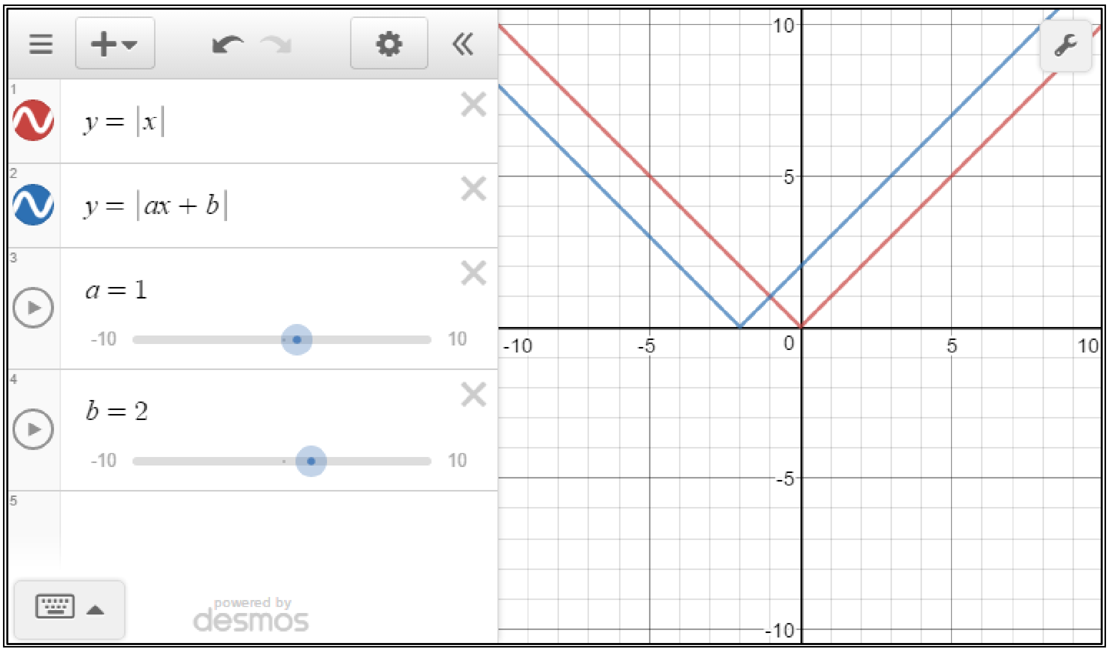
MEI Desmos Tasks for A2 Core

Task 1: Functions – The Modulus Function

1. Plot the graph: $y = |x|$
2. Plot the graph: $y = |ax + b|$



Questions for discussion

- What combination of transformations maps the graph of $y = |x|$ onto the graph of $y = |ax + b|$?
- Where is the vertex on the graph of $y = |ax + b|$?
- Where does the graph of $y = |ax + b|$ intersect the y-axis?

Problem (*Try the problem with pen and paper first then check it on your software*)

Sketch the graph of $y = |3x + 2| - 3$ and find the points of intersection with the axes.

Further Tasks

- Investigate the graphs of
  - $y = |\mathrm{f}(x)|$
  - $y = \mathrm{f}(|x|)$

  for different functions $\mathrm{f}(x)$, e.g. $\mathrm{f}(x) = \sin x$ or $\mathrm{f}(x) = x^3 - x^2$
- Investigate the solutions to the inequality $|x + a| + b > 0$.

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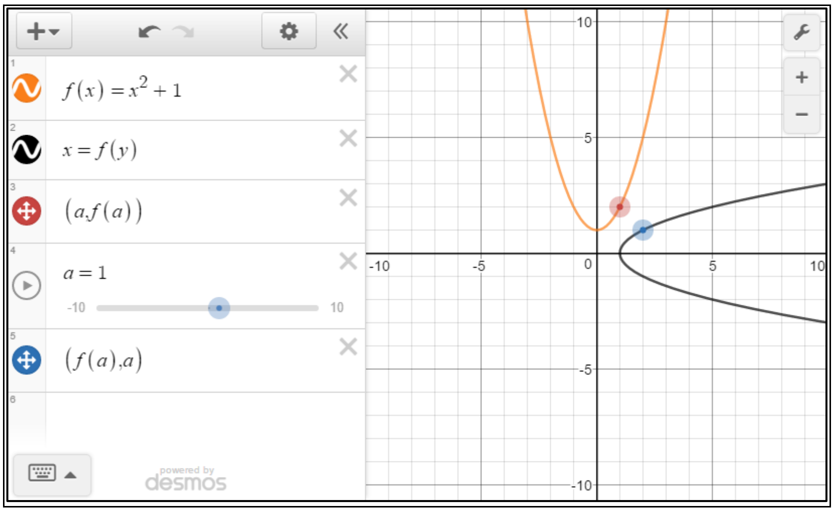
Task 2: Inverse functions

1. Plot function: $f(x) = x^2 + 1$
2. Plot the inverse: $x = f(y)$
3. Plot the point: $(a, f(a))$
4. Plot the point: $(f(a), a)$



Questions for discussion

- What graphical transformation maps the graph of the original function onto its inverse?
- How could you restrict the domain of the original function so that inverse is is the graph of a function of x?
- What is the equation of the reflected curve when it is rewritten to make y the subject?

Try finding the equation of the reflected curve for the graphs of some other functions.

Problem *(Try the problem with pen and paper first then check it on your software)*

Find inverses of the following functions:

$f(x) = (x + 3)^2, x \geq -3$ $\quad$ $g(x) = x^3$ $\quad$ $h(x) = \frac{1}{x-2}$

Further Tasks

- Find the inverse of the function $f(x) = x^2 + 6x + 1, x \geq -3$. Can you always find the inverse of a quadratic function $f(x) = ax^2 + bx + c$?
- Investigate the graphs of the inverse trigonometric functions (you might find radians more convenient for this).

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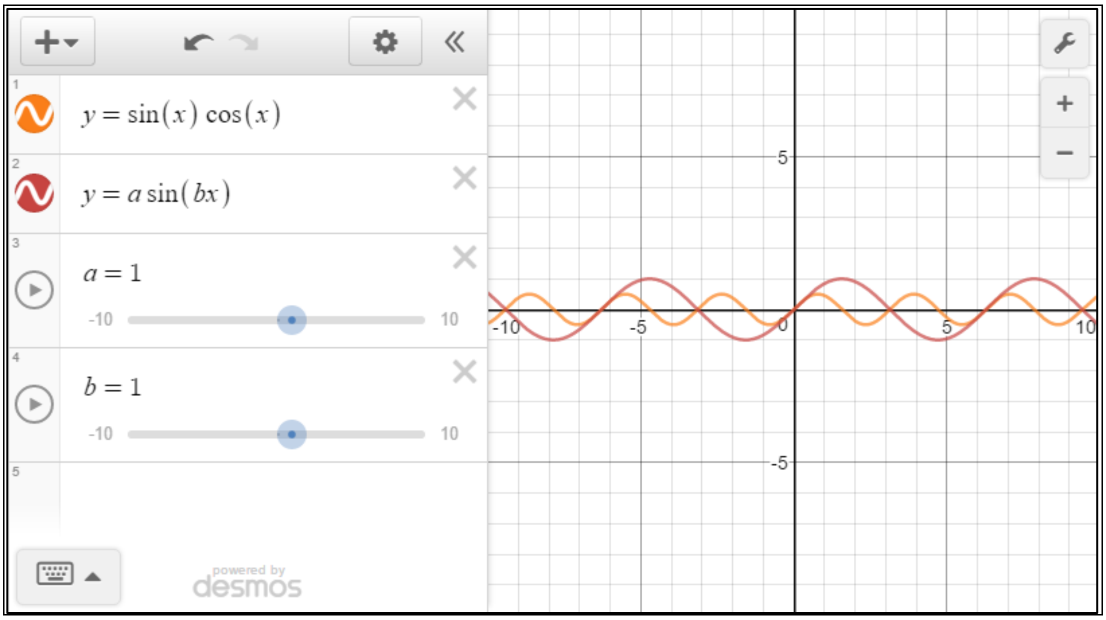
Task 3: Trigonometry – Double Angle formulae

1. Plot the graph: $y = \sin(x)\cos(x)$ $y = \sin(x)\cos(x)$
2. Plot the graph: $y = a\sin(bx)$



Questions for discussion

- For what values of a and b does $\sin x \cos x = a \sin bx$?
- Find values of a, b and c so that:
  - $\cos^2 x = a\cos(bx) + c$
  - $\sin^2 x = a\cos(bx) + c$
- How do these relationships link to the double angle formulae for sin and cos?

Problem *(Try the problem with pen and paper first then check it on your software)*

Sketch the curves $y = \sin 2\theta$ and $y = \cos\theta$ on the same axes.

Solve the equation $\sin 2\theta - \cos\theta = 0$ in the range $0 \le \theta < 2\pi$.

Further Tasks

- Plot the curve with equation $y = \frac{2\tan\theta}{1-\tan^2\theta}$. By considering specific points on the curve show that this is the same as the graph of $y = \tan 2\theta$.
- Find expressions for $\sin 4\theta$ and $\cos 4\theta$ in terms of $\sin\theta$ and $\cos\theta$.

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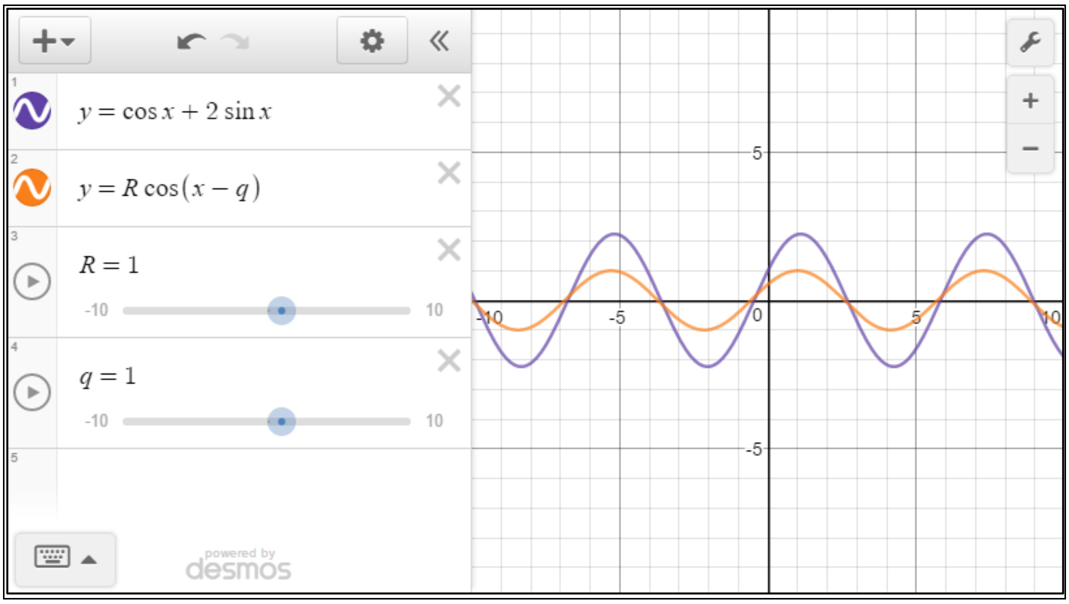
Task 4: Trigonometry: *R*cos(*θ*–*α*)

NB q is being used as a substitute for α when plotting graphs in the software.

1. Plot the graph: $y = \cos x + 2\sin x$
2. Plot the graph: $y = R\cos(x - q)$



Questions for discussion

- For what values of q and R does $\cos x + 2\sin x = R\cos(x - q)$?
- In general, given values for a and b, how can you find values of α and R so that $a\cos x + b\sin x \equiv R\cos(x - \alpha)$?
- How can you explain the relationship using $R\cos(x - \alpha) \equiv R\cos x\cos\alpha + R\sin x\sin\alpha$?

Problem *(Try the problem with pen and paper first then check it on your software)*

Express $4\cos x + 3\sin x$ in the form $R\cos(x - \alpha)$ where $0 < \alpha < \frac{\pi}{2}$.

Further Tasks

- Explore how the form $R\cos(x - \alpha)$ can be used to find the maximum value of $a\cos x + b\sin x$ and the angle at which it occurs.
- Explore the forms $R\sin(x + \alpha)$, $R\sin(x - \alpha)$ and $R\cos(x + \alpha)$.

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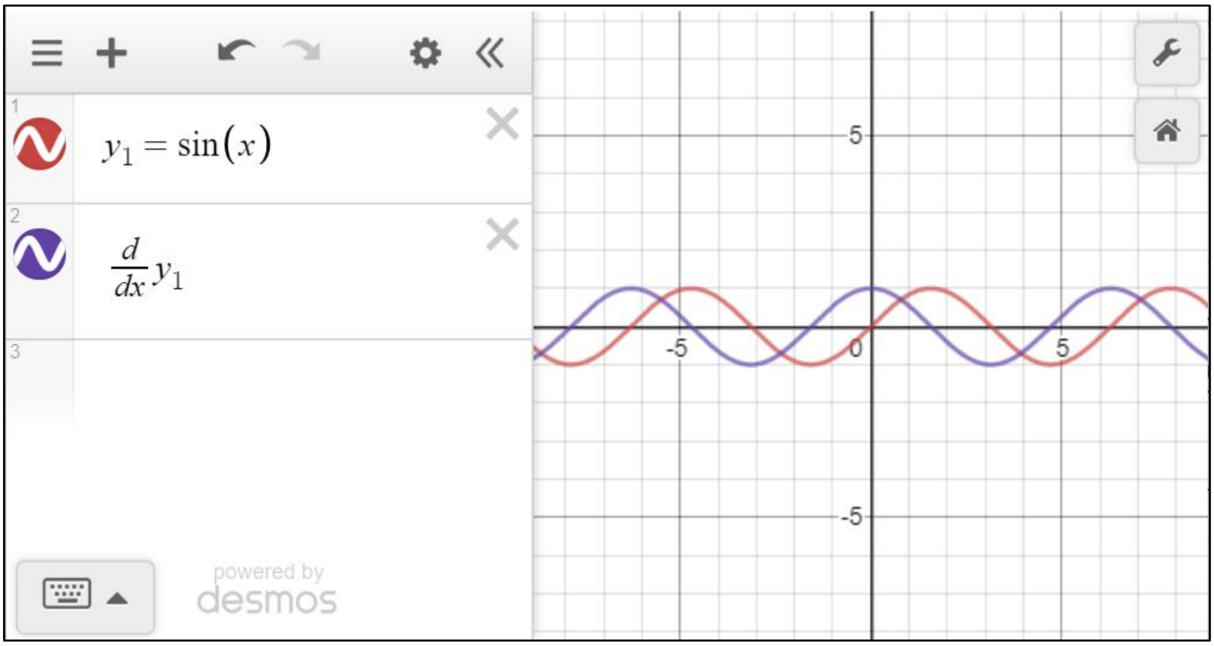
Task 5: Differentiation – Trigonometric functions

NB It is essential that the Graph Settings are set as Radians (not Degrees)

1. Plot the graph of: $y_1 = \sin(x)$
2. Enter: $\frac{d}{dx} y_1$

Type: ***y1=*** *to get the subscript form*

You can type ***d/dx*** *or use the d/dx key in: funcs > misc > d/dx*



Questions for discussion

- How does the gradient function relate to the original graph of $y = \sin x$:
  - What are its maximum and minimum values?
  - When is the value of the gradient function 0?
  - For what values of x do these (max, min or 0) occur?
- What properties will the gradient function of $y = \cos\ x$ have?

Problem *(Try the problem with pen and paper first then check it on your software)*

Find the equation of the tangent to the curve $y = \sin x$ at the point $x = \frac{\pi}{3}$.

Further Tasks

- Investigate the derivatives of $y = \sin ax$ and $y = b \sin x$.
- Explain why this wouldn't work as neatly if the angle was measured in degrees.

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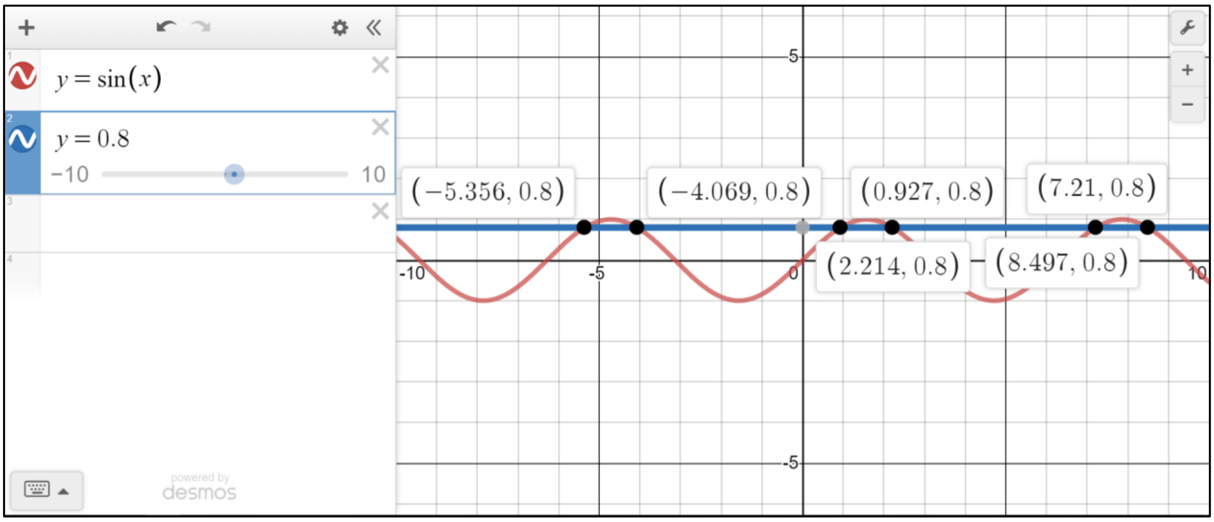
Task 6: Trigonometry – Solving equations in radians

1. Add the graph: $y = \sin x$
2. Add the graph: $y = 0.8$
3. Click on the points of intersection to see their values



Questions for discussion

- What symmetries are there in the positions of the points of intersection as you vary k?
- How can you use these symmetries to find the other intersection points based on the value of $\sin^{-1} k$? This is known as the "principal value".

Problem *(Try the question just using the sin^{-1} function on your calculator first then check it using your software)*

Solve the equation $\sin x = 0.5$ $(-2\pi < x \leq 4\pi)$, giving your answers in terms of π.

Further Tasks

- Investigate the symmetries of the solutions to $\cos x = k$ and $\tan x = k$.
- Investigate the symmetries of the solutions to $\sin 2x = k$.

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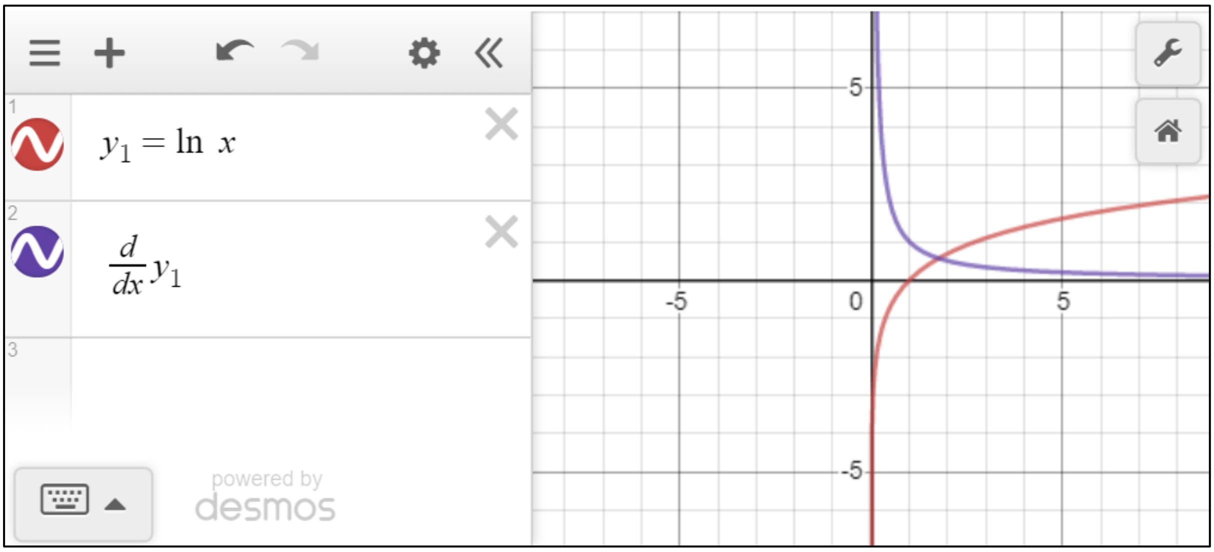
Task 7: Derivative of the natural logarithm $y = \ln x$

1. Plot the graph of $y_1 = \ln x$
2. Enter: $\frac{d}{dx} y_1$

Type: ***y1=*** *to get the subscript form*

You can type ***d/dx*** *or use the d/dx key in: funcs > misc > d/dx*



Questions for discussion

- What is derivative of $y = \ln x$?
- What is the derivative of $y = \ln kx$?
  How can you explain this algebraically and graphically?

Problem (Check your answer by plotting the graph and the derivative on your software)

Find the equation of the tangent to the curve $y = \ln x$ at the point $x = 2$.

Further Tasks

- Find the tangent to $y = \ln x$ that passes through the origin.
- Explain the relationship between the derivatives of $y = e^x$ and $y = \ln x$.
  Hint: consider the point (a, b) on $y = e^x$ and the reflected point (b, a) on $y = \ln x$.

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Task 8: Converting parametric equations to cartesian equations

1. Plot the parametric curve $x = t^2$, $y = t^3$ by entering: (t^2, t^3)
2. Change the domain to $-10 \le t \le 10$
3. Plot the curve $y = x^{\frac{3}{2}}$
4. Plot the curve $y = x^{\frac{2}{3}}$

Questions for discussion

- Which of the cartesian equations gives the same graph as the parametric equation and how can you explain this algebraically?
- How would you find a cartesian equation for these parametric curves:
    - $x = 2t + 1, y = \frac{1}{t}$
    - $x = \cos t\,, y = \sin t$

Problem *(Try the question with pen and paper first then check it on your software)*

Find a cartesian equation of the curve $x = \mathrm{e}^t, y = t^2 + 1$.

Further Tasks

- Compare finding the points of intersection with the x and y axes for the same curve written in parametric and cartesian form.
- Explore converting from cartesian to parametric form.

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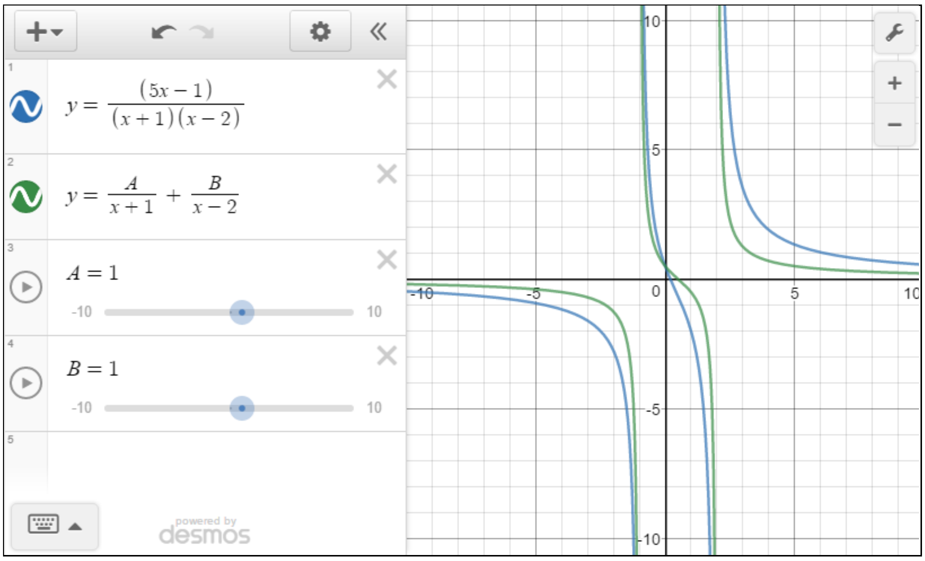
Task 9: Partial Fractions

1. Enter the function $y=\dfrac{5x-1}{(x+1)(x-2)}$
2. Enter the function $y=\dfrac{A}{x+1}+\dfrac{B}{x-2}$ and add sliders for A and B



Find values of A and B so that the graphs of the functions are the same.

Questions for discussion

- How could you find the values algebraically using $\dfrac{5x-1}{(x+1)(x-2)}=\dfrac{A}{x+1}+\dfrac{B}{x-2}$?
- Does this method work for $\dfrac{2x+7}{(x+2)(x+3)}=\dfrac{A}{x+2}+\dfrac{B}{x+3}$?

Problem *(Check your answers by plotting the graphs on your software)*

Find values of *A* and *B* to make the following an identity:

$$\frac{7x-14}{(x+3)(x-4)}\equiv\frac{A}{x+3}+\frac{B}{x-4}$$

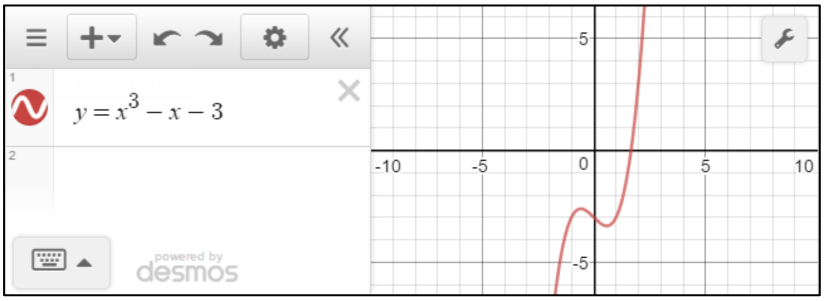
Further Tasks

- Find *A*, *B* and *C* such that $\dfrac{3x+7}{(x+2)^2}=\dfrac{A}{x+2}+\dfrac{B}{(x+2)^2}$
- Find *A*, *B* and *C* such that $\dfrac{7x^2+29x+28}{(x-1)(x+3)^2}=\dfrac{A}{x-1}+\dfrac{B}{x+3}+\dfrac{C}{(x+3)^2}$

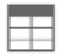
MEI Desmos Tasks for A2 Core

Task 10 – Numerical Methods: Change of sign

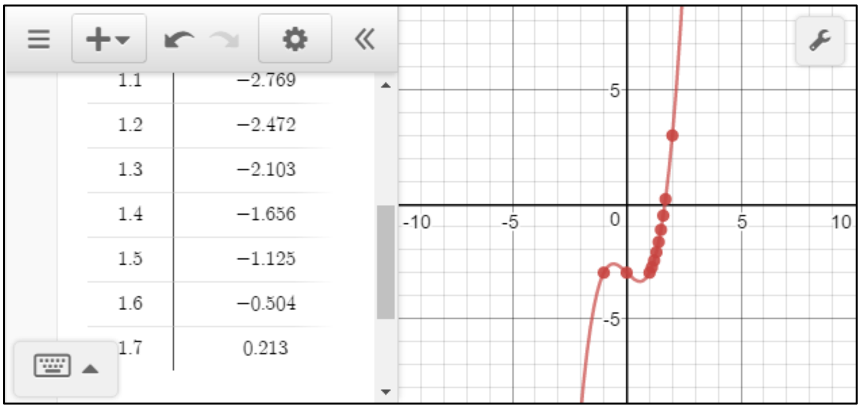
1. Plot the graph of $y = x^3 - x - 3$



In this example you can see that the root lies between $x = 1$ and $x = 2$.

2. With the curve selected press Edit  and Convert to table 

3. In the x-column enter the values 1.1, 1.2, 1.3, … until you observe a change of sign in y.



In this example there is a change of sign between x = 1.6 and x = 1.7.

4. You can now investigate further by entering x-values from 1.6 to 1.7 in steps of 0.01.

5. You can check your answer by zooming-in and selecting the point of intersection with the x-axis.

Try using your software to find the roots of other equations using the change of sign method.

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Teacher guidance

Using these tasks

These tasks are designed to help students in understanding mathematical relationships better through exploring dynamic constructions. They can be accessed using the computer-based version of Desmos or the tablet/smartphone app. Each task instruction sheet is reproducible on a single piece of paper and they are designed to be an activity for a single lesson or a single homework task (approximately).

The tasks have been designed with the following structure –

- **Construction:** step-by-step guidance of how to construct the objects in Desmos. Students will benefit from learning the rigorous steps need to construct objects and this also removes the need to make prepared files available to them. If students become confident with using Desmos they can be encouraged to add additional objects to the construction to aid their exploration.

- **Questions for discussion:** This discussion can either be led as a whole class activity or take place in pairs/small groups. The emphasis is on students being able to observe mathematical relationships by changing objects on their screen. They should try to describe what happens, and explain why.

- **Problem:** Students are expected to try the problem with pen and paper first then check it on their software. The purpose is for them to formalise what they have learnt through exploration and discussion and apply this to a “standard” style question. Students could write-up their answers to the discussion questions and their solution to this problem in their notes to help consolidate their learning and provide evidence of what they’ve achieved. This problem can be supplemented with additional textbook questions at this stage if appropriate.

- **Further Tasks:** Extension activities with less structure for students who have successfully completed the first three sections.

Task 1: The Modulus Function

Students should consider how this relates to the graph of $y = ax+b$

Problem solution:

$x = -\frac{5}{3}, \frac{1}{3}\ y = -1$

Students might need some help structuring the investigation into $|x + a|+b > 0$. One strategy is to fix either a or b and investigate changing the other parameter.

Task 2: Inverse functions

The aim of this task is to reinforce the link between the reflection in the line $y = x$ and rearranging $y = \mathrm{f}(x)$ to express x in terms of y. The software can plot this using $x = \mathrm{f}(y)$

Problem solutions:

$f^{-1}(x) = \sqrt{x} - 3$ $\quad g^{-1}(x) = \sqrt[3]{x}$ $\quad h^{-1}(x) = \frac{1}{x} + 2$

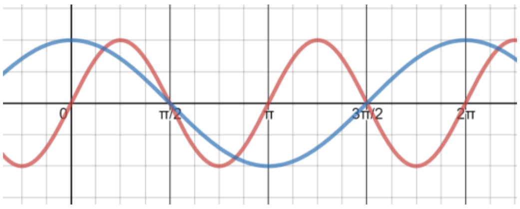
It is important to emphasise that the domain of the original function needs to be restricted so that it is one-to-one for the inverse to be a function.

Task 3: Trigonometry – Double Angle formulae

Students might need some help structuring the investigation into $\sin x \cos x = a \sin(bx)$. One strategy is to fix b and investigate changing a first to find a curve with the correct amplitude.

Use of the compound angle formulae for $\sin(a+b)$ and $\cos(a+b)$ might be useful for some students to verify their results.

Problem solution:



$\theta = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$

Task 4: Trigonometry: *R*cos(*θ*–*α*)

Students are expected to be able to relate their findings to the expansion of $R\cos(x - A) = R\cos x \cos A + R\sin x \sin A$.

Problem solution:
$4\cos\theta + 3\sin\theta = 5\cos(\theta - 0.644)$.

Task 5: Differentiation – Trigonometric functions

By considering key points the students should be able to observe that this has the same shape as cos(x).

Problem solution:

$y = \frac{x}{2} + \frac{\sqrt{3}}{2} - \frac{\pi}{6}$ or $y = 0.5x + 0.342$

Task 6: Trigonometry – Solving equations in radians

This task encourages students to think about the symmetries of the trigonometric graphs and use these in finding solutions to equations.

Problem solution:
$x = \frac{-11\pi}{6}, \frac{-7\pi}{6}, \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$

Task 7: Derivative of the natural logarithm $y=\ln x$

This task can be done on its own or with task 6. The aim of this task is for students to be able to find the gradients and equations of tangents to the natural logarithm function.

For the second discussion point students might be surprised that the result doesn't change but they should be encouraged to think of this in terms of laws of logs.

Problem solution:

$y = 0.5x - 0.307$

Task 8: Converting parametric equations to cartesian equations

In the discussion questions students could also consider why the cartesian version does not plot the full curve given by the parametric version.

Solutions to discussion questions:

$x = 2t+1,\ y=\frac{1}{t}\ :\quad y=\frac{2}{x-1}$ $\qquad x=\cos t, y=\sin t:\quad x^2+y^2=1$

Trig-based parametric equations will often require identities to convert to cartesian form.

Problem solution:

$y = (\ln x)^2 + 1$

Task 9: Partial Fractions

This task can be used as an introduction to partial fractions or as a consolidation exercise. Students should be encouraged to express their methods algebraically.

Solutions to partial fractions:

$$\frac{5x-1}{(x+1)(x-2)} = \frac{2}{x+1} + \frac{3}{x-2}$$

$$\frac{2x+7}{(x+2)(x+3)} = \frac{3}{x+2} - \frac{1}{x+3}$$

$$\frac{7x-14}{(x+3)(x-4)} = \frac{5}{x+3} + \frac{2}{x-4}$$

$$\frac{5x^2+3x+7}{(x+2)(x^2+3)} = \frac{3}{x+2} + \frac{2x-1}{x^2+3}$$

$$\frac{7x^2+29x+28}{(x-1)(x+3)^2} = \frac{4}{x-1} + \frac{3}{x+3} - \frac{1}{(x+3)^2}$$

Task 10: Numerical Methods – Change of sign

This task is a set of instructions for how to implement the change of sign method on the software. Students are encouraged to work through these instructions and then try solving some equations of their own.

It is useful to have some additional equations for students to be finding the roots of once they have completed this sheet.