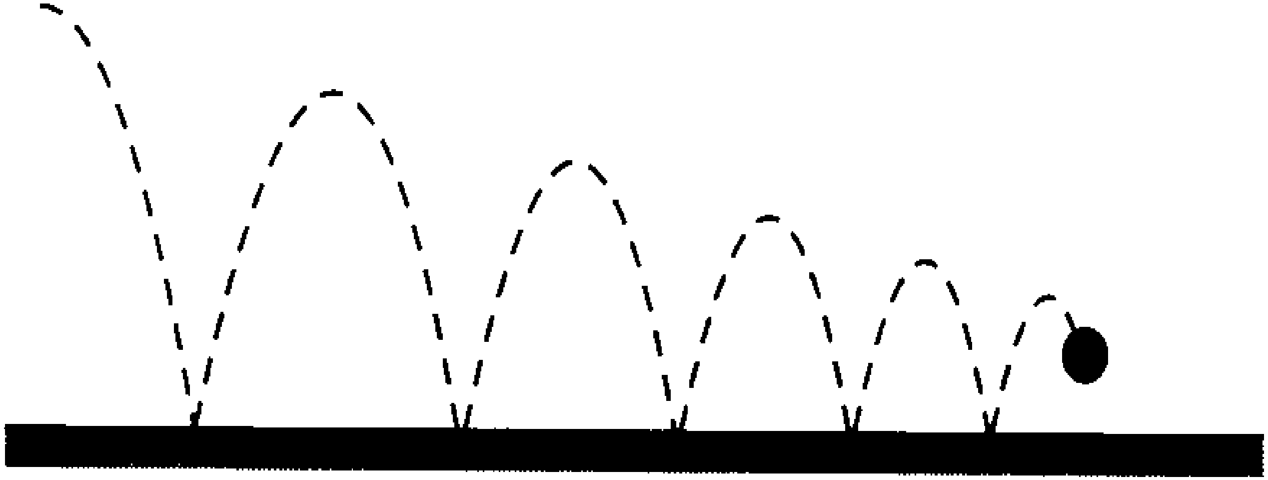
Bouncing balls

- Loss of kinetic energy in a collision
- Coefficient of restitution, e, relates the speed of separation to the speed of approach
- For a particle in direct collision with a fixed plane

$$e = \frac{v}{u}$$



Bouncing balls



- If we know % energy loss in each bounce we can work out how long ball takes to stop bouncing.

References

- Mechanics in Action

stem.org.uk/resources/elibrary/resource/26065/mechanics-action

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