MEI Conference 2018

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Using graphing technology for teaching Further Mathematics

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Task 2 – Roots of polynomial equations

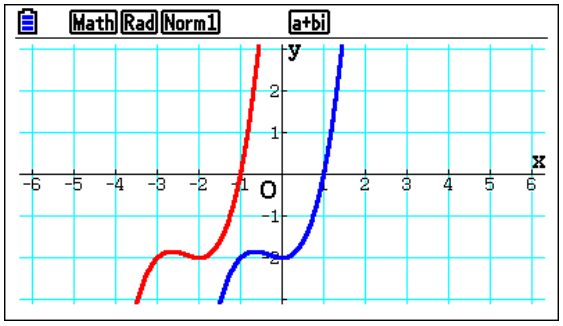
1. In Graphs mode plot the graphs $y = x^3 + x^2 - 2$ and $y = x^3 + 7x^2 + 16x + 10$.

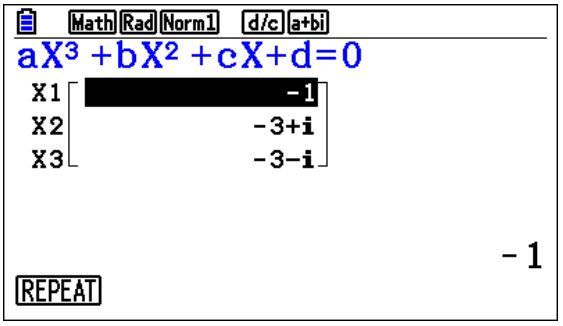
2. In Equation mode set the Complex output to **a+bi**.
   SHIFT > SET UP > Complex Mode: a+bi

3. Select Type: Polynomial (F2) and set the Degree to 3.

4. Set **a = 1**, **b = 1**, **c = 0**, **d = –2** and solve the equation.

5. Set **a = 1**, **b = 7**, **c = 16**, **d = 10** and solve the equation.





Questions for discussion

- What is the relationship between the graphs of $y = x^3 + x^2 - 2$ and $y = x^3 + 7x^2 + 16x + 10$?
- What is the relationship between the roots of $x^3 + x^2 - 2 = 0$ and $x^3 + 7x^2 + 16x + 10 = 0$?
- If $\mathrm{f}(x) = x^3 + x^2 - 2$ then $\mathrm{f}(x + a) = x^3 + 7x^2 + 16x + 10$. What is the value of a?
- Would the relationships hold if you solved $\mathrm{f}(x + a) = 0$ for other values of a?

Problem *(Try the problem with pen and paper first then check it on your software)*

The cubic equation $x^3 - 2x^2 + 3x + 1 = 0$ has roots α, β and γ. Find a cubic equation with integer coefficients that has roots $\alpha + 3$, $\beta + 3$ and $\gamma + 3$.

Further Tasks

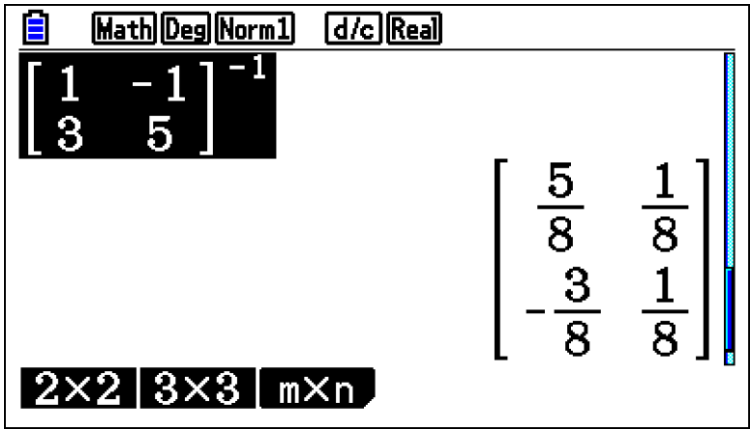
- For a given cubic equation, with roots $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$, find a cubic equation with integer coefficients that has roots $\boldsymbol{k\alpha}$, $\boldsymbol{k\beta}$ and $\boldsymbol{k\gamma}$ for different values of $\boldsymbol{k}$.
- Investigate the coefficients of related quartic equations.

Task 3 – Matrices: Determinants and inverse of 2×2 matrices

1. In Run-Matrix mode enter: $\mathrm{Det}\begin{pmatrix}1 & -1\\ 3 & 5\end{pmatrix}$

   Determinant: Option > MAT/VCT (F2) > Det (F3).
   Insert matrix: Press EXIT twice then Math (F4) > Mat (F1) > 2×2 (F1):

2. To find the inverse enter: $\begin{pmatrix}1 & -1\\ 3 & 5\end{pmatrix}^{-1}$.



Questions for discussion

- What is the relationship between the matrix, the determinant and the inverse?
- What is the answer when a matrix is multiplied by its inverse?
- Are there any matrices that don't have an inverse?

Problem *(Try the question with pen and paper first then check it on your calculator)*

For the matrices $\mathrm{A} = \begin{pmatrix}3 & 2\\ 1 & 4\end{pmatrix}$ and $\mathrm{B} = \begin{pmatrix}2 & -1\\ -5 & 4\end{pmatrix}$ find A^{-1}, B^{-1} and $(\mathrm{AB})^{-1}$.

Further Tasks

- For other 2×2 matrices, A and B, investigate the relationship between A^{-1}, B^{-1} and $(\mathrm{AB})^{-1}$.
- Investigate the determinants and inverse of matrices for standard transformations: reflection, rotation and stretches.

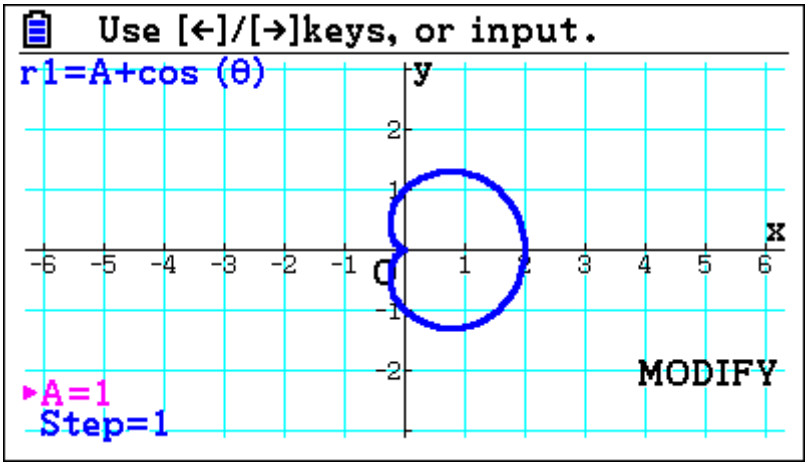
Task 5 – Polar curves

1. Go into Graphs mode and check the angle is set to radians:
   SHIFT > SET UP and set **Angle: Rad**

2. Select **TYPE (F3)** and **r= (F2)** to set the graph type to polar.

3. Enter the function $r1 = A + cos(\theta)$

4. Plot the curve using **MODIFY (F5)**.



Use ◀/▶ to vary **A**.

Questions for discussion

- What is the maximum/minimum distance from the pole and for what values of θ does this occur?
- How is this polar curve related to the Cartesian curve $y = A + \cos x$?

Problem *(Try the question with pen and paper first then check it on your calculator)*

Plot the following curves:

$r = 2 + \cos\theta$ $\quad$ $r = 2$ $\quad$ $r = 1 + \sin 2\theta$ $\quad$ $r = 3 + \cos\frac{\theta}{2}$ $\quad$ $r = 3 - 2\sin\theta$

Further Tasks

- For what values of θ does r take its maximum and minimum values? How can these be deduced from the polar equation?

- For which parts of the graph does r take negative values? What are the conditions such that $r = a + b\cos\theta$ and $r = a + b\sin\theta$ doesn't take negative values?

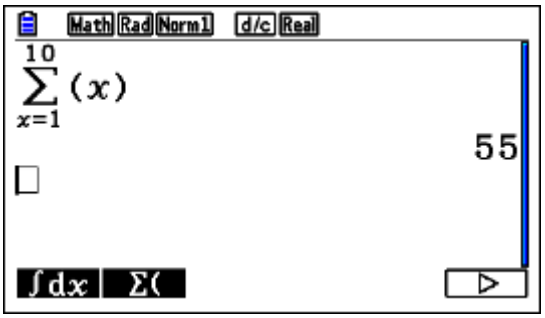
- The default setting is to plot values of θ from 0 to 2π (this can be changed with V-Window). Are there any curves for which this results in the same graph being traced over again? Are there any graphs for which the graph is incomplete using this range?

Task 6 – Summation of simple finite series

1. Go into Run-Matrix mode.
2. Add a summation sign:

   MATH (F4) > **►** **(F6)** > **Σ((F2)**

3. Calculate $\sum_{x=1}^{10} x$.



Investigate $\sum_{x=1}^{n} x$ for different values of n.

Questions for discussion

- How can $\sum_{x=1}^{n} x$ be expressed in terms of n?
- How can $\sum_{x=1}^{n} x^2$ be expressed in terms of n?
- How would you find an expression in terms of n for $\sum_{x=1}^{n} x(x+2)$?

Problem (*Try the question with pen and paper first then check it on your calculator*)

Find an expression for $\sum_{x=1}^{n} (x-1)(x+3)$ in terms of n and hence find $\sum_{x=1}^{10} (x-1)(x+3)$.

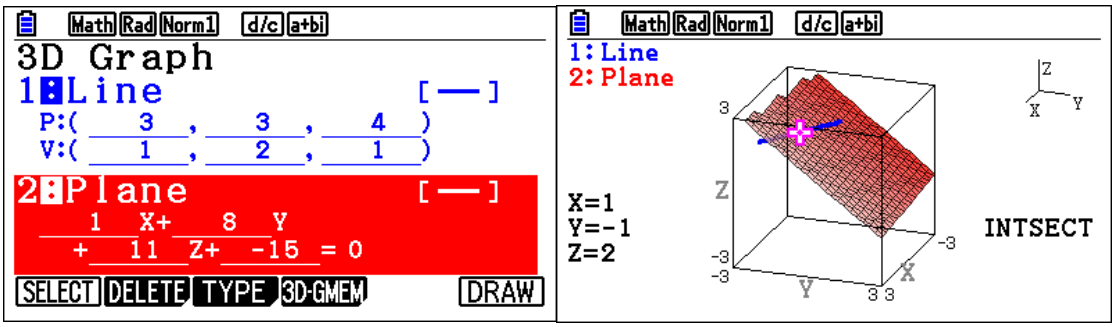
Further Tasks

- Investigate sums of the form $\sum_{x=m}^{n}$, i.e. sums that don't start from 1.
- Investigate $\sum_{x=1}^{n} x^3$ and find a relationship between this and $\sum_{x=1}^{n} x$.

Task 7 – Vector Geometry: Intersection of a line and a plane in 3D

1. Go into 3D Graph Mode.
2. Select **TYPE (F3) > Line > P&V (F3)** and enter the equation $\mathbf{r} = \begin{pmatrix}3\\3\\4\end{pmatrix} + \lambda\begin{pmatrix}1\\2\\1\end{pmatrix}$.

   Press **SET (F6)** to store it as equation 1.
3. Select **TYPE (F3) > Plane** and enter the equation $x + 8y + 11z - 15 = 0$.
   Press **SET (F6)** to store it as equation 1.
4. Press **DRAW (F6)** to plot the line and the plane.
5. Press **G-Solv (F6) > INTSECT (F2)** to find the point of intersection.



Questions for discussion

- How would you find this point of intersection algebraically?
- Are there any lines that don't intersect this plane?

Problem (*Try the question with pen and paper first then check it on your calculator*)

Find the point of intersection of the line $\mathbf{r} = \begin{pmatrix}-6\\4\\6\end{pmatrix} + \lambda\begin{pmatrix}4\\-3\\-3\end{pmatrix}$ and the plane $7x + 2y - 11z = 10$.

Further Tasks

- Investigate the line of intersection of two planes.
- Investigate the angle between a line and a plane.