MEI Further Pure with Technology – Suggested Scheme of Work (2019-2020)

This template shows how Integral Resources can be used to support Further Mathematics students and teachers. This is for the optional Further Pure with Technology component of A level Further Mathematics. Students are required to study the compulsory pure and either a major or two more minors alongside FPT. Students can also take FPT as an additional option (i.e. four minors or two minors and a major) and have the best allowable combination of minor marks count towards their grade. It is examined in A level paper Y436.

This content makes up 16¾% of A Level FM content

Integral Resources include a wide range of resources for both teacher and student use in learning and assessment. Interactive resources and ideas for using technology are featured throughout. Sample resources are available via: integralmaths.org/.

- Teacher access to the Integral Resources (integralmaths.org/) for Further Mathematics is available free of charge to all schools/colleges that register with the Advanced Mathematics Support Programme: amsp.org.uk/

- Individual student access to the full range of Integral Resources for Further Mathematics is available at a cost of £30 per student, for a Video+ account, or via a full school/college subscription to Integral. Teachers will get access to the management system so they can monitor their students' progress. For more details see: amsp.org.uk/teachers/a-level-further/resources

A textbook for Further Pure with Technology is also available (published by Hodder Education).
### MEI Further Pure with Technology – Suggested Scheme of Work (2019-2020)

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|      | **Number Theory:** Prime numbers | • Be able to write, adapt and interpret programs to solve number theory problems.  
• Be able to identify the limitations of a short program and suggest refinements to it. | MEI_FM_FPT  
► Number theory  
► Number theory 1: Programming | Exercise level 1  
Exercise level 2 | Using `if` and `for` in Python to solve number theory problems. |
|      | **Number Theory:** Prime numbers | • Know and use the unique prime factorisation of natural numbers. | MEI_FM_FPT  
► Number theory  
► Number theory 2: Prime numbers | Exercise level 1  
Exercise level 2  
Section test N2 | Students should be able to create a prime testing routine. |
|      | **Number Theory:** Congruences and modular arithmetic | • Be able to solve problems using modular arithmetic.  
• Know and use Fermat’s little theorem.  
• Know and use Euler’s totient function, \( \varphi(n) \), Euler’s theorem.  
• Know and use Wilson’s theorem. | MEI_FM_FPT  
► Number theory  
► Number theory 3: Congruences and modular arithmetic | Exercise level 1  
Exercise level 2  
Section test N3 | Using the notation \( a \equiv b \pmod{m} \).  
Students should understand and use the theorems but are not expected to know formal proofs of them. |
|      | **Number Theory:** Diophantine equations | • Be able to find Pythagorean triples and use related equations.  
• Be able to solve Pell’s equation and use solutions to solve related problems.  
• Be able to solve other Diophantine equations and use solutions to solve related problems. | MEI_FM_FPT  
► Number theory  
► Number theory 4: Diophantine equations | Exercise level 1  
Exercise level 2  
Section test N4 | Students should be able to use a program to search for integer solutions to problems. |

**MEI_FM_FPT**  
► Number theory  
► Number theory: Topic assessment
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|        | **Investigation of curves: Equations and properties of curves**        | • Be able to plot a family of curves in graphing software, in cartesian, polar and parametric forms.  
  • Be able to use CAS to work with equations of curves.  
  • Know and use the vocabulary associated with curves.  
  • Be able to find, describe and generalise properties of curves.  
  • Be able to convert equations between cartesian and polar forms in all cases. Be able to convert equations from polar to parametric form, and parametric to cartesian form. | **MEI_FM_FPT**  
  ► Investigation of curves  
  ► Investigation of curves 1: Equations and properties of curves | Exercise level 1  
  Exercise level 2  
  Section test C1 | Students should be able to sketch curves based on the plots obtained from software and conjecture the properties of them.  
  CAS can be used for any solving, substituting, differentiating, integrating and finding limits but evidence of how it has been used should be clearly presented. |
|        | **Investigation of curves: Derivatives of curves**                     | • Be able to find the gradient of the tangent to a curve at a point.  
  • Be able to find and work with equations of chords, tangents and normals. | **MEI_FM_FPT**  
  ► Investigation of curves  
  ► Investigation of curves 2: Derivatives of curves | Exercise level 1  
  Exercise level 2  
  Section test C2 |                                                                                                                                       |
|        | **Investigation of curves: Derivatives of curves**                     | • Be able to use the limit of \( f(x) \) as \( x \to a \) or \( x \to \infty \) to investigate and deduce properties of a curve.  
  • Be able to determine asymptotes.  
  • Be able to identify cusps by examining the behaviour nearby. | **MEI_FM_FPT**  
  ► Investigation of curves  
  ► Investigation of curves 3: Limiting behaviour | Exercise level 1  
  Exercise level 2  
  Section test C3 | Students should be able to determine the position of asymptotes and cusps based algebraically (though CAS can be used for finding derivatives and limits). |
|        | **Investigation of curves: Envelopes and arc lengths**                | • Be able to calculate arc length using cartesian, parametric and polar coordinates.  
  • Understand the meaning of an envelope of a family of curves. Be able to find an envelope of a family of curves. | **MEI_FM_FPT**  
  ► Investigation of curves  
  ► Investigation of curves 4: Envelopes and arc lengths | Exercise level 1  
  Exercise level 2  
  Section test C4 | Students should be able to form integrals and derivatives (CAS can be used to evaluate them). |

**MEI_FM_FPT / ► Investigation of curves ◄ Investigation of curves: Topic assessment**
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<td><strong>Differential equations:</strong> Tangent fields</td>
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<td>MEI_FM_FPT ► Exploring differential equations  ► Exploring differential equations 1: Tangent fields</td>
<td>Exercise level 1 Exercise level 2 Section test D1 Students should be able to sketch a tangent field based on a plot obtained from the software and interpret properties of the DE based on this.</td>
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<td>Be able to use software to produce a tangent field for a differential equation.  Be able to use software to produce analytical solutions to (families of) first order differential equations, when this is possible.  Be able to sketch a tangent field for a first order differential equation and be able to interpret it.</td>
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<td><strong>Differential equations:</strong> Analytical solutions of differential equations</td>
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<td>MEI_FM_FPT ► Exploring differential equations  ► Exploring differential equations 2: Analytical solutions of differential equations</td>
<td>Exercise level 1 Exercise level 2 Section test D2 Students should be able to use CAS to solve a DE analytically (where possible) and used the techniques from the Investigation of curves topic to determine properties of a family of solutions.</td>
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<td>Be able to verify a given solution of a differential equation.  Be able to work with particular solutions and families of particular solutions.</td>
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<td><strong>Differential equations:</strong> Numerical solutions of differential equations</td>
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<td>MEI_FM_FPT ► Exploring differential equations  ► Exploring differential equations 3: Numerical solutions of differential equations</td>
<td>Exercise level 1 Exercise level 2 Section test D3 Students should be able to generate spreadsheets to solve DEs numerically for each of the three methods as well as simple variations of these.  Students should understand how the methods are generated, based on lines or curves at the points, but are not expected to provide formal proofs of the formulas.</td>
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<td>Be able to construct, adapt or interpret a spreadsheet to solve first order differential equations numerically.  Know how to solve a given first order differential equation $\frac{dy}{dx} = f(x,y)$ with initial conditions to any required degree of accuracy by repeated application of the Euler method or a modified Euler method.  Understand that a smaller step length usually gives a more accurate answer. Understand that a modified Euler method usually gives a more accurate solution than an Euler method for a given step length.  Understand the concepts underlying Runge-Kutta methods.  Be able to solve first order differential equations using Runge-Kutta methods.</td>
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**MEI_FM_FPT ► Exploring differential equations ► Exploring differential equations: Topic assessment**