MEI Conference 2018

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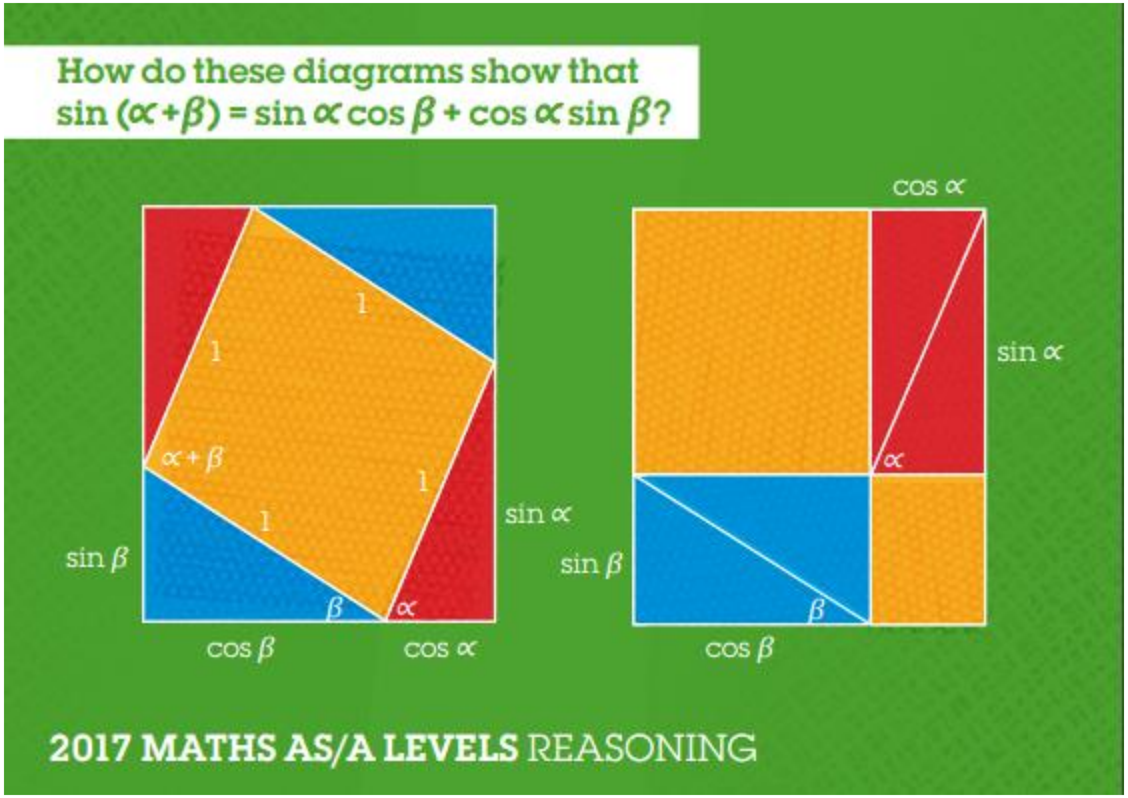
Embedding argument, language and proof in A level Maths



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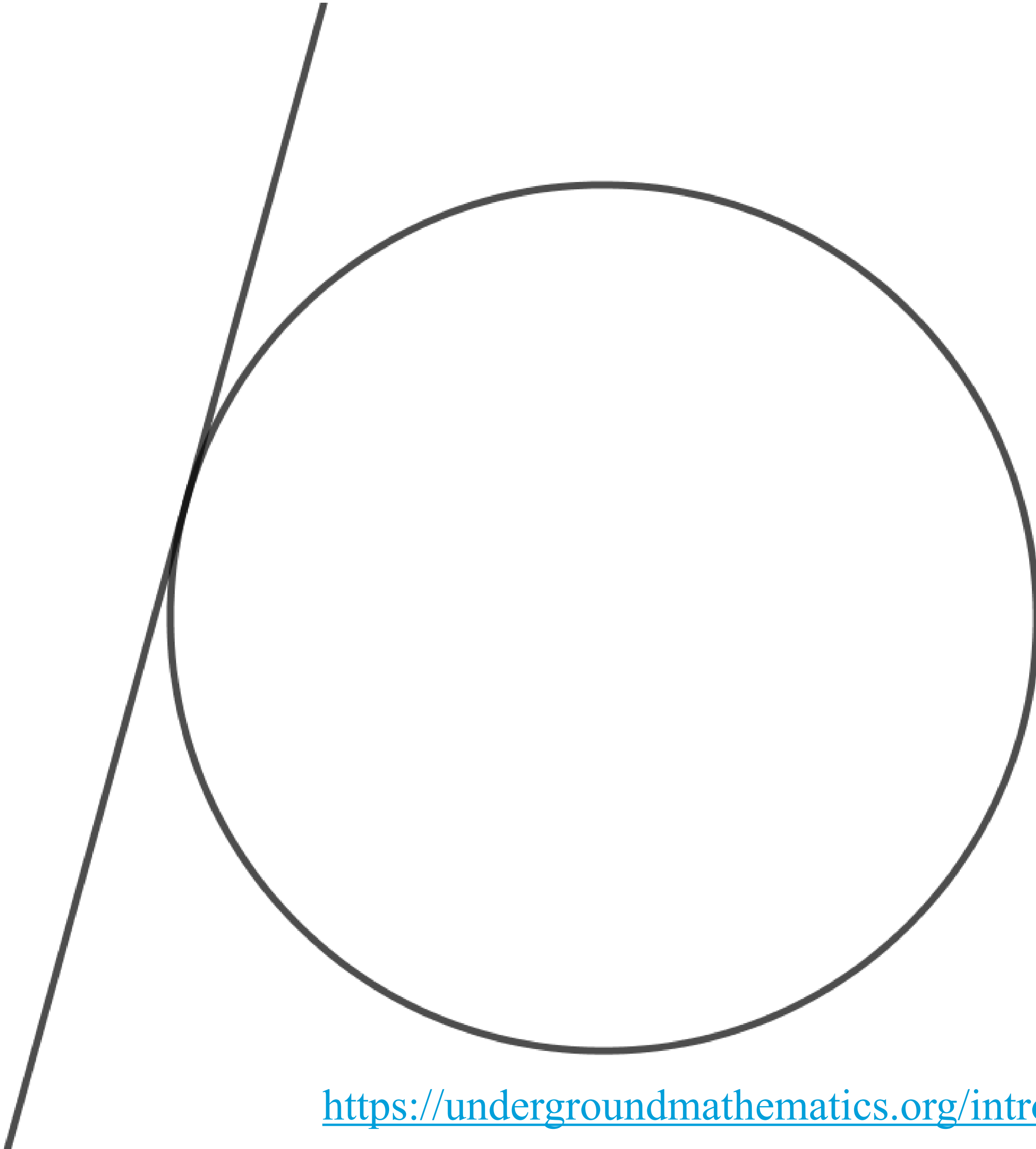


https://undergroundmathematics.org/introducing-calculus/a-tangent-is

DfE: Mathematics AS and A level content

OT1 Mathematical argument, language and proof

AS and A level mathematics specifications must use the mathematical notation set out in appendix A and must require students to recall the mathematical formulae and identities set out in appendix B.

	Knowledge/Skill
OT1.1	**[Construct and present mathematical arguments through appropriate use of diagrams; sketching graphs; logical deduction; precise statements involving correct use of symbols and connecting language, including: constant, coefficient, expression, equation, function, identity, index, term, variable]**
OT1.2	**[Understand and use mathematical language and syntax as set out in the content]**
OT1.3	**[Understand and use language and symbols associated with set theory, as set out in the content]** **[Apply to solutions of inequalities]** and probability
OT1.4	Understand and use the definition of a function; domain and range of functions
OT1.5	**[Comprehend and critique mathematical arguments, proofs and justifications of methods and formulae, including those relating to applications of mathematics]**

Assessment Objective 2

Reason, interpret and communicate mathematically

Learners should be able to:

- construct rigorous mathematical arguments (including proofs);
- make deductions and inferences;
- assess the validity of mathematical arguments;
- explain their reasoning; and
- use mathematical language correctly.

25%/20% of marks in A/AS level

DfE: Mathematics AS and A level content

	Content
A1	**[Understand and use the structure of mathematical proof, proceeding from given assumptions through a series of logical steps to a conclusion; use methods of proof, including proof by deduction, proof by exhaustion]** **[Disproof by counter example]** Proof by contradiction (including proof of the irrationality of $\sqrt{2}$ and the infinity of primes, and application to unfamiliar proofs)

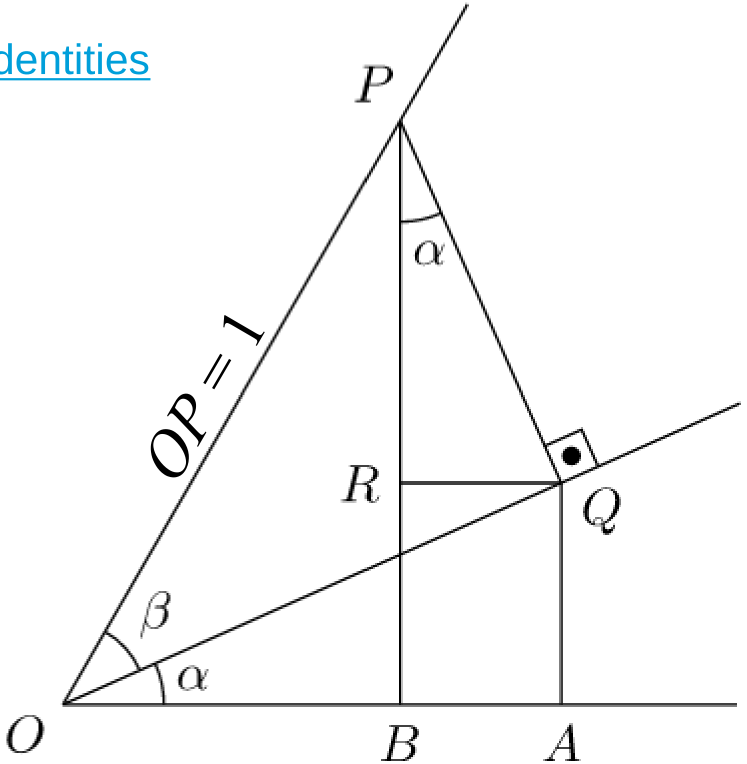
DfE: Mathematics AS and A level content

E6	Understand and use double angle formulae; use of formulae for $\sin(A \pm B)$, $\cos(A \pm B)$ and $\tan(A \pm B)$; understand geometrical proofs of these formulae Understand and use expressions for $a\cos\theta + b\sin\theta$ in the equivalent forms of $r\cos(\theta \pm \alpha)$ or $r\sin(\theta \pm \alpha)$
E7	
E8	Construct proofs involving trigonometric functions and identities

Question 13 from
https://filestore.aqa.org.uk/resources/mathematics/AQA-73571-SQP.PDF
with solution at
https://filestore.aqa.org.uk/resources/mathematics/AQA-73571-SMS.PDF

Trig angle sum identities

https://en.wikipedia.org/wiki/Proofs_of_trigonometric_identities



Cosine [edit]

Using the figure above,

$$OP = 1$$

$$PQ = \sin\beta$$

$$OQ = \cos\beta$$

$$\frac{OA}{OQ} = \cos\alpha, \text{ so } OA = \cos\alpha\cos\beta$$

$$\frac{RQ}{PQ} = \sin\alpha, \text{ so } RQ = \sin\alpha\sin\beta$$

$$\cos(\alpha+\beta) = OB = OA - BA = OA - RQ = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

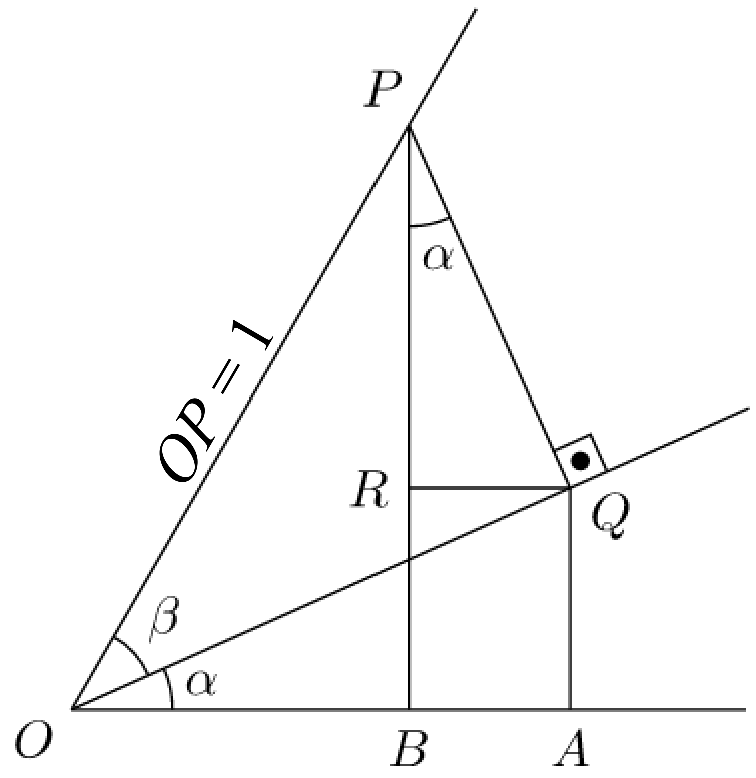
Trig angle sum identities

Where is $\sin(\alpha + \beta)$ on the diagram?

How can you write length *PQ*?

How can you write length *PR*?

Investigate length *QA*.



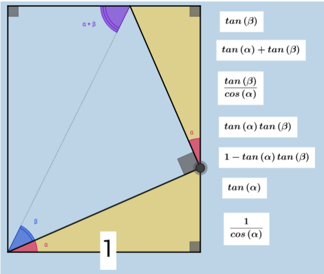
Comparing two approaches

- Which approach do you prefer?
- Is there room for both?

Three activities

There are 22 cards. They contain, muddled up, two different proofs that $\sqrt{2}$ is irrational. One of the cards has a mistake on it. Sort the cards into the right order so that you have two proofs.

https://www.geogebra.org/m/bR97TU37



Match the scatter diagrams, correlation coefficients and variables

Irrationality of $\sqrt{2}$

- There are 22 cards. They contain, muddled up, two different proofs that $\sqrt{2}$ is irrational. One of the cards has a mistake on it.
- Sort the cards into the right order so that you have two proofs.
- Compare the proofs. Which do you prefer, and why? Do you want to improve either of the proofs e.g. by adding extra lines or explanations?
- Adapt one of the proofs to show that $\sqrt{3}$, [$\sqrt{10}$, $\sqrt{12}$] is irrational.
- Where does each proof break down if you try to show that $\sqrt{4}$ is irrational?

Irrationality of $\sqrt{2}$

- You focus on the structure of the proof first – which cards go at the beginning and the end? – so you are learning about proof by contradiction before worrying about details.
- You have to think about the hcf=1 requirement in the standard proof and why you don't need it in the other proof.
- Working in pairs on this means that you are 'comprehending and critiquing mathematical arguments', both those on the cards and what your partner is saying.
- You get to compare proofs, which is an important follow up activity; this includes thinking how to extend the proofs to the irrationality of $\sqrt{3}$, $\sqrt{10}$ and $\sqrt{12}$ (all subtly different) and thinking about why they fail to prove that $\sqrt{4}$ is irrational.

Irrationality of $\sqrt{2}$

Reflections

What would you do during Years 12 and 13 so that your students would cope with this proof?

Irrational, primes and factorisation

A	A
G	G
J	D
E	J
P	E
H*	C
K	O
M	N
I	F
	B
	L
	M
	I
* Error in H	

A

Suppose, for a contradiction, that $\sqrt{2}$ is rational

G

That is, we can write $\sqrt{2} = \frac{m}{n}$ where m and n are integers and where $n \neq 0$

J

Squaring, we have $2 = \frac{m^2}{n^2}$

E

Multiply across to get $2n^2 = m^2$

P

In the prime factorisation of m^2 and n^2, 2 occurs to an even power

H

In the prime factorisation of $2n^2$, 2 occurs to an even power

K

But prime factorisations are unique, so 2 should appear to the same power in both $2n^2$ and m^2

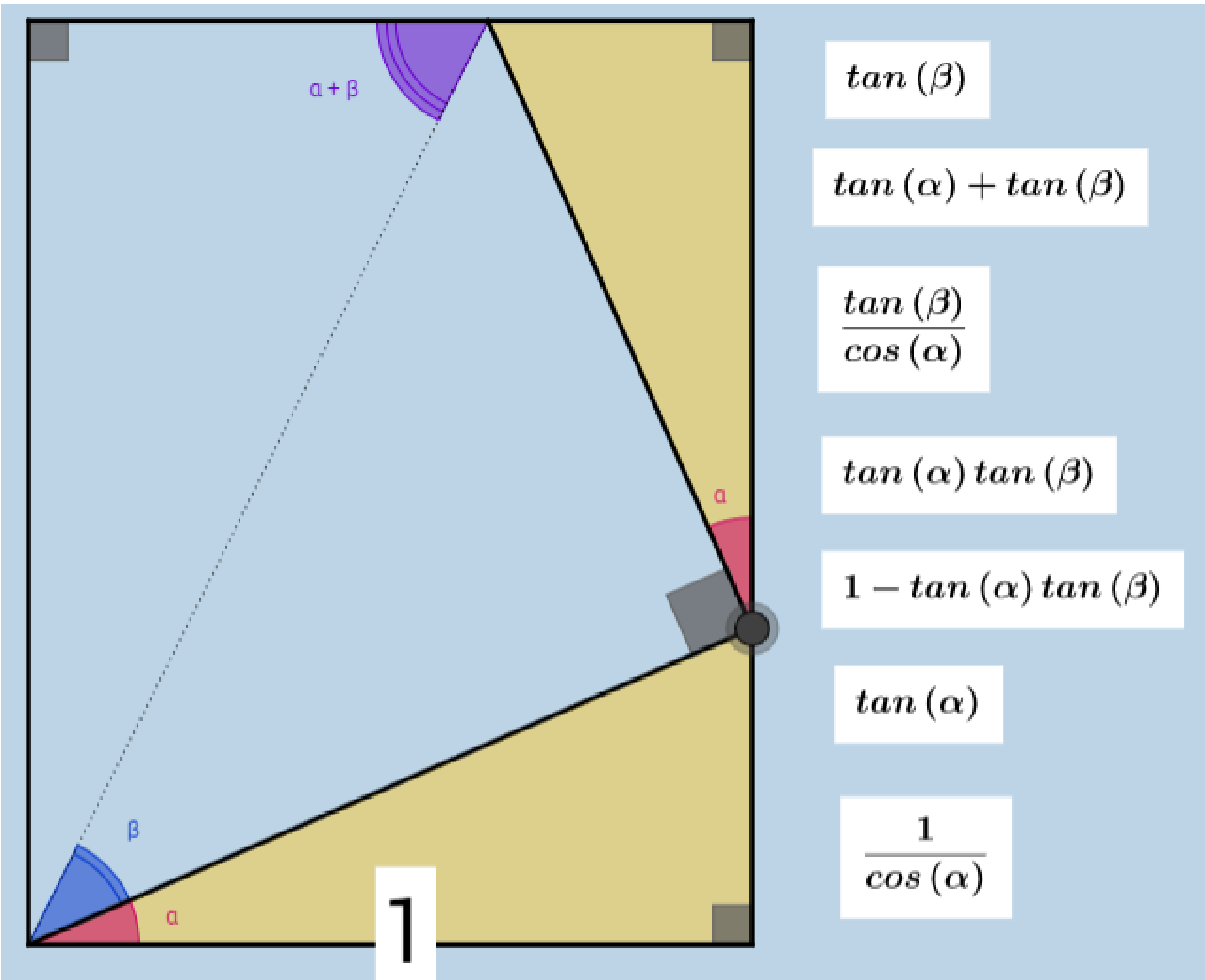
M

This is a contradiction, so our original assumption that $\sqrt{2}$ is rational must be wrong

I

So $\sqrt{2}$ is irrational

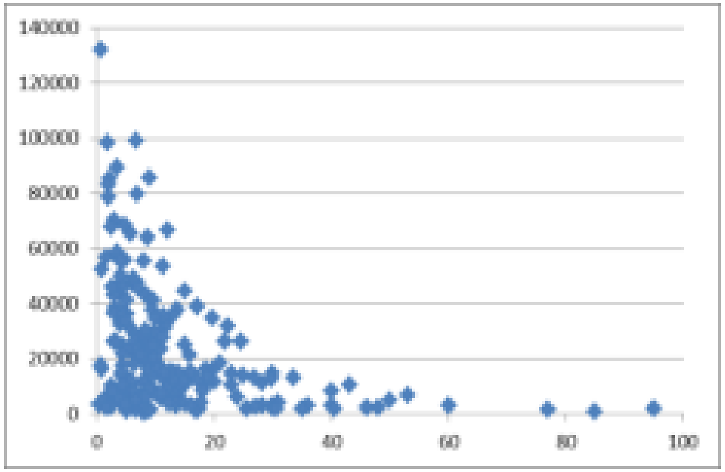
Trig angle sum identities (3)



https://www.geogebra.org/m/bR97TU37

Correlation

Land area v population



0.6302

Infinity of primes – developing a proof

Write down a few prime numbers $p_1, p_2, p_3, \ldots, p_n$.

Work out $P = p_1 p_2 p_3 \ldots p_n + 1$, the product of all your prime numbers plus 1.

What can you say about P?

Infinity of primes – developing a proof

Suppose, for a contradiction, that there is a finite number of prime numbers. Let $S = \{p_1, p_2, p_3, \ldots, p_n\}$ be the set of all the prime numbers.

Define $P = p_1 p_2 p_3 \ldots p_n + 1$

P is not divisible by any prime in S

Infinity of primes – an alternative proof

Suppose, for a contradiction, that there is a largest prime number, p.

Define $P = p! + 1$

P is not divisible by any prime number.

Observations

- The journey to formal written proof starts with discussion in the classroom
- Reasoning is always *about* something – it doesn't stand in isolation
- Look out for opportunities for reasoning and proof when solving problems
- The task itself and how a task is presented can affect the quality of discussion
- Expect slightly stricter mark schemes for setting out arguments
- Do some work on rational/irrational numbers, prime factorisation
- Don't forget statistical reasoning

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