

MEI Further Pure with Technology – Suggested Scheme of Work (2018-2019)

This template shows how **Integral Resources** can be used to support Further Mathematics students and teachers. This is for the optional Further Pure with Technology component of A level Further Mathematics. You will need to deliver either a major or 2 more minors & Further Pure alongside this. It is examined in A level paper Y436.

This content makes up 16 $\frac{2}{3}$ % of A Level FM content

Integral Resources include a wide range of resources for both teacher and student use in learning and assessment. Interactive resources and ideas for using technology are featured throughout. Sample resources are available via: integralmaths.org/.

- **Teacher access** to the **Integral Resources** (integralmaths.org/) for Further Mathematics is available **free of charge** to all schools/colleges that register with the Advanced Mathematics Support Programme: amsp.org.uk/
- **Individual student access** to the full range of **Integral Resources** for Further Mathematics is available at a cost of £30 per student, for a *Video+* account, or via a full school/college subscription to Integral. Teachers will get access to the management system so they can monitor their students' progress. For more details see: amsp.org.uk/teachers/a-level-further/resources

A textbook for Further Pure with Technology is also available (published by Hodder Education).

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Date	Topic	Specification statements	Integral Resources	Exercises & Assessment (Integral)	Notes
	Number Theory: Programming; Prime numbers	<ul style="list-style-type: none"> Be able to write, adapt and interpret programs to solve number theory problems. Be able to identify the limitations of a short program and suggest refinements to it. Know and use the unique prime factorisation of natural numbers. 	MEI_FM_FPT ▶ Number theory ▶ Number theory 1: Programming	Exercise level 1 Exercise level 2	Using <code>if</code> and <code>for</code> in Python to solve number theory problems. Students should be able to create a prime testing routine.
			MEI_FM_FPT ▶ Number theory ▶ Number theory 2: Prime numbers	Exercise level 1 Exercise level 2 Section test N2	
	Number Theory: Congruences and modular arithmetic	<ul style="list-style-type: none"> Be able to solve problems using modular arithmetic. Know and use Fermat's little theorem. Know and use Euler's totient function, $\varphi(n)$, Euler's theorem. Know and use Wilson's theorem. 	MEI_FM_FPT ▶ Number theory ▶ Number theory 3: Congruences and modular arithmetic	Exercise level 1 Exercise level 2 Section test N3	Using the notation $a \equiv b \pmod{m}$. Students should understand and use the theorems but are not expected to know formal proofs of them.
	Number Theory: Diophantine equations	<ul style="list-style-type: none"> Be able to find Pythagorean triples and use related equations. Be able to solve Pell's equation and use solutions to solve related problems. Be able to solve other Diophantine equations and use solutions to solve related problems. 	MEI_FM_FPT ▶ Number theory ▶ Number theory 4: Diophantine equations	Exercise level 1 Exercise level 2 Section test N4	Students should be able to use a program to search for integers solutions to problems.
				MEI_FM_FPT ▶ Number theory ▶ Number theory: Topic assessment	

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	Investigation of curves: Equations and properties of curves	<ul style="list-style-type: none"> Be able to plot a family of curves in graphing software, in cartesian, polar and parametric forms. Be able to use CAS to work with equations of curves. Know and use the vocabulary associated with curves. Be able to find, describe and generalise properties of curves. Be able to convert equations between cartesian and polar forms in all cases. Be able to convert equations from polar to parametric form, and parametric to cartesian form. 	MEI_FM_FPT ▶ Investigation of curves ▶ Investigation of curves 1: Equations and properties of curves	Exercise level 1 Exercise level 2 Section test C1	Students should be able to sketch curves based on the plots obtained from software and conjecture the properties of them. CAS can be used for any solving, substituting, differentiating, integrating and finding limits but evidence of how it has been used should be clearly presented.
	Investigation of curves: Derivatives of curves; Limiting behaviour	<ul style="list-style-type: none"> Be able to find the gradient of the tangent to a curve at a point. Be able to find and work with equations of chords, tangents and normals. Be able to use the limit of $f(x)$ as $x \rightarrow a$ or $x \rightarrow \infty$ to investigate and deduce properties of a curve. Be able to determine asymptotes. Be able to identify cusps by examining the behaviour nearby. 	MEI_FM_FPT ▶ Investigation of curves ▶ Investigation of curves 2: Derivatives of curves	Exercise level 1 Exercise level 2 Section test C2	Students should be able to determine the position of asymptotes and cusps based algebraically (though CAS can be used for finding derivatives and limits).
		MEI_FM_FPT ▶ Investigation of curves ▶ Investigation of curves 3: Limiting behaviour	Exercise level 1 Exercise level 2 Section test C3		
	Investigation of curves: Envelopes and arc lengths	<ul style="list-style-type: none"> Be able to calculate arc length using cartesian, parametric and polar coordinates. Understand the meaning of an envelope of a family of curves. Be able to find an envelope of a family of curves. 	▶ MEI_FM_FPT ▶ Investigation of curves ▶ Investigation of curves 4: Envelopes and arc lengths	Exercise level 1 Exercise level 2 Section test C4	Students should be able to form integrals and derivatives (CAS can be used to evaluate them).
				MEI_FM_FPT / ▶ Investigation of curves ▶ Investigation of curves: Topic assesment	

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	Differential equations: Tangent fields; Analytical solutions of differential equations	<ul style="list-style-type: none"> Be able to use software to produce a tangent field for a differential equation. Be able to use software to produce analytical solutions to (families of) first order differential equations, when this is possible. Be able to sketch a tangent field for a first order differential equation and be able to interpret it. Be able to verify a given solution of a differential equation. Be able to work with particular solutions and families of particular solutions. 	MEI_FM_FPT ▶ Exploring differential equations ▶ Exploring differential equations 1: Tangent fields	Exercise level 1 Exercise level 2 Section test D1	Students should be able to sketch a tangent field based on a plot obtained from the software and interpret properties of the DE based on this.
			MEI_FM_FPT ▶ Exploring differential equations ▶ Exploring differential equations 2: Analytical solutions of differential equations	Exercise level 1 Exercise level 2 Section test D2	Students should be able to use CAS to solve a DE analytically (where possible) and used the techniques from the Investigation of curves topic to determine properties of a family of solutions.
	Differential equations: Numerical solutions of differential equations	<ul style="list-style-type: none"> Be able to construct, adapt or interpret a spreadsheet to solve first order differential equations numerically. Know how to solve a given first order differential equation $\frac{dy}{dx} = f(x, y)$ with initial conditions to any required degree of accuracy by repeated application of the Euler method or a modified Euler method . Understand that a smaller step length usually gives a more accurate answer. Understand that a modified Euler method usually gives a more accurate solution than an Euler method for a given step length. Understand the concepts underlying Runge-Kutta methods. Be able to solve first order differential equations using Runge-Kutta methods. 	MEI_FM_FPT ▶ Exploring differential equations ▶ Exploring differential equations 3: Numerical solutions of differential equations	Exercise level 1 Exercise level 2 Section test D3	<p>Students should be able to generate spreadsheets to solve DEs numerically for each of the three methods as well as simple variations of these.</p> <p>Students should understand how the methods are generated, based on lines or curves at the points, but are not expected to provide formal proofs of the formulas.</p>
				MEI_FM_FPT ▶ Exploring differential equations ▶ Exploring differential equations: Topic assessment	