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Fractals

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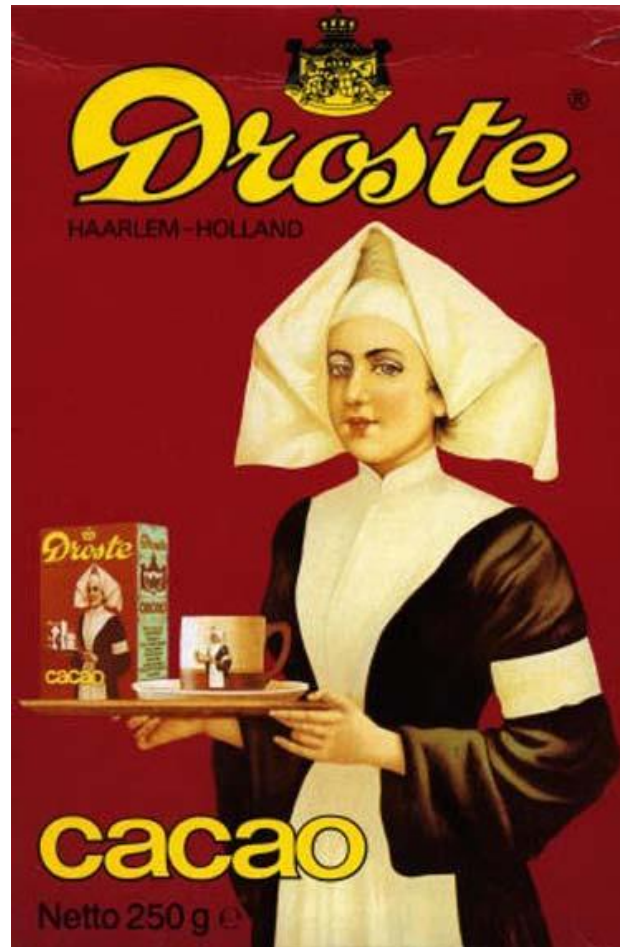
Fractals

- A fractal is a shape made of parts similar to the whole in some way
- Fractals are natural phenomena or a mathematical sets that exhibit a repeating pattern that displays at every scale.

Self Similarity

- a **self-similar** object is exactly or approximately similar to a part of itself (i.e. the whole has the same shape as one or more of the parts).
- many objects/structures in the real world are approximately self-similar at different scales, such as trees, vegetables, river systems and coastlines.

The Droste Effect

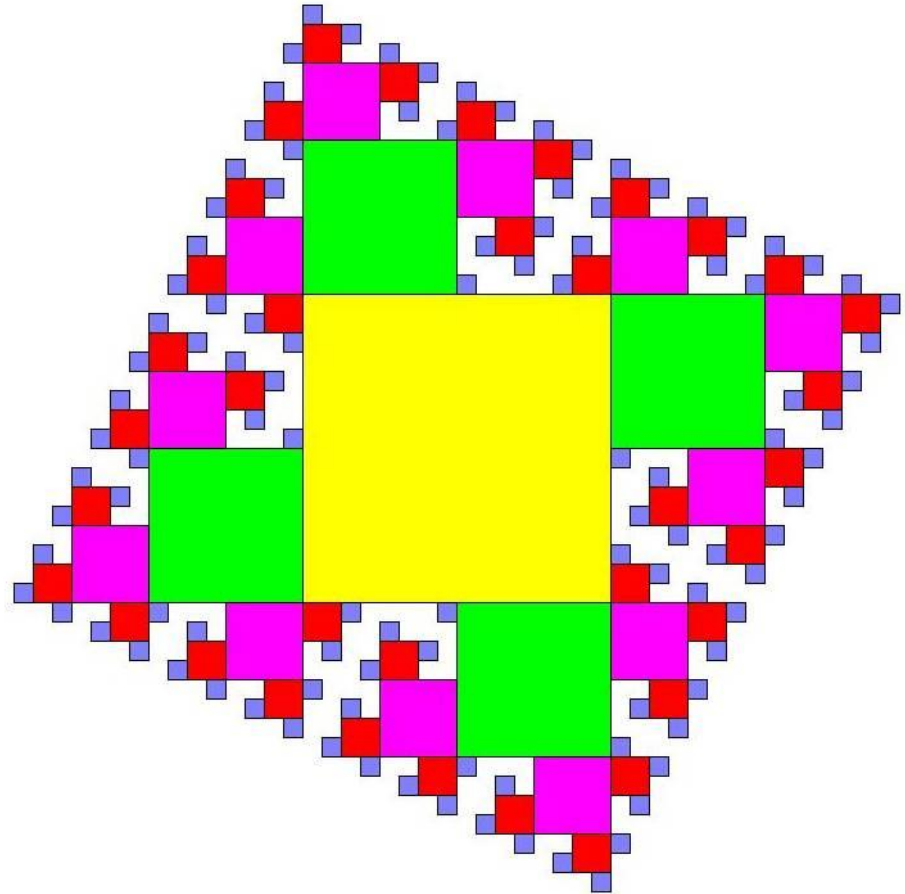


Fractal Patterns

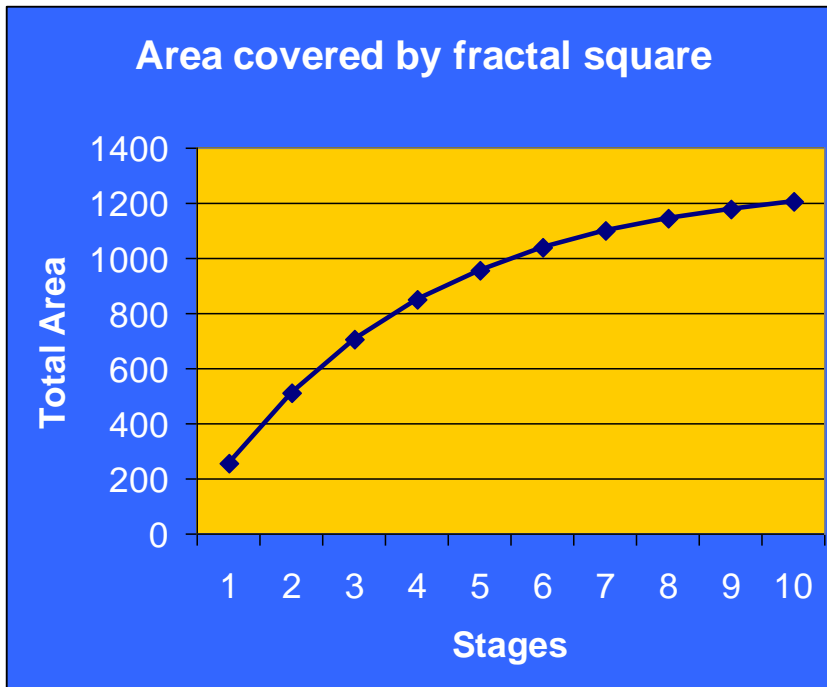
- Sequences
- Fractions and proportions
- Area
- Perimeter
- Limits
- Concept of Infinity

Fractal Patterns

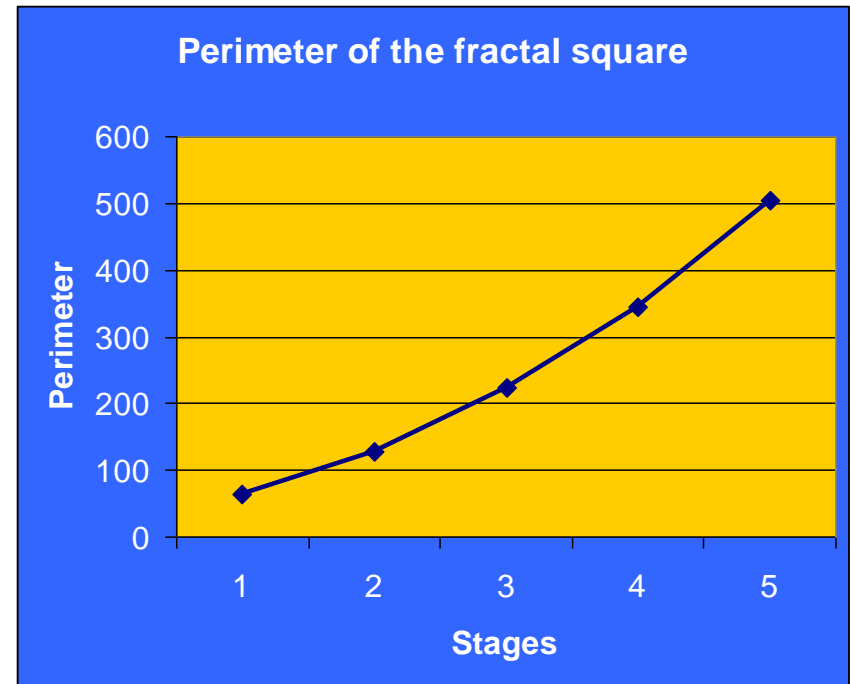
- Cantor Set
- Sierpinski Carpet
- Square Pattern



Conclusion



The area is leveling off to a limit which is 5 times that of the original square.



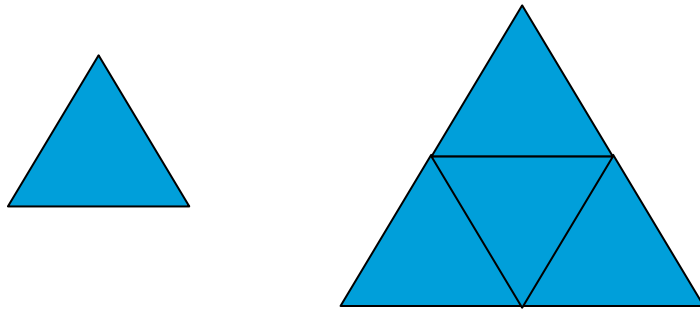
The perimeter is growing more and more and has no limit. The further you go the longer it becomes.

How long is the UK coastline?



Fractal Dimensions

- measure for “roughness”



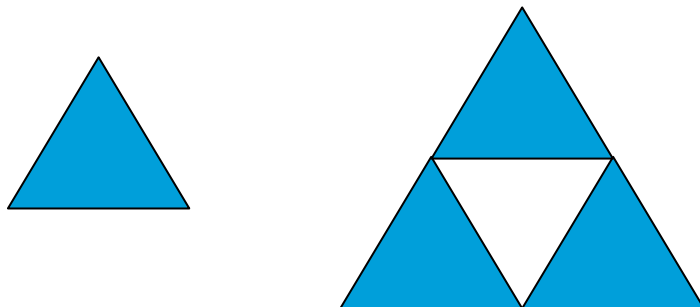
Scaling Factors

$$L \text{ sf} = 2$$

$$A \text{ sf} = 4$$

$$(L)^D = A$$

$$D = 2$$



Scaling Factors

$$L \text{ sf} = 2$$

$$A \text{ sf} = 3$$

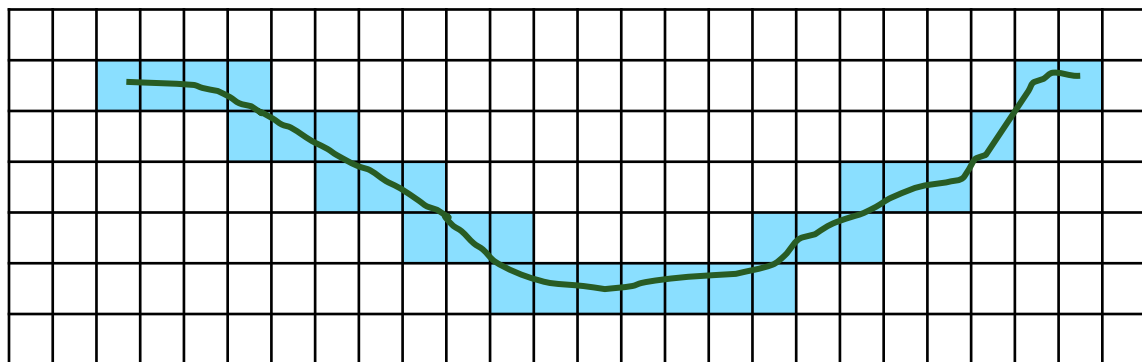
$$(L)^D = A$$

$$D = 1.58$$

Fractional Dimension

- Cantor Set
- Sierpinski Carpet
- Koch Curve
- Coastline

Counting Squares



Estimating Fraction Dimension

- Number of squares (original) = N_0
- Number of squares (scaled version) = N_1

- Scale Factor = S
- Dimension = D

$$\frac{N_1}{N_0} = S^D$$

Estimating Fraction Dimension

- Perimeter of Triangle $D = 1$
- Area of Circle $D = 2$
- Length of South Coast of England $D = 1.21$

$$\frac{N_1}{N_0} = S^D$$

Fractional Dimension

- Cantor Set $D = 0.63$
- Sierpinski Carpet $D = 1.89$
- Koch Curve $D = 1.26$
- Coastline - England $D = 1.21$
- Coastline - Norway $D = 1.52$

Useful Links

- Chaos Game – GeoGebra file by Ben Sparks
<https://ggbm.at/yr2XXPms>
- *Fractals are typically not self-similar*
a youtube video by 3blue1brown - Grant Sanderson
<https://www.youtube.com/watch?v=gB9n2gHsHN4>
- Think Maths
<http://www.think-maths.co.uk/downloads/fractal-activity-sheets>
- Cut the Knot www.cut-the-knot.org

About MEI

- Registered charity committed to improving mathematics education
- Independent UK curriculum development body
- We offer continuing professional development courses, provide specialist tuition for students and work with employers to enhance mathematical skills in the workplace
- We also pioneer the development of innovative teaching and learning resources

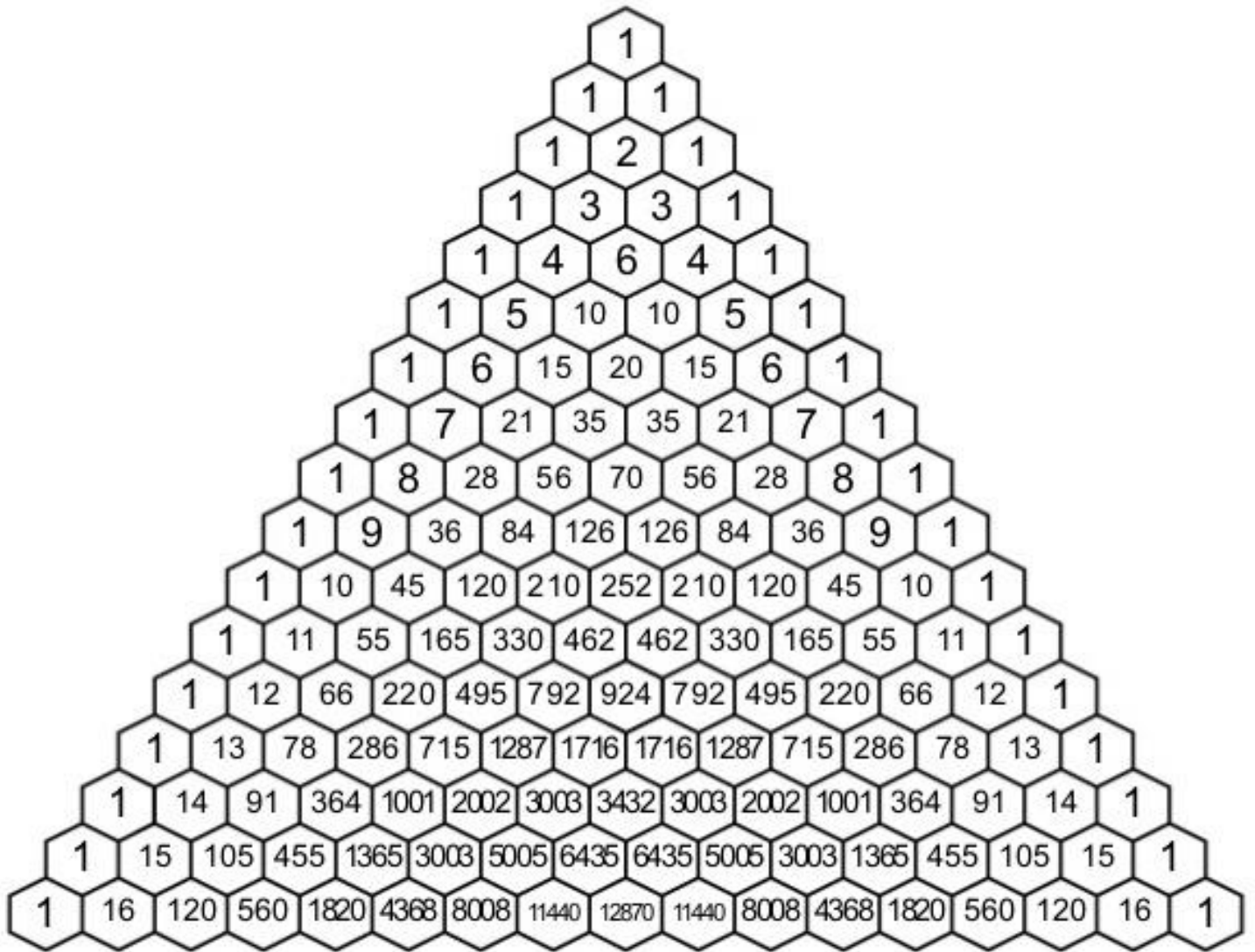


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Fractals

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1 or 2

[Pascal's Triangle](#)

[Chaos Game](#)

Self-similar patterns

Cantor Set

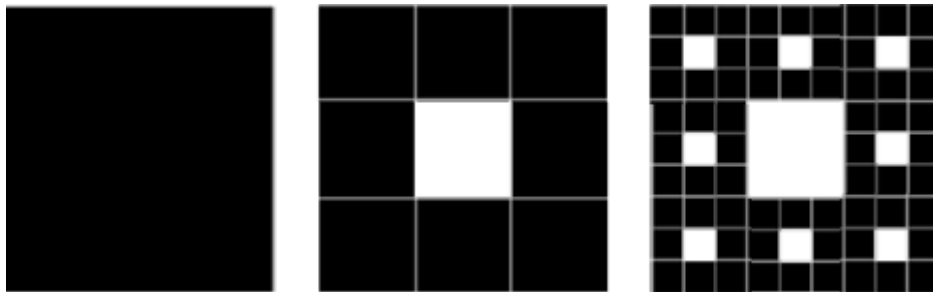
Starting with a straight line of any length, remove the middle third. Repeat this with the remaining line segments.



1. Continue drawing the patterns in the Cantor Set.
2. What fraction of the original length of the line remains after each step?
3. How many steps is it until less than 10% of the line remains?
4. How much of the line will be left if you were to continue removing the middle third of each line segment forever?

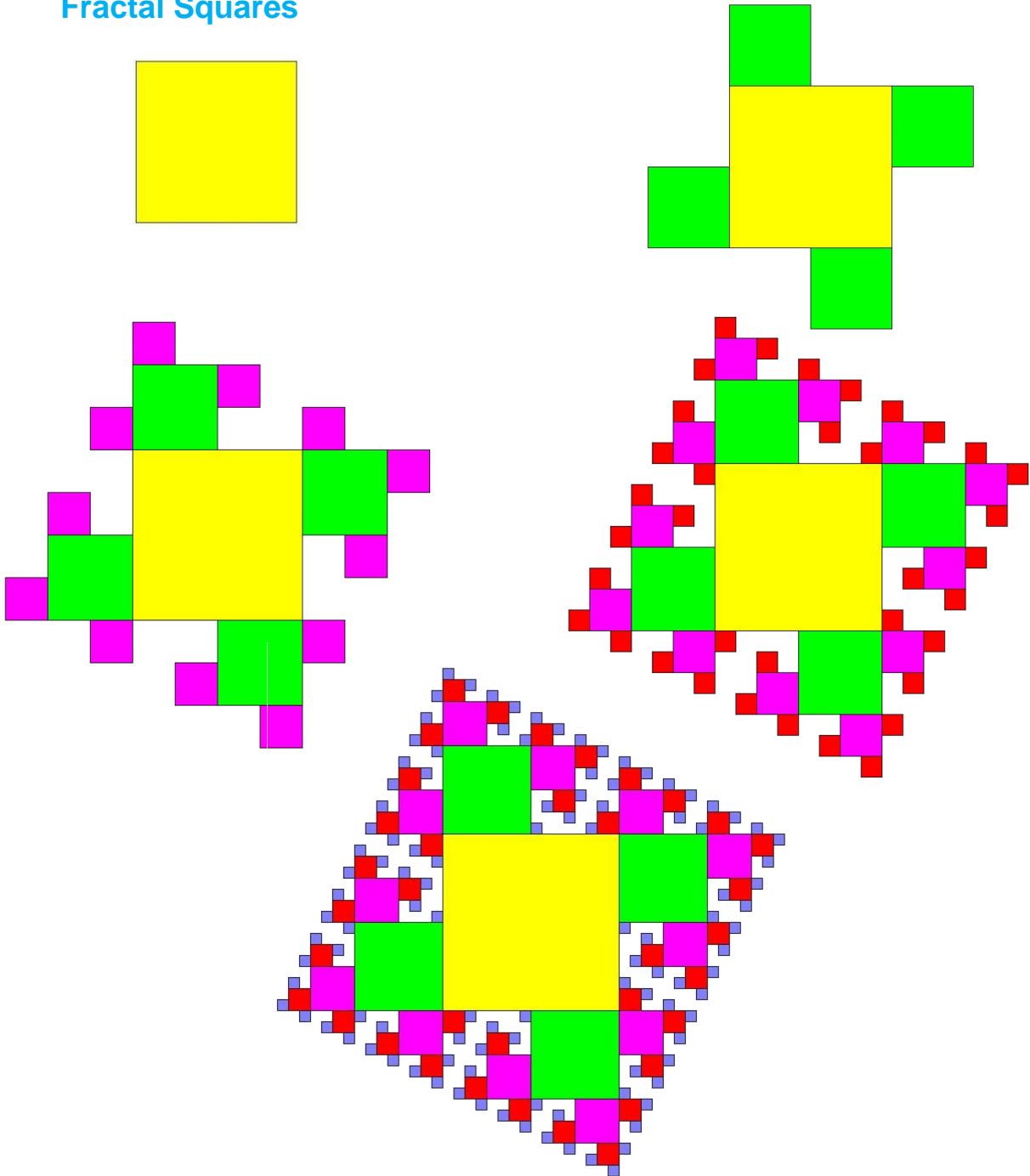
Sierpinski Carpet

Starting with a square, divide it into 9 smaller squares and remove the central square. Repeat this with each of the smaller squares.



1. Continue drawing the patterns in the Sierpinski carpet.
2. What fraction of the original area of the square remains after each step?
3. After how many steps is there less than 10% of the area remaining?
4. How much of the area will be left if you were to continue removing squares forever?

Fractal Squares

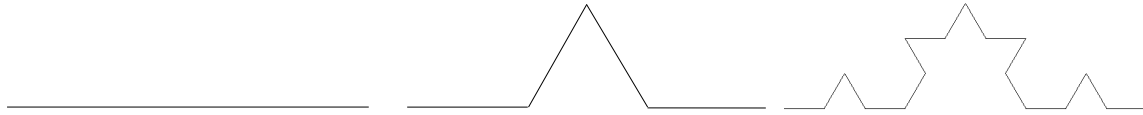


This pattern is created by adding smaller squares of side length half that of the squares in the previous stage. Suppose the first square is 32cm by 32cm.

1. What is the area covered by the squares in each stage?
2. What is the perimeter of the shape in each stage?
3. If you were to continue to add smaller and smaller squares forever, what would be the area and perimeter of the resulting shape?

Koch Curve

Starting with a straight line of any length, replace the middle third with two sides of an equilateral triangle. Repeat this with each resulting line segment.



1. Draw the next pattern in the sequence.
2. What fraction of the original length of the line remains after each step?
3. After how many steps is the length of the curve double the original length?

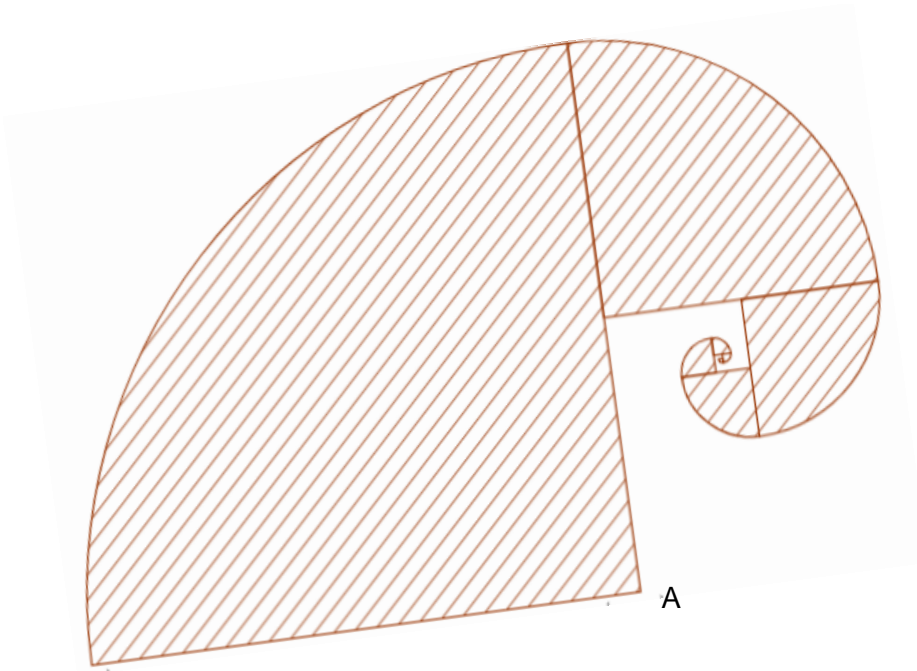
Menger Sponge

A Menger sponge is a 3D version of the Sierpinski Carpet. It is made by splitting a cube into 27 smaller cubes and removing the cube from the centre and the cubes from the centre of each face.



1. What fraction of the volume of the original cube remains at each stage?
2. If the original cube has sides of length 27 units, what is its surface area?
3. What is the surface area of the Menger sponges at each stage?
4. If you were to continue removing smaller and smaller cubes forever, what would happen to the volume and surface area of the sponge?

Square Spiral



The shape is made up of quarter-circles stuck together, each successive one is half the radius of the previous one. The radius of the first quarter-circle is 1 unit.

There is a smooth outer spiral and an inner rectilinear spiral.

1. What is the length of the smooth outer spiral?
2. What is the length of the rectilinear inner spiral?
3. What is the area enclosed between the two curves?
4. How far is the point where the two spirals converge from A?

Fractal Dimension

