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Further Pure with Technology

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FPT

Further Pure with Technology is an MEI A level Further Maths minor option with three topics:

- Investigation of curves (graphing/CAS)
- Differential Equations (graphing/CAS/spreadsheet)
- Number Theory (programming language)

More details:

mei.org.uk/fpt

Sample assessment

Conjecturing and proving

- There is a strong emphasis on students conjecturing based on the output of their software.
- They are then expected to be able to prove these results.

Plot the curves with equations:

$$x = a\cos t + 3\cos\frac{2t}{3}$$

$$y = a\sin t - 3\sin\frac{2t}{3}$$

where $0 \leq t < 6\pi$ for the cases $a = 2$, $a = 3$ and $a = 4$.

State any features:

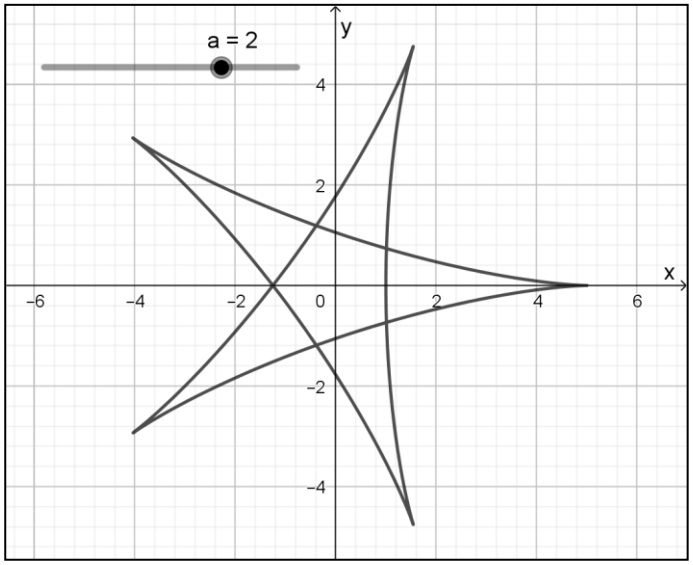
- common to all these curves;
- unique to one of these cases.

Features of curves

$$x = a\cos t + 3\cos\frac{2t}{3}$$

$$y = a\sin t - 3\sin\frac{2t}{3}$$

where $0 \leq t < 6\pi$ for the cases $a = 2$, $a = 3$ and $a = 4$.



- Important points: intersections with the axes and stationary points.
- Any symmetry.
- Whether the curve is bounded.
- Any asymptotes.
- Cusps or loops.

Computer Algebra System (CAS)

- A CAS can perform basic algebraic techniques automatically, such as:
  - solving;
  - factorising/expanding;
  - differentiating/integrating;
  - finding limits.

- This allows the focus to be on selecting techniques and interpreting the output.

Plot the tangent field for the differential equation:

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1 - 2xy}{x^2}$$
$$(x > 0, y > 0)$$

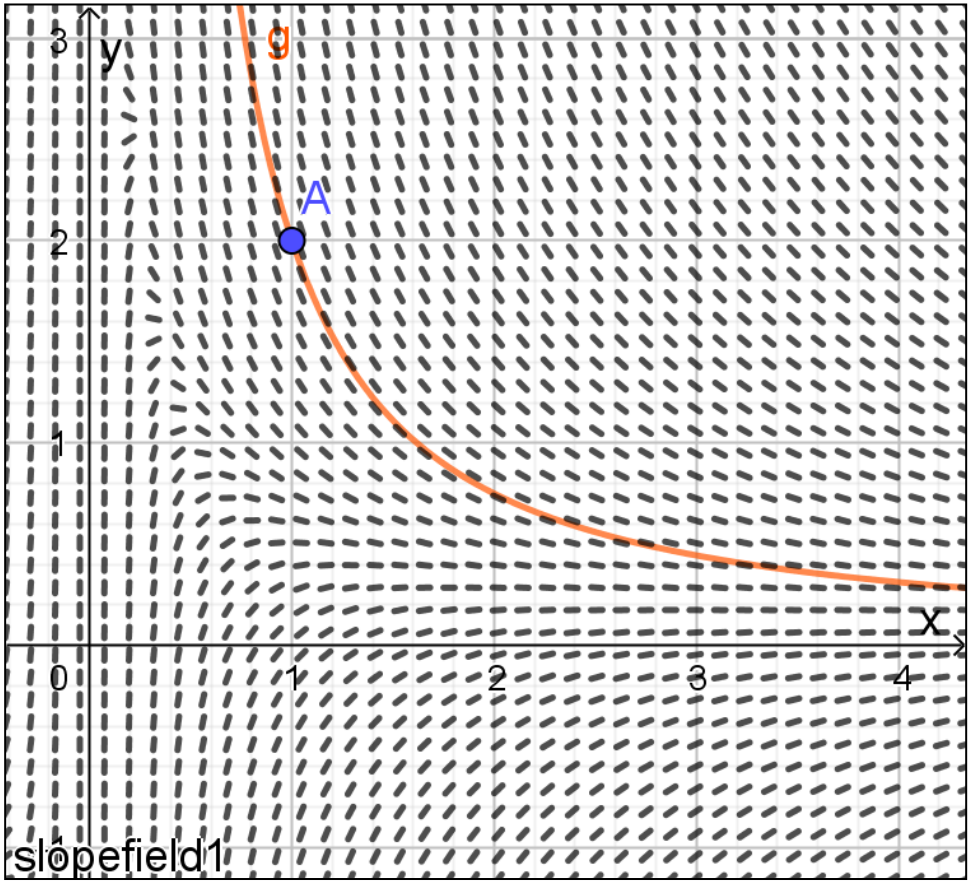
For what region of the plane will the particular solution through (x_1, y_1) be a decreasing function?

Differential equations in GeoGebra

- **Slopefield(f)**
- **SolveODE(f,A)**
- In CAS:
  SolveODE(y'=f)

Try these for

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1 - 2xy}{x^2}$$



Performing lots of calculations

- Technology can perform a large number of calculations quickly and accurately.
- This allows for systematic searches.

Write a program to determine whether a positive integer n can be written as the sum of the squares of two integers a and b: $n = a^2 + b^2$.

Use your program to investigate which odd positive integers can be written as the sum of the squares of two integers.

Using Python for number theory

- Python can be downloaded from: www.python.org/

- Online editor at: repl.it/languages/python3

```
def squares(n):
    for a in range(0,n):
        for b in range(a,n):
            if (a*a+b*b)==n:
                print(n,a,b)
```

Investigation of curves

- Plotting families of curves (cartesian, polar or parametric) and investigating properties (using CAS).
- Working with chords, tangents and normals to curves.
- Using limits to determine asymptotes and cusps.
- Finding arc lengths on curves.
- Finding envelopes of families of curves.

Differential Equations

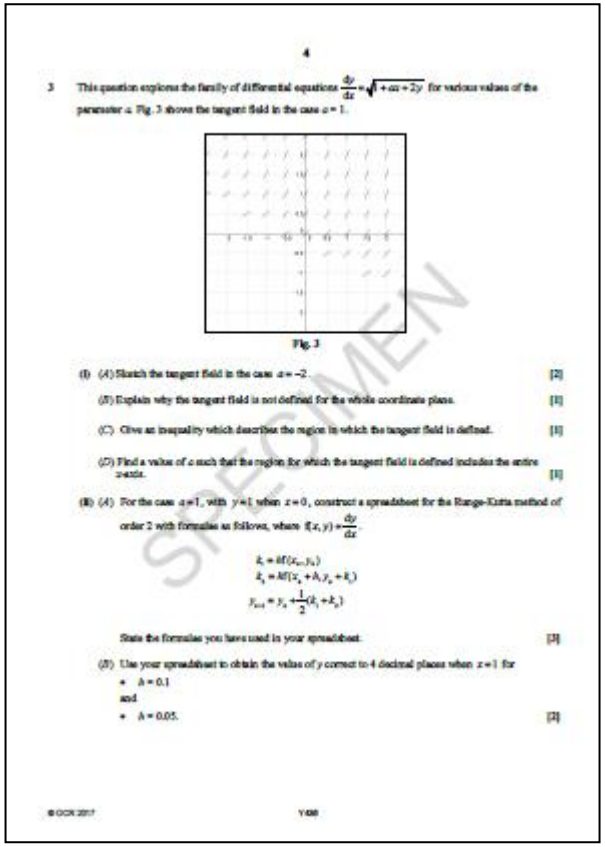
- Plotting and interpreting tangent fields for differential equations.
- Solving differential equations analytically (using CAS) and working with families of particular solutions.
- Solving differential equations numerically using the Euler and Runge-Kutta methods (using a spreadsheet).

Number theory

- Writing programs to search for solutions to number theory problems (using Python).
- Use unique prime factorisation.
- Modular arithmetic (including Fermat's Little Theorem, Euler's totient function, Euler's theorem and Wilson's theorem).
- Diophantine equations (including Pythagoras' theorem and Pell's equations).

Assessment

- A timed written paper that assumes that students have access to the technology.
- For the examination each candidate needs access to a computer with appropriate software and no communication ability.



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3 This question explores the family of differential equations $\frac{dy}{dx} = \sqrt{x^2 + ax + 2y}$ for various values of the parameter a. Fig. 3 shows the tangent field in the case $a = 1$.

Fig. 3

(i) (A) Sketch the tangent field in the case $a = -2$. [2]

(B) Explain why the tangent field is not defined for the whole coordinate plane. [1]

(C) Give an inequality which describes the region in which the tangent field is defined. [1]

(D) Find a value of a such that the region for which the tangent field is defined includes the entire x-axis. [1]

(ii) (A) For the case $a = 1$, with $y = 1$ when $x = 0$, construct a spreadsheet for the Runge-Kutta method of order 2 with formulas as follows, where $f(x, y) = \frac{dy}{dx}$.

$k_1 = hf(x_n, y_n)$

$k_2 = hf(x_n + h, y_n + k_1)$

$y_{n+1} = y_n + \frac{1}{2}(k_1 + k_2)$

State the formulas you have used in your spreadsheet. [3]

(B) Use your spreadsheet to obtain the value of y correct to 4 decimal places when $x = 1$ for

- $h = 0.1$

and

- $h = 0.05$. [2]

Offering FPT

FPT can be offered:

- As one of three minor options (preferably in year 13)
- As an additional minor option with the best scores counting
- Extension course (not certificated) for non-MEI centres

Resources:

- Textbook (June 2018)
- Integral resources (summer 2018)
- OCR practice papers (via OCR interchange)
- Old FPT resources (past papers and Integral)
- Possible live online course (2018/19)

About MEI

- Registered charity committed to improving mathematics education
- Independent UK curriculum development body
- We offer continuing professional development courses, provide specialist tuition for students and work with employers to enhance mathematical skills in the workplace
- We also pioneer the development of innovative teaching and learning resources