MEI Conference 2018

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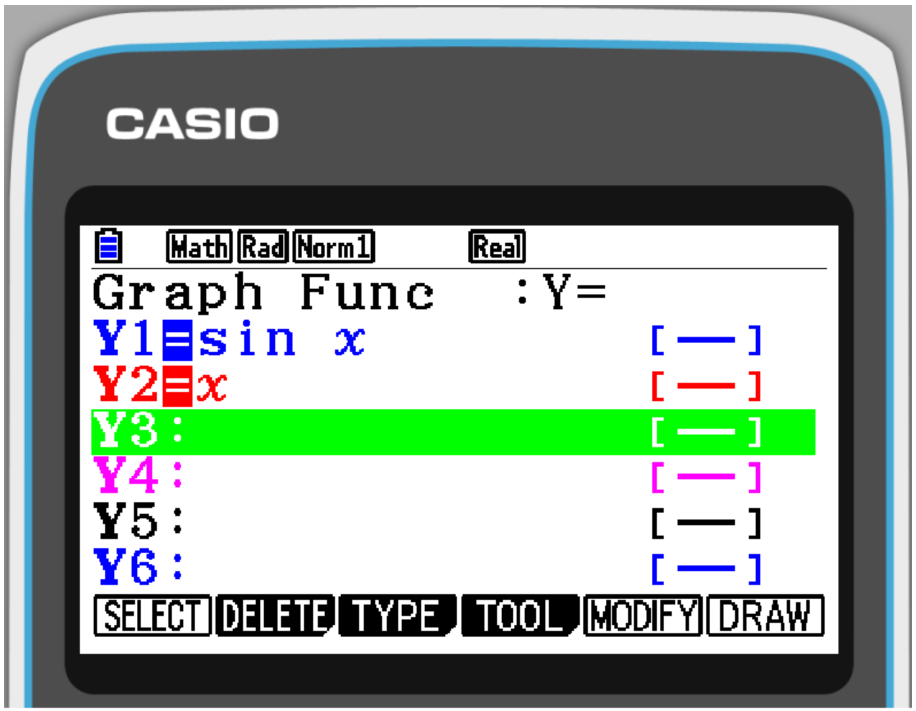
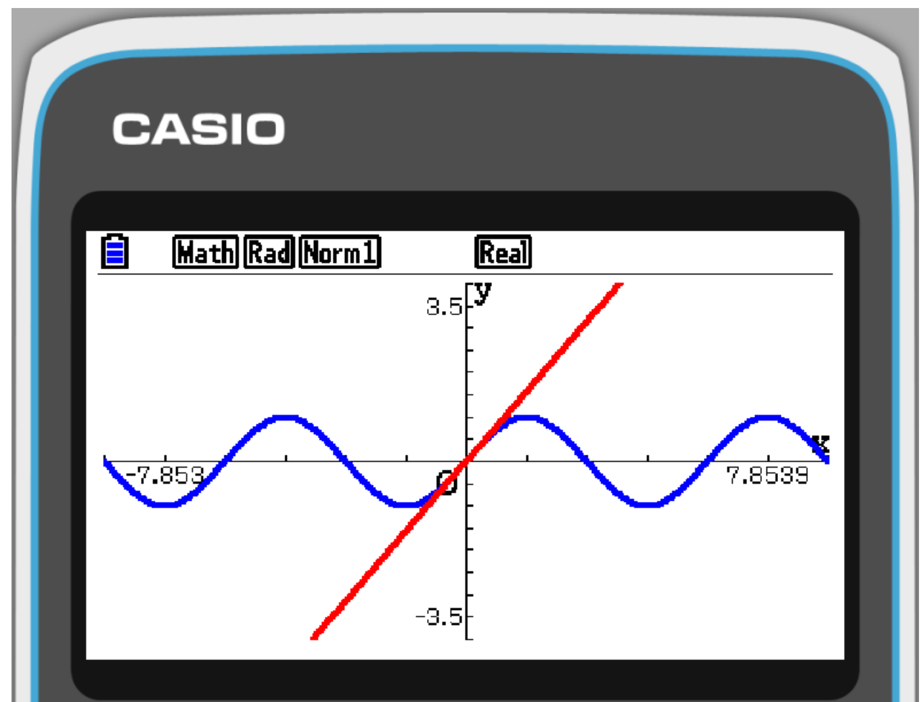
Using graphing technology for teaching calculus

In this session, we will look at ways of incorporating student use of graphing technology into the learning of a range of calculus topics from across A level Mathematics. Suitable for all teachers who have taught the pure content of A level Mathematics. Graphical calculators will be provided for use during the session.

Sketch $y=x\sin x$ showing turning points





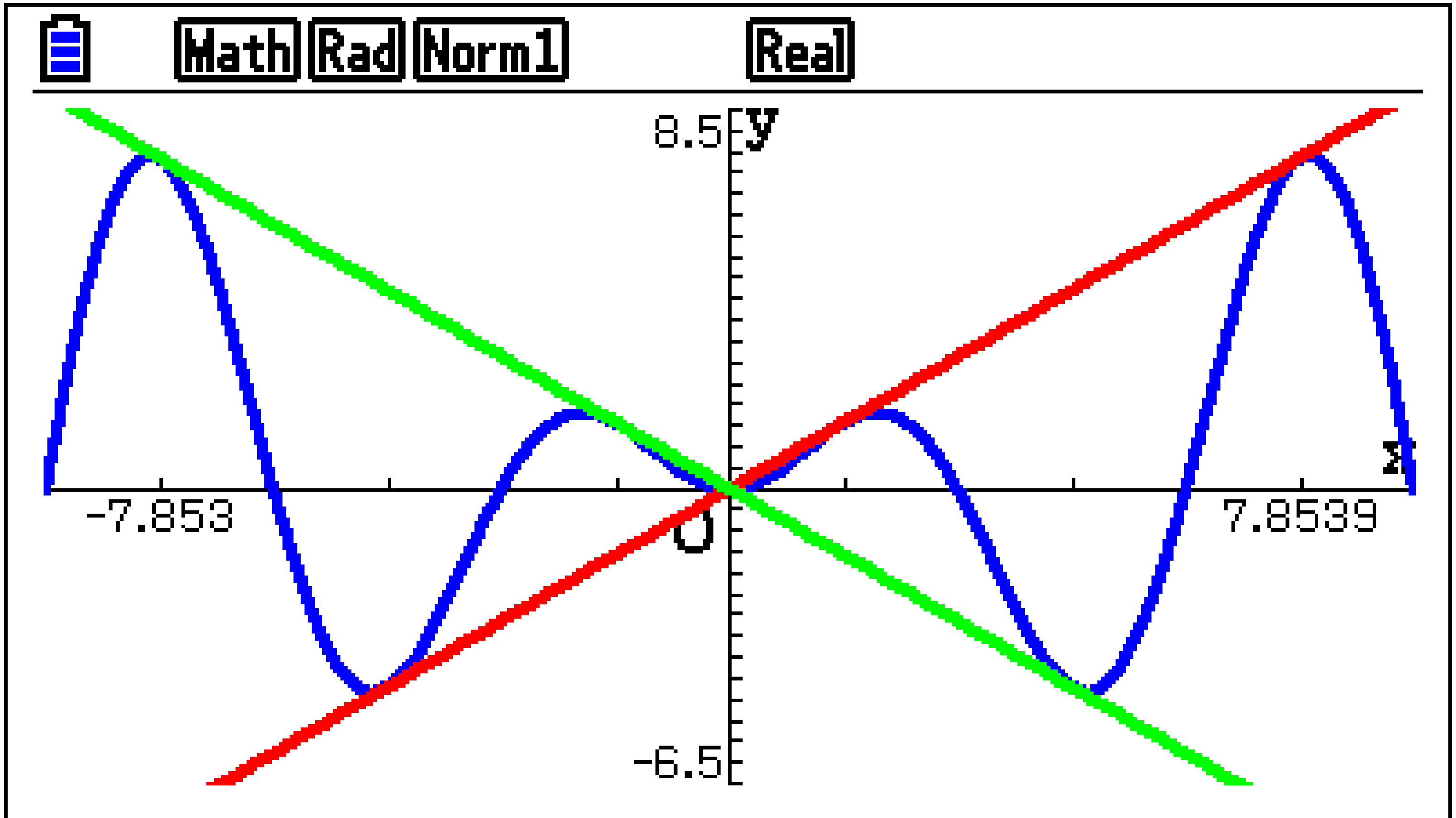
First reset your calculator to factory settings:
[MENU] [ALPHA] [⌫] [F5] [F2] [F1] [EXIT]

Add a new Graphs screen: [MENU] [5]

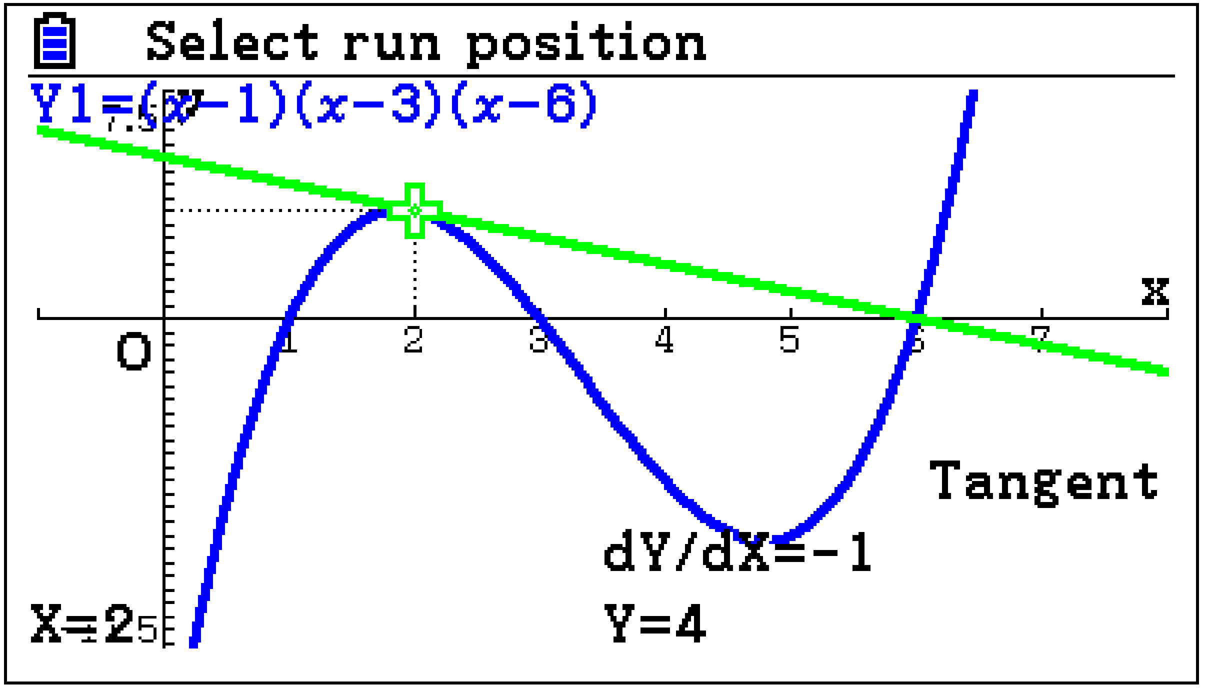
Set Angle: Radians:
[SHIFT] [MENU] scroll down using ▼ to Angle then [F2] [EXIT]

Draw the graph $y = x\sin x$: [X,θ,T] [sin] [X,θ,T] [EXE] [F6]

Use V-Window [SHIFT] [F3] to set appropriate ranges for x and y using the ▼ cursor, [EXE] after each entry and [EXIT] to finish.


Math
Rad
Norm1
Real
8.5
y
-7.853
7.8539
x
O
-6.5

A

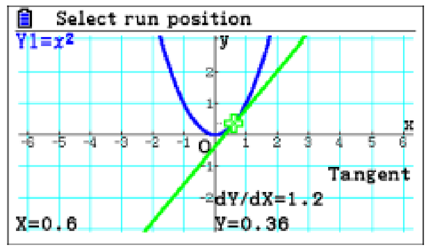


The point has been chosen as the midpoint of two roots.

Does that tangent pass through the other root?

Task A: Differentiation – Exploring the gradient on a curve

1. Add a new Graphs screen: [MENU] [5]
2. Switch the derivative on: [SHIFT] [MENU] ▼ ▼ ▼ ▼ ▼ ▼ [F1] [EXIT]
3. Add the function Y1 = x² and draw it: [X,θ,T] [x^2] [EXE] [F6]
4. Add the tangent at the point: Sketch > Tangent [F4] [F2]



Use ◀ / ▶ to move the position of the point on the curve.

Question

- How is the gradient of the tangent to the curve at a point related to the point?

Verify your comments by trying some other curves of the form $y = ax^2 + b$. You might find it useful to examine a table of values of the gradient: [MENU] [7] [F6]. (Compare X and Y'1).

To add/edit curves toggle the Graphs/Text screens with [F6].

Problem

What is the relationship between a point on the curve and the gradient of the tangent to the curve at that point for:

$y = x^3$ $y = x^4$ $y = x^5$ $y = x^n$

Further Tasks

Investigate the relationship between a point on the curve and the gradient of the tangent to the curve at that point for:

- $y = ax^2$ or $y = ax^3$
- $y = ax^2 + bx + c$ or $y = ax^3 + bx^2 + cx + d$

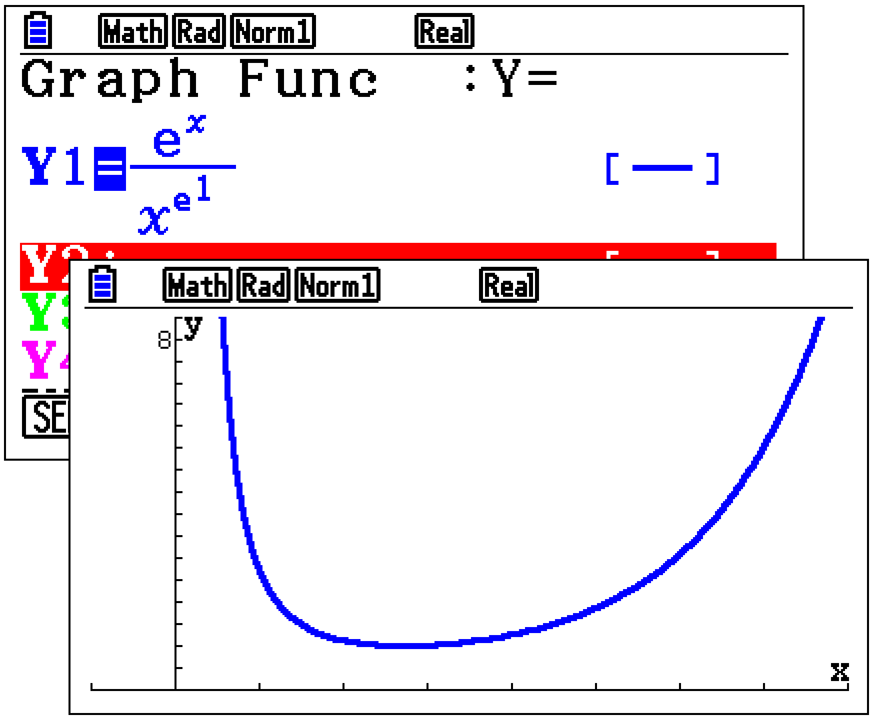
A: $e^{\pi} > \pi^{e}$

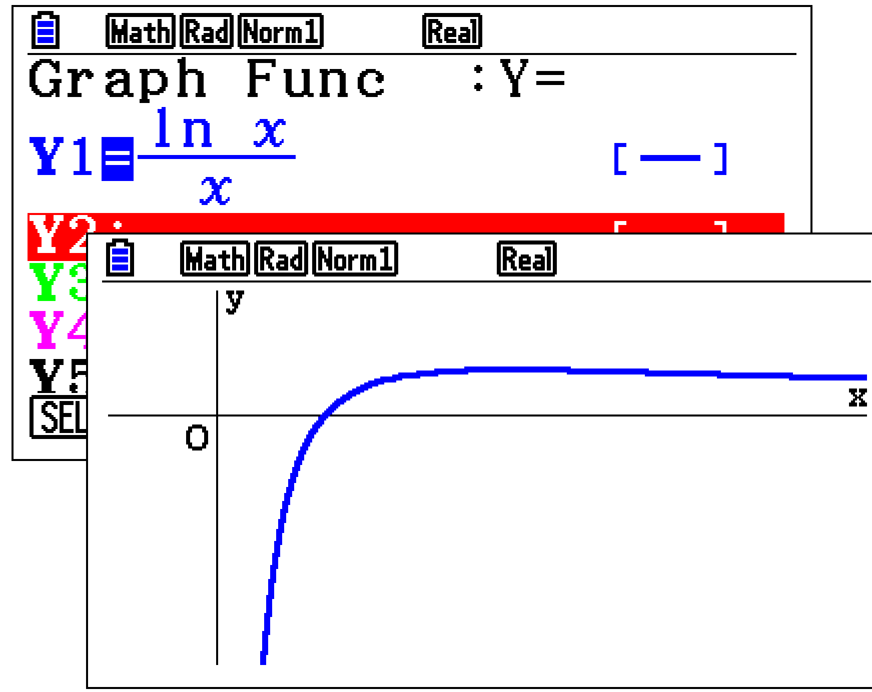
B: $e^{\pi} < \pi^{e}$

C: $e^{\pi} = \pi^{e}$

B

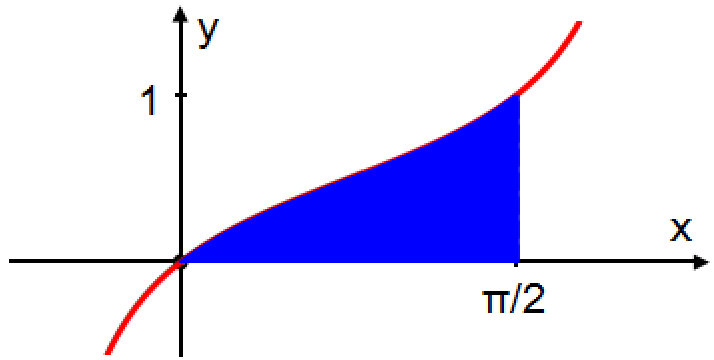
$$e^{\pi} > \pi^{e} \quad \text{or} \quad e^{\pi} < \pi^{e}\,?$$



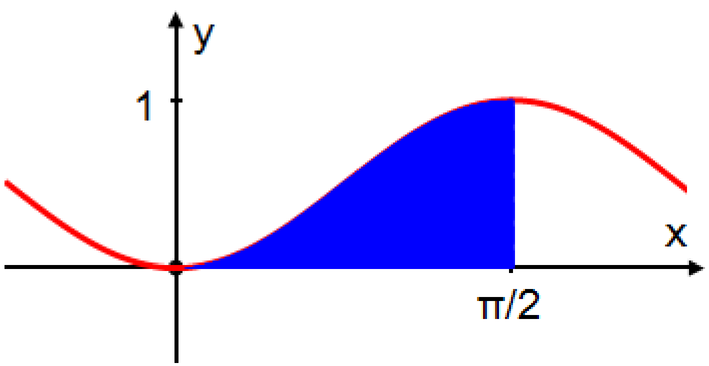


Which region has the largest area?

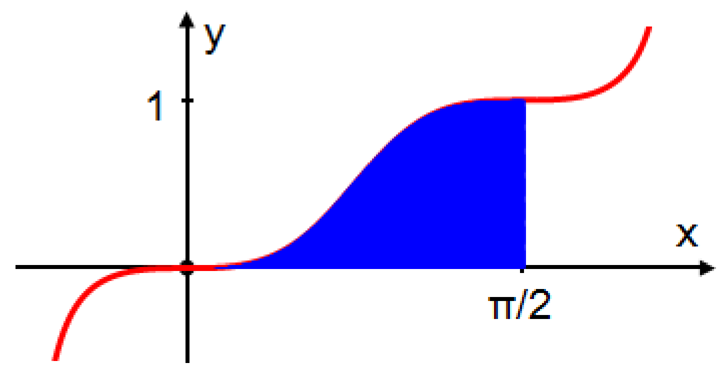
C



Equation 1: $y=\sin x/(\sin x+\cos x)$



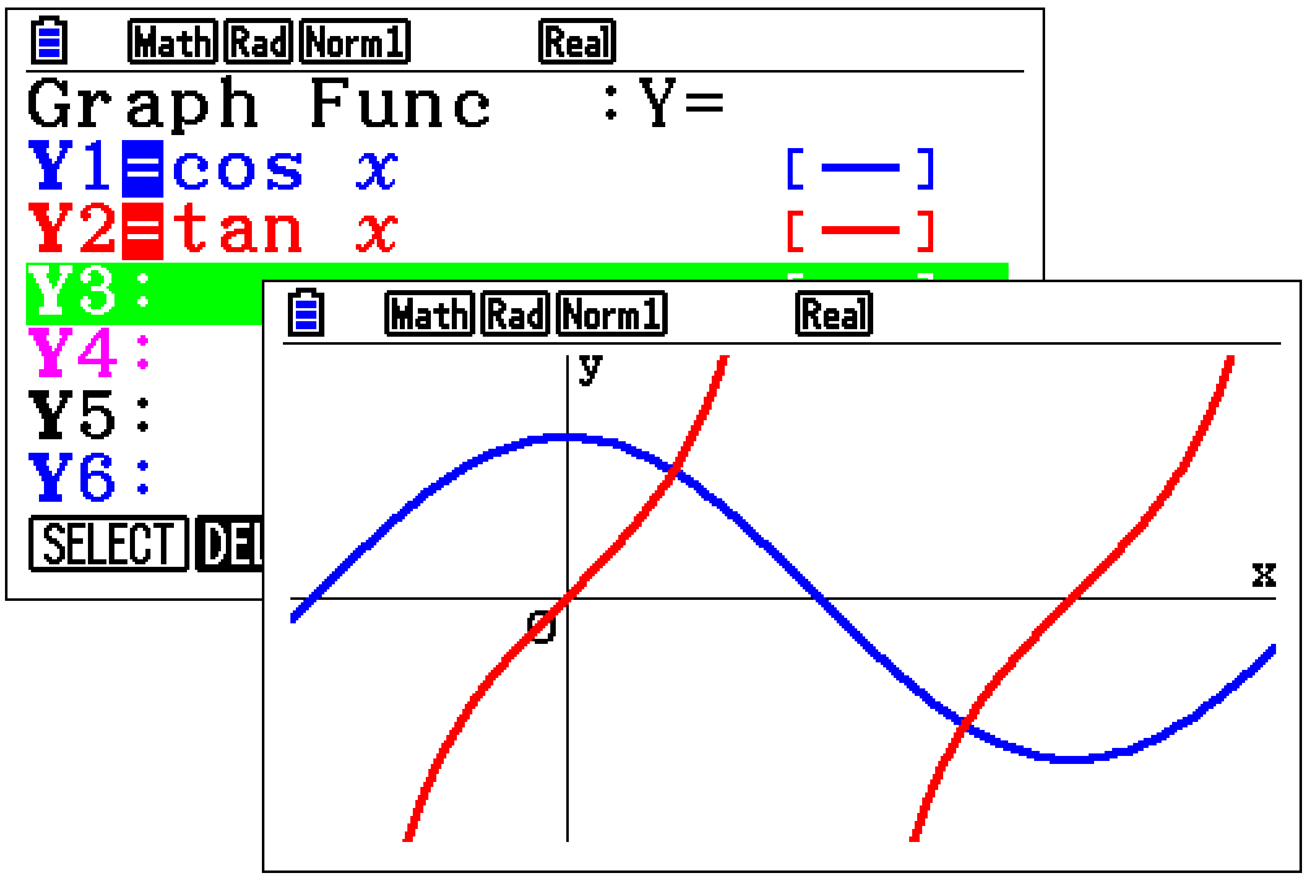
Equation 1: $y=\sin^2 x/(\sin^2 x+\cos^2 x)$



Equation 1: $y=\sin^3 x/(\sin^3 x+\cos^3 x)$

At the intersection points, are the two tangents perpendicular?

D



E

At $t=1, \frac{dy}{dx}=6$

At $t=1, \frac{dy}{dx}=\frac{1}{\ldots}$

At $t=1, \frac{dy}{dx}$

$x = 2t + \frac{1}{t}$

$x = 4\sqrt{t}$

$x = t^2$

$\frac{dy}{dt} = 2t$

$\frac{dy}{dt} = \ldots$

$\frac{dx}{dt} = 4 + 2t$

$\frac{dx}{dt} = 3$

$\frac{dx}{dt} = 4 - \frac{1}{t^2}$

$t=1$, gives the point (5 ,1)

$t=1$, gives the point (4 ,1)

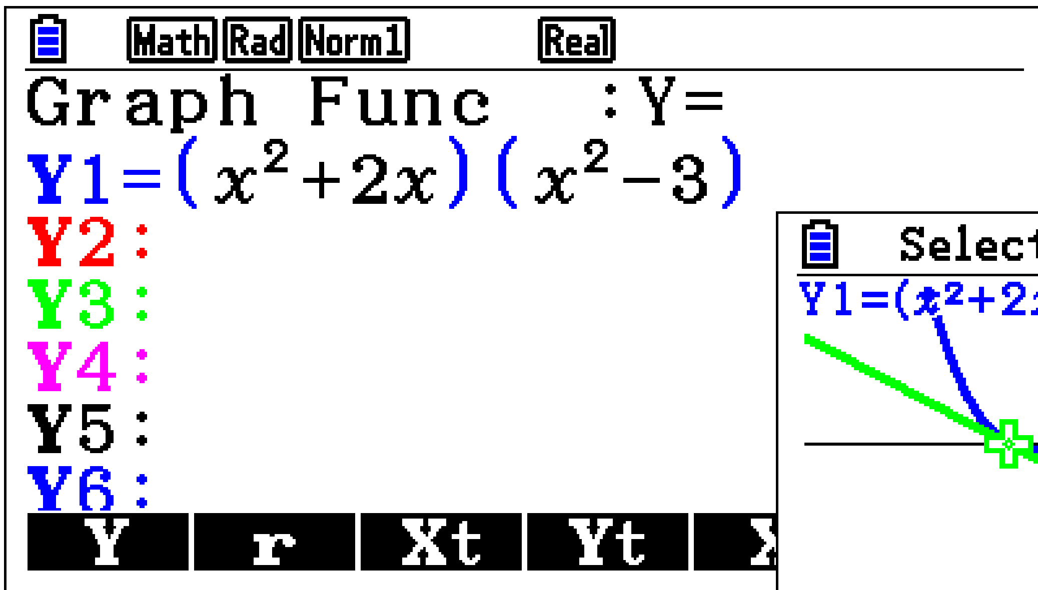
$t=1$, gives the point (5 ,2)

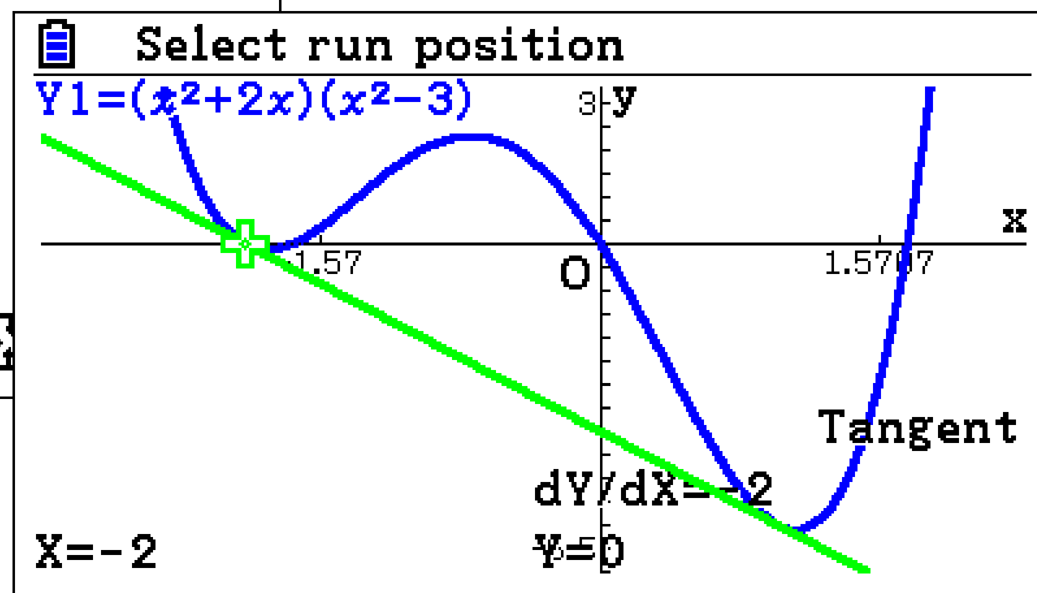
$y = \ln t$

$y = t^3$

$y = t^2 + 1$

Is that a tangent to the curve at two points, or a very near miss?





About MEI

- Registered charity committed to improving mathematics education
- Independent UK curriculum development body
- We offer continuing professional development courses, provide specialist tuition for students and work with employers to enhance mathematical skills in the workplace
- We also pioneer the development of innovative teaching and learning resources

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Using graphing technology for teaching calculus

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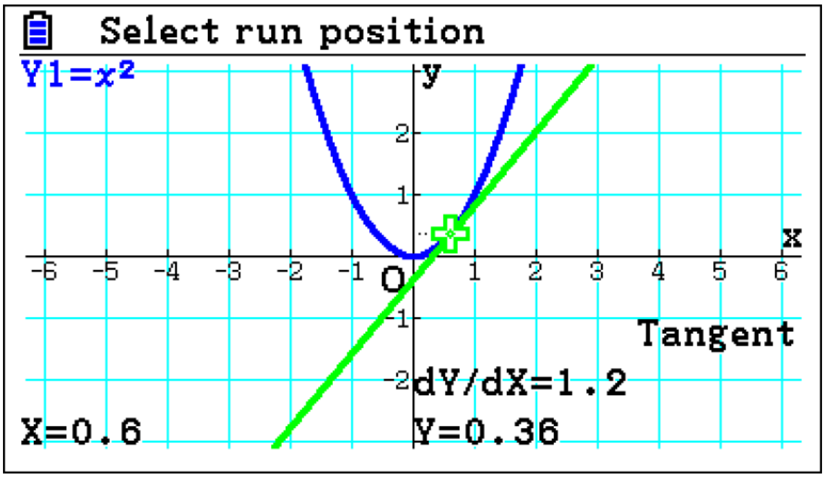
Task A: Differentiation – Exploring the gradient on a curve

1. Add a new Graphs screen: [MENU] [5]
2. Switch the derivative on: [SHIFT] [MENU] [▼] [▼] [▼] [▼] [▼] [▼] [F1] [EXIT]
3. Add the function **Y1 = x²** and draw it: [X,θ,T] [x²] [EXE] [F6]
4. Add the tangent at the point: Sketch > Tangent [F4] [F2]



Use [◀]/[▶] to move the position of the point on the curve.

Question

- How is the gradient of the tangent to the curve at a point related to the point?

Verify your comments by trying some other curves of the form $y = ax^2 + b$. You might find it useful to examine a table of values of the gradient: [MENU] [7] [F6]. (Compare **X** and **Y'1**).

To add/edit curves toggle the Graphs/Text screens with [F6].

Problem

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Further Tasks

Investigate the relationship between a point on the curve and the gradient of the tangent to the curve at that point for:

- $y = ax^2$ or $y = ax^3$
- $y = ax^2 + bx + c$ or $y = ax^3 + bx^2 + cx + d$

Task B: Differentiation – Gradient graphs

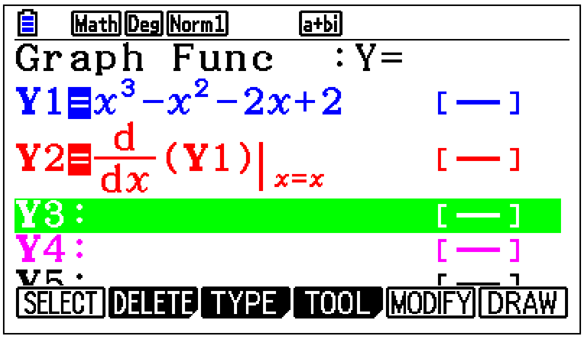
1. Add a new Graphs screen: [MENU] [5]

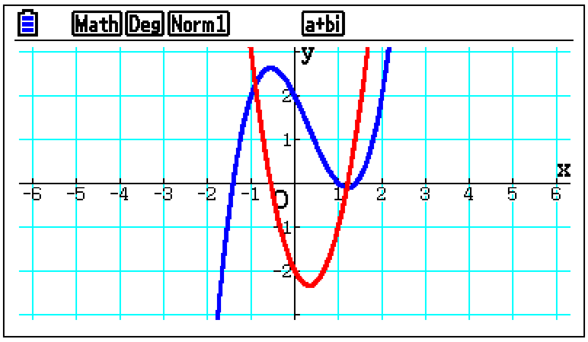
2. Add a cubic function as Y1, e.g. **Y1=x^3 – x^2 – 2x+2** :
   [X,θ,T] [^] [3] [▶] [−] [X,θ,T] [x^2] [−] [2] [X,θ,T] [+] [2] [EXE]

3. Add the derivative, $\frac{dy}{dx}$, as Y2: $\mathbf{Y2=\frac{d}{dx}(Y1)|_{x=x}}$
   [OPTN] [F2] [F1] [F1] [1] [▶] [X,θ,T] [EXE]

   The derivative is in Option > Calc

4. Plot the curves: [F6]





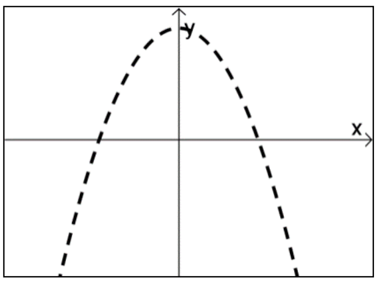
Question for discussion

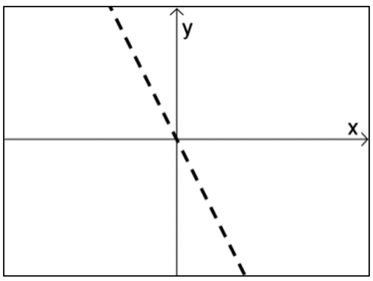
- How is the shape of the gradient graph related to the gradient of the tangent to the curve?

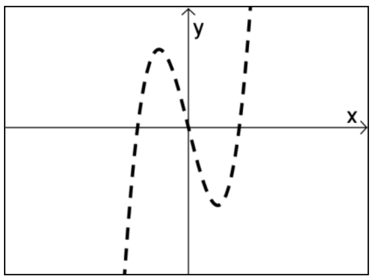
Verify your comments by trying some other functions for Y1.

Problem

Change your original function so the gradient function has one of the following graphs:







Extension Task

Find the point on the function $y = x^3 - 6x^2 + 9x - 1$ where the tangent has its maximum downwards slope. Investigate the point with maximum downward slope for other cubic functions.

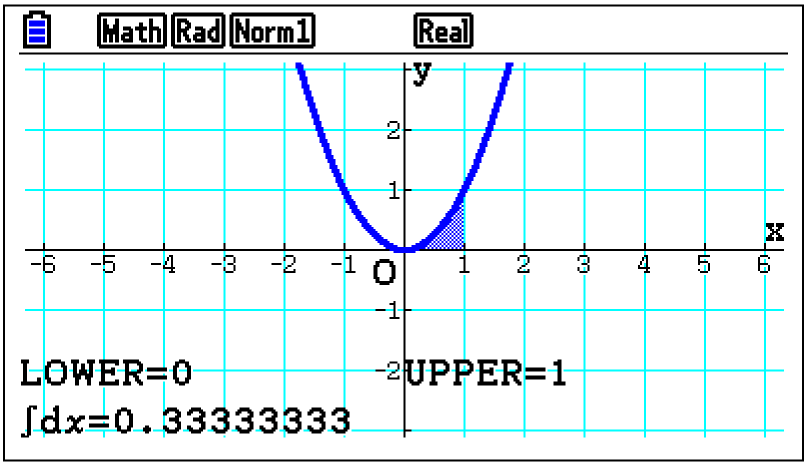
Task C: Integration – Area under a curve

1. Add a new Graphs screen: [MENU] [5]

2. Add the function **Y1 = x²** and draw it: [X,θ,T] [x²] [EXE] [F6]

3. Find the area under the curve using the Integral function: G-Solv > ∫**dx** > ∫**dx**:
   [F5] [F6] [F3] [F1]

4. Set the lower limit to 0 and find the area under the curve for different values of the upper limit, e.g. to find the area between 0 and 1 press: [0] [EXE] [1] [EXE]



Questions

- What is the relationship between the area and upper limit?
- What is the relationship if f(x) is changed to a different power of x?

Problem (*Try the problem with pen and paper first then check it on your calculator)*

Find the area under $y = x^3$ between $x = 0$ and $x = 3$.

Further Tasks

- Investigate the area under $y = x^2$ between $x = a$ and $x = b$.
- Investigate the areas under functions that are the sums of powers of x:
  e.g. $y = x^3 + 3x^2 + 4x + 1$

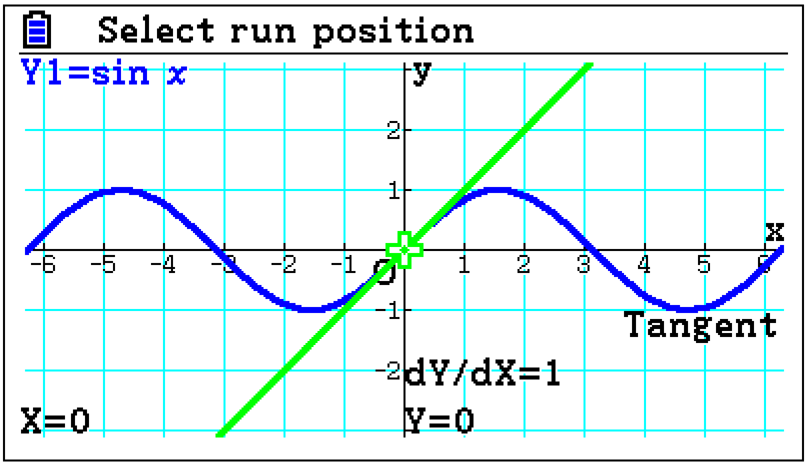
Task D: Differentiation – Trigonometric functions

1. Add a new Graphs screen: [MENU] [5]

2. In SET UP set **Derivative: On** and **Angle:Radians:**
   [SHIFT] [MENU] [▼] [▼] [▼] [▼] [▼] [▼] [F1] [▼] [▼] [▼] [▼] [F2] [EXIT]

3. Draw the graph $y = \sin x$, **Y1=sin(x)**: [sin] [X,θ,T] [EXE] [F6]

4. Add the tangent at the point (*Sketch* > *Tangent)*: [F4] [F2]



Use [◀]/[▶] to move the position of the point on the curve.

Questions

- How does the gradient of the tangent vary as x varies:
  - What are its maximum and minimum values?
  - When is the gradient of the tangent 0?
  - For what values of x do these occur?
- Can you suggest a function for the derivative?
- Can you suggest a function for the derivative of $y = \cos(x)$?

Problem (C*heck your answer by plotting the graph and the tangent on your calculator)*

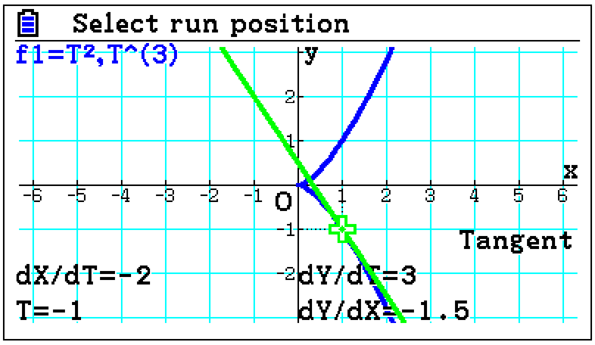
Find the equation of the tangent to the curve $y = \sin x$ at the point $x = \frac{\pi}{3}$.

Further Tasks

- Investigate the derivatives of $y = \sin ax$ and $y = b \sin x$.
- Explain why this wouldn't work as neatly if the angle was measured in degrees.

Task E: Tangents to parametric curves

1. Add a new Graphs screen: MENU 5
2. In SET UP set **Derivative: On** and **Angle: Radians**:
   SHIFT MENU ▼ ▼ ▼ ▼ ▼ ▼ F1 ▼ ▼ ▼ ▼ F2 EXIT
3. Set the graph Type to Parametric: F3 F3
4. Use V-Window to set the range of T to **Tmin: –4**, **Tmax: 4**, **pitch: 0.05**
   SHIFT F3 ▼ ▼ ▼ ▼ ▼ ▼ ▼ (–) 4 EXE 4 EXE 0 • 0 5 EXE EXIT
5. Draw the graph $x = t^2$, $y = t^3$. **X1t=T², Y1t=T³**: X,θ,T x^2 EXE X,θ,T ^ 3 EXE F6
6. Add the tangent at the point (*Sketch* > *Tangent)*: F4 F2



Use ◀ / ▶ to move the position of the point on the curve.

Questions

- What is the relationship between $\frac{\mathrm{d}y}{\mathrm{d}x}$, $\frac{\mathrm{d}x}{\mathrm{d}t}$ and $\frac{\mathrm{d}y}{\mathrm{d}t}$?
- Does this relationship for other parametric curves?
  e.g. $x = 2t + 1,\ y = \frac{1}{t}$ or $x = \cos t, y = \sin t$.

Problem *(Check your answer by plotting the graph and the tangent on your calculator)*

Find the coordinates of the points on the curve $x = 2t + \cos t, y = \sin t, -2\pi < t \le 2\pi$ for which the tangent to the curve is parallel to the *x*-axis.

Further Tasks

- Explore how you can find the equation of the tangent to a parametric curve at a point.
- Describe how to find the tangent to a parametric curve that passes through a specific point that is not on the curve.