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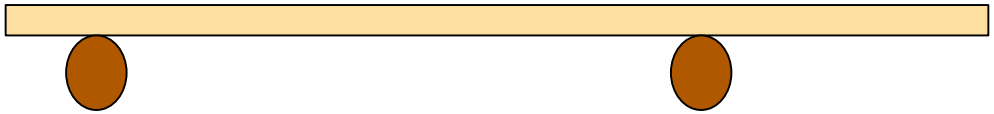
# While we are waiting to start…

- Balance the ruler horizontally on two finger tips:



- Predict what will happen when you try to move both fingers towards each other.
- Were you correct?
- Try starting with your fingers at different points. Can you explain what happens?

# A level Mechanics: Moments

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# Reformed A level content

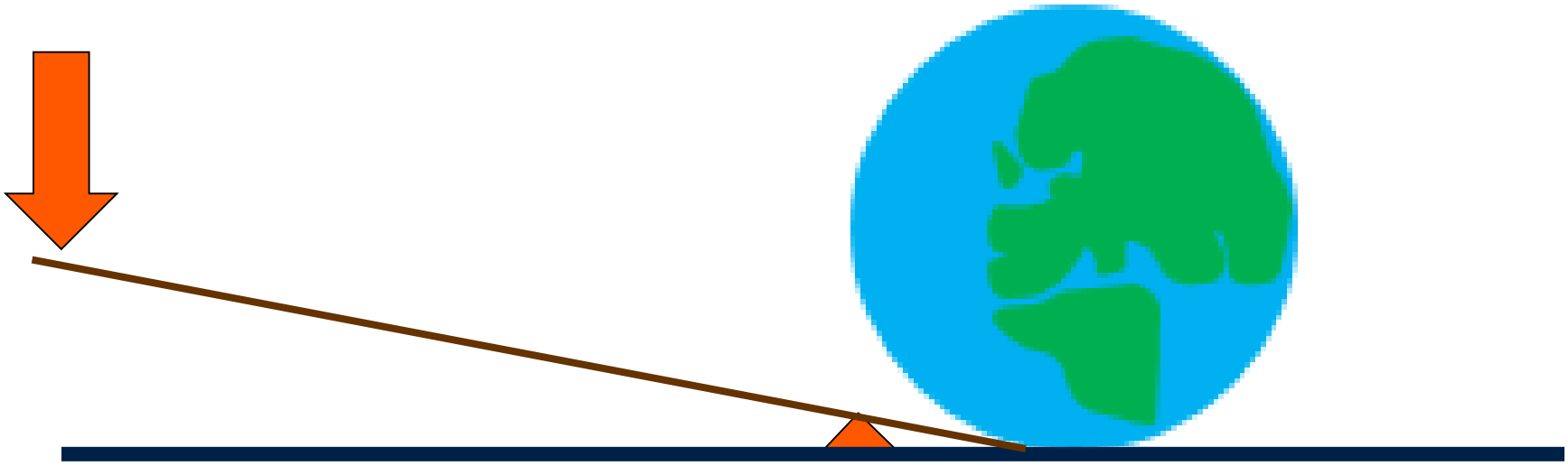
S: Moments

| Ref | Content description |
|---|---|
| **S1** | Understand and use moments in simple static contexts |

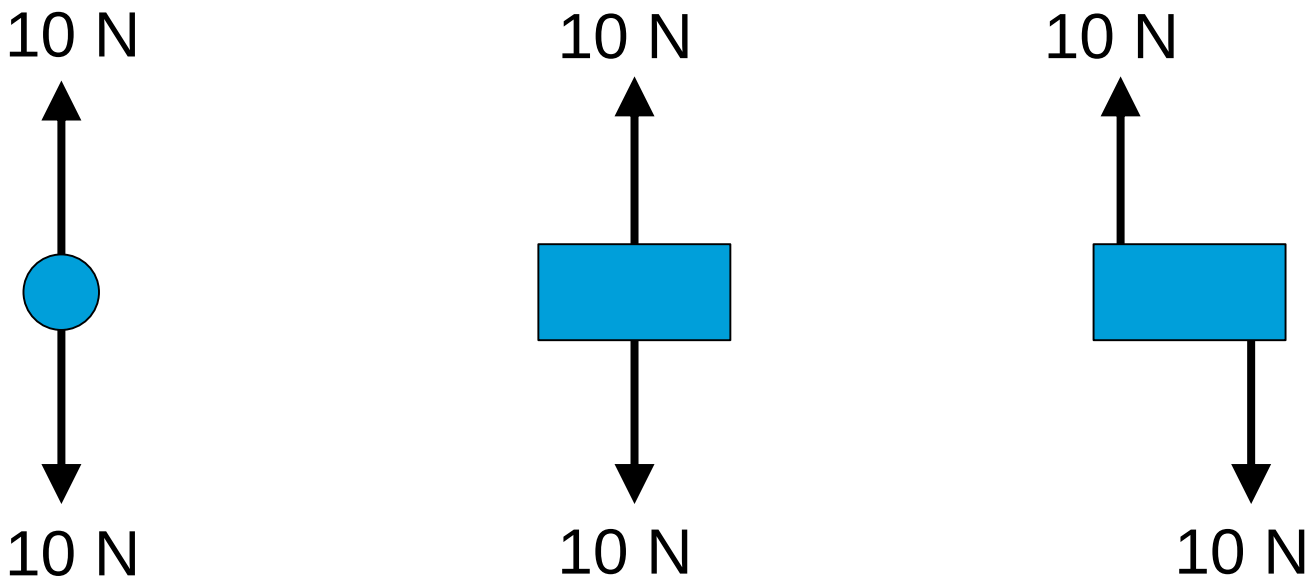
https://www.gov.uk/government/publications/gce-as-and-a-level-mathematics

# Archimedes

"Give me a place to stand and I will move the Earth"



# Equilibrium?

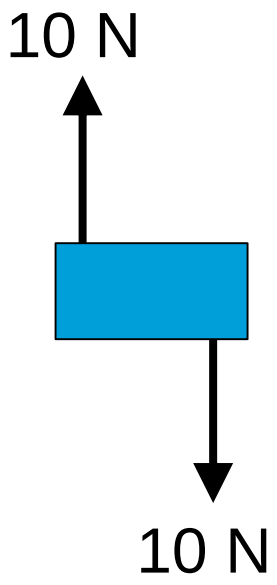


# Equilibrium?

The forces on the body are not passing through the same point.

We say they are not **concurrent**.

This causes the body to turn;
it stays in the same position but its orientation changes.



# Limitations of the particle model

- Situations which may be successfully modelled by particles might include
  - a book at rest on a slope
  - a book sliding down a slope
  - a squash ball thrown through the air

- Situations where a particle model might not be appropriate include
  - a ladder leaning against a wall
  - a tall lorry driving around a sharp bend
  - a board rubber flying through the air

# Key terminology

- Particle

  An object which has mass but is of negligible size.

- Rigid body

  An object with mass and size which does not change shape when forces are applied.

- Uniform body

  A body with the mass evenly distributed throughout. If a uniform body is symmetrical we can consider its weight to act at its geometric centre.

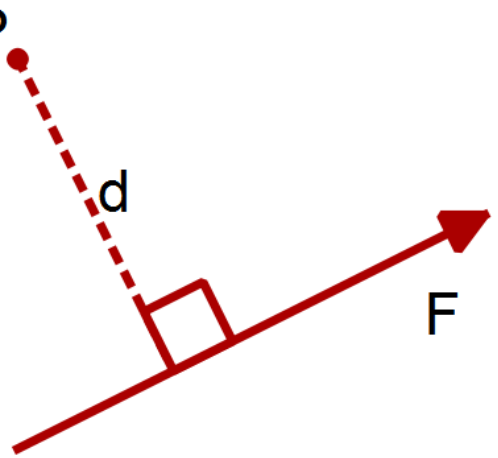
# Moments

- The moment of a force, about a given point or axis, is defined as the product of the force and the perpendicular distance of the line of action of the force from the axis.
- The force is in Newtons, the distance is in metres.
- The SI units of moments are Nm, Newton-metres

The moment of F about P is Fd.

Moment has sense.

In this case it is anti-clockwise.



# Conditions for static equilibrium

- For a particle
  - Total resultant force in any direction equals zero.

- For a rigid body
  - Total resultant force in any direction equals zero.
  - Total moment about any point equals zero.

# Moment calculations

A uniform wooden beam of mass 20 kg and length 3 m rests on two supports as shown. Find the reaction forces at each of the points A and B.

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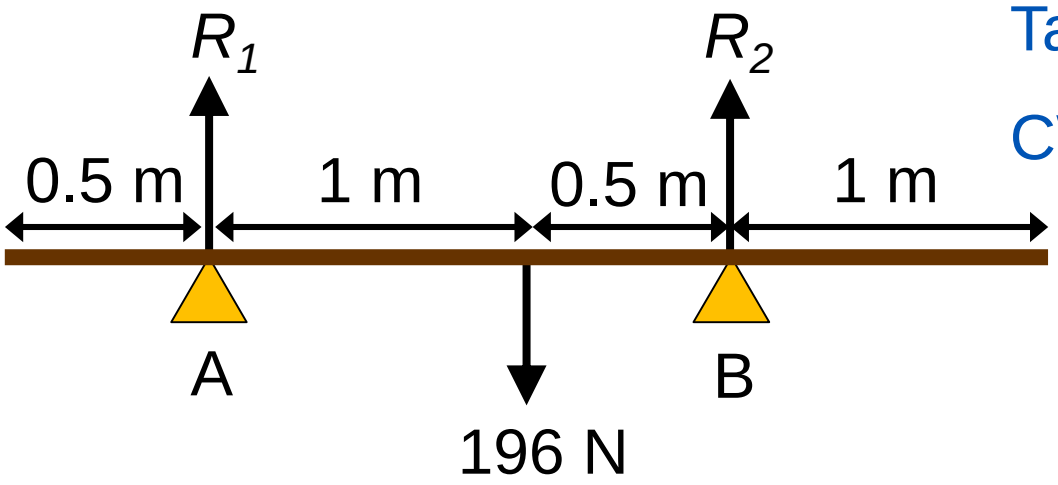


Taking moments about A:

CW moment = ACW moment

$196 \times 1 = R_2 \times 1.5$

$$R_2 = \frac{196}{1.5}$$

$= 131$ N

# Moment calculations

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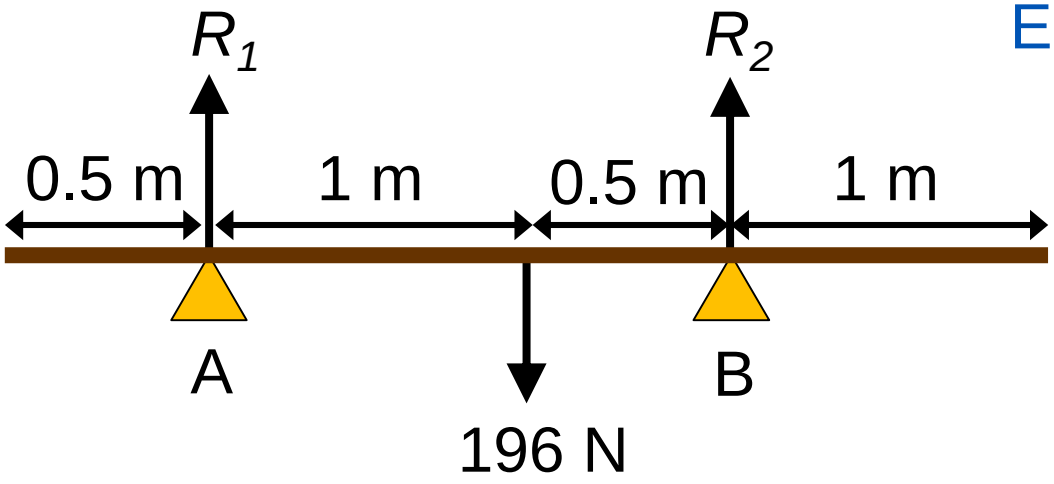



Equating vertical forces:

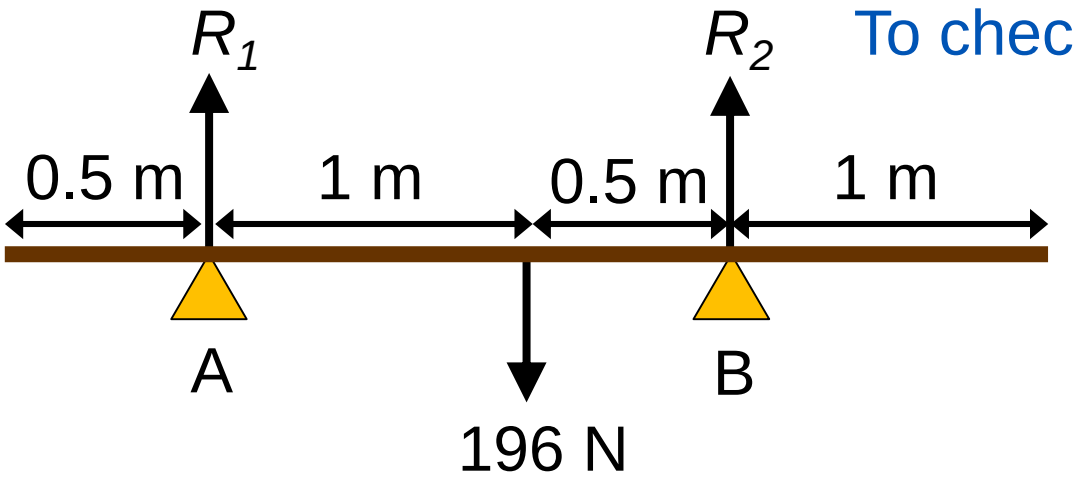
$R_1 + R_2 = 196$

$R_1 = 196 - 131$

$= 65$ N

# Moment calculations

A uniform wooden beam of mass 20 kg and length 3 m rests on two supports as shown. Find the reaction forces at each of the points A and B.




To check, take moments about B:

$196 \times 0.5 = R_1 \times 1.5$

$$R_1 = \frac{196 \times 0.5}{1.5}$$

$= 65$ N

# Moment calculations

A block of mass *m* kg is placed on the end of the beam as shown. The beam is now on the point of turning about A. Find *m*.

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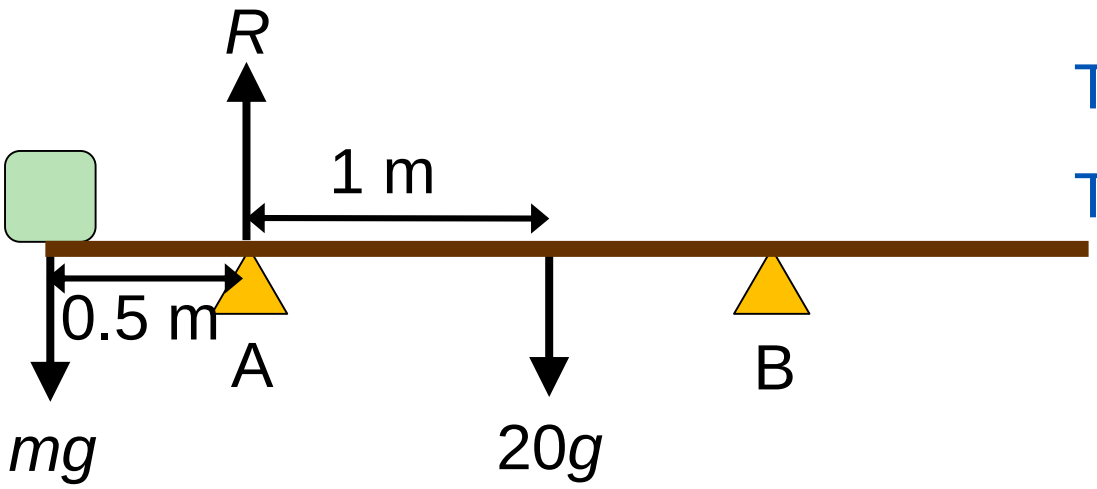
A block of mass $m$ kg is placed on the end of the beam as shown. The beam is now on the point of turning about A. Find $m$.




The reaction at B is now 0.

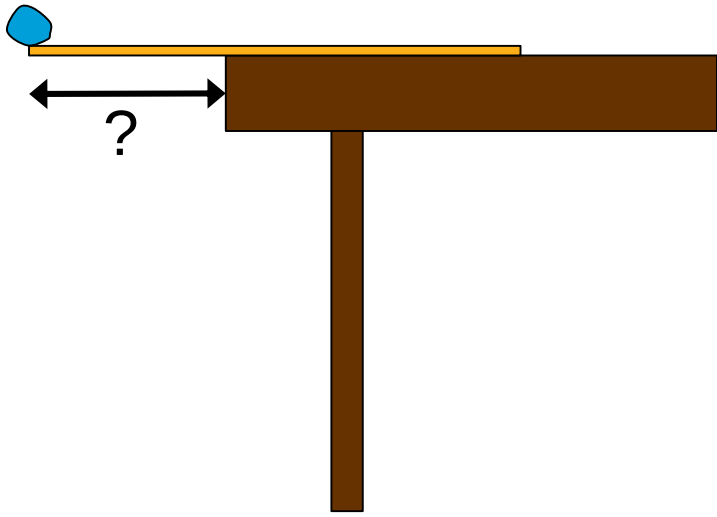
Taking moments about A:

$0.5mg = 20g$

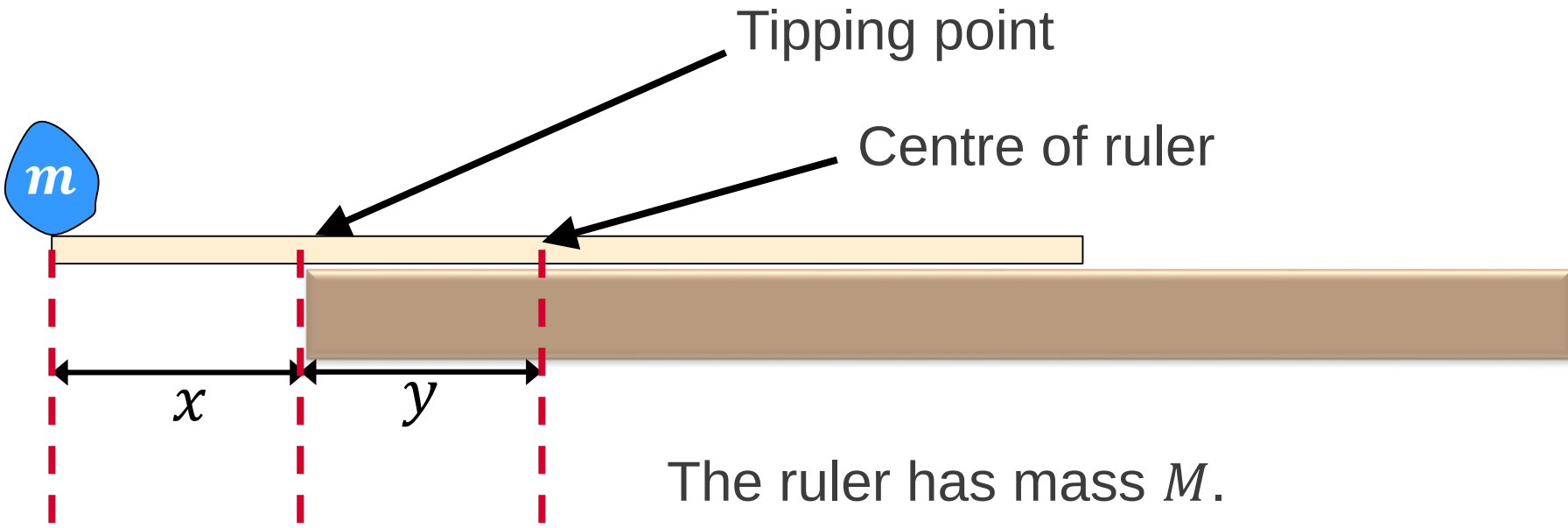
$m = 40$ kg

# Ruler practical activity

- Find the mass of the ruler.
- Check that the ruler balances at its centre.
- Place the ruler on the table edge with a lump of Blu-tack stuck to the end.
- Find the point at which the ruler is about to topple.
- Calculate the mass of the Blu-tack and check your answer.



# Ruler practical activity



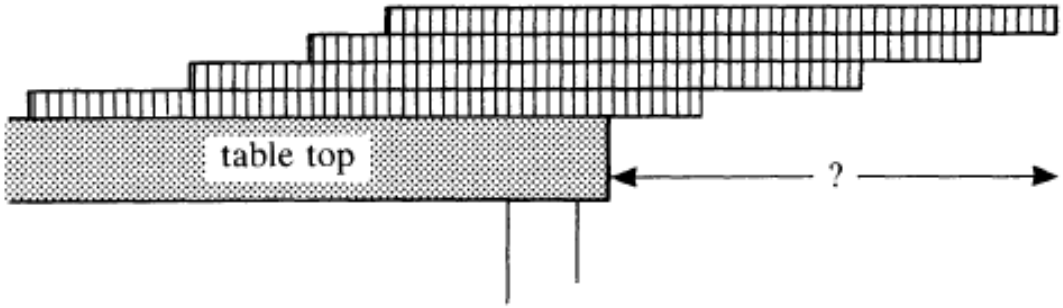

The ruler has mass $M$.

The Blu-tack has mass $m$

In theory,

$$mx = My$$

# Ruler practical activity 2

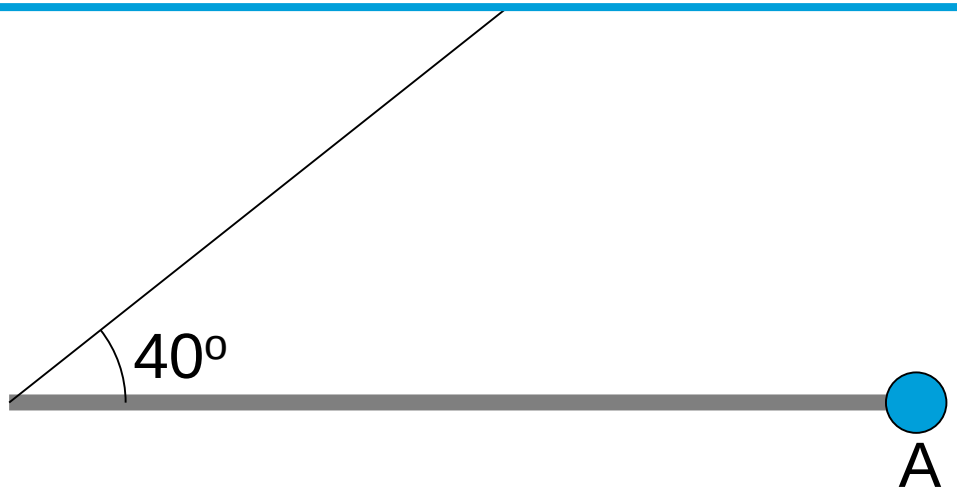



**Problem: how much overhang can you get?**

Extract from ‘Mechanics in Action’ (1990). A fantastic free resource available from the Stem Centre; you have to set up a free account and log in to access it.
https://www.stem.org.uk/system/files/elibrary-resources/legacy_files_migrated/3635-Mechanics%20in%20action.pdf

# Moment calculations

A uniform rigid steel rod of mass 10 kg is free to rotate around a fixed hinge at A. It is held horizontally by a light inextensible string as shown. Find the tension in the string.



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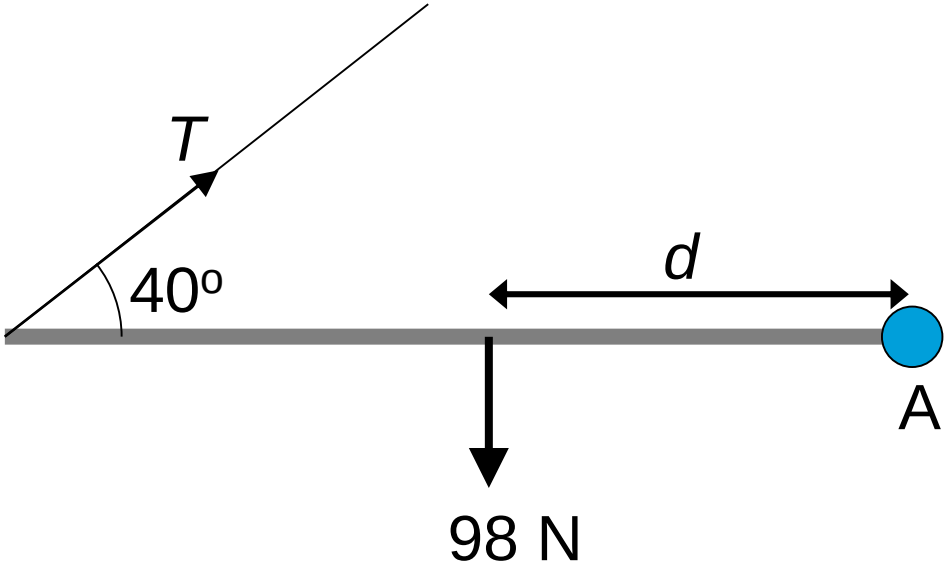



Taking moments about A:

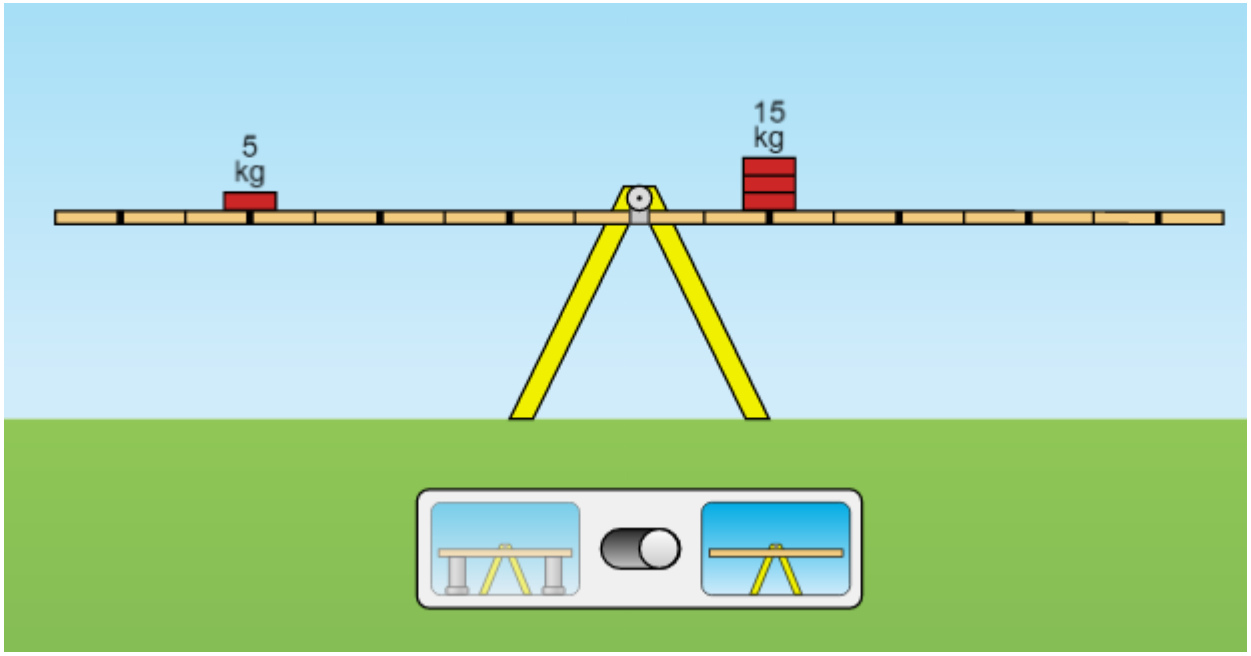
CW moment = ACW moment

$T \sin 40 \times 2d = 98 \times d$

$$T = \frac{98}{2 \sin 40}$$

$= 76.2$ N

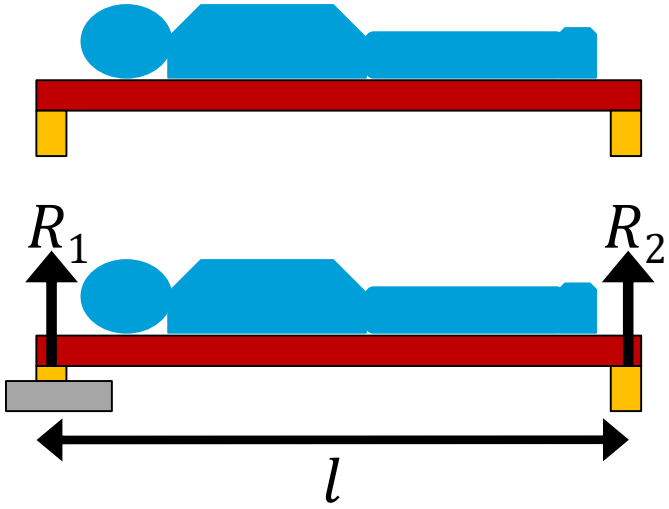
# PhET Colorado



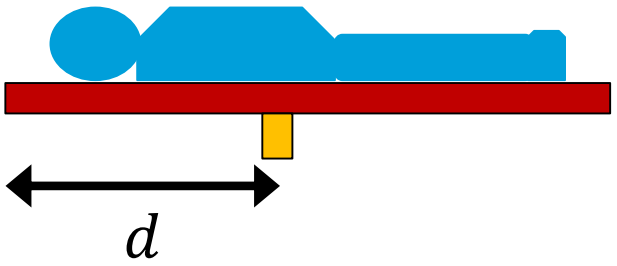

https://phet.colorado.edu/en/simulation/balancing-act

# Balancing a body

- Your volunteer lies on a plank of wood supported by two small blocks at either end. They must be very still!
- At each end in turn carefully remove the blocks and use smaller blocks and scales to measure the reaction forces $R_1$ and $R_2$.
- Take moments about the head end to locate the distance, $d$, of the centre of mass from the head end of the board.
- Place block supports at this point and carefully remove the supports from either end. The body should balance!




$$d = \frac{R_2 \times l}{R_1 + R_2}$$



# AMSP sustained courses

- Teaching Mechanics 1

Covers the mechanics content in A level Mathematics.

- Teaching Mechanics 2

Covers the mechanics content in A level Further Mathematics.

- For more information please visit:

http://furthermaths.org.uk/teaching-mechanics

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- We also pioneer the development of innovative teaching and learning resources