MEI Desmos Tasks for GCSE Mathematics

Using these tasks

These tasks are designed to help students in understanding mathematical relationships better through exploring dynamic constructions. They can be accessed using the computer-based version of Desmos or the tablet/smartphone app. Each task is reproducible on a single piece of paper and they are designed to be an activity for a single lesson or a single homework task (approximately).

The algebra tasks have been designed with the following structure –

- **Construction:** Step-by-step guidance of how to construct the objects in Desmos. Students will benefit from learning the rigorous steps need to construct objects and this also removes the need to make prepared files available to them. If students become confident with using Desmos they can be encouraged to add additional objects to the construction to aid their exploration.

- **Questions for discussion:** This discussion can either be led as a whole class activity or take place in pairs/small groups. The emphasis is on students being able to observe mathematical relationships by changing objects on their screen. They should try to describe what happens, and explain why.

- **Problem:** Students are expected to try the problem with pen and paper first then check it on their software. The purpose is for them to formalise what they have learnt through exploration and discussion and apply this to a “standard” style question. Students could write-up their answers to the discussion questions and their solution to this problem in their notes to help consolidate their learning and provide evidence of what they’ve achieved. This problem can be supplemented with additional textbook questions at this stage if appropriate.

- **Further Tasks:** Extension activities with less structure for students who have successfully completed the first three sections.

The geometry tasks have been designed with the following structure –

- **Construction:** Step-by-step guidance of how to construct the objects in Desmos. Students will benefit from learning the rigorous steps need to construct objects and this also removes the need to make prepared files available to them. If students become confident with using Desmos they can be encouraged to add additional objects to the construction to aid their exploration.

- **Questions for discussion:** This discussion can either be led as a whole class activity or take place in pairs/small groups. The emphasis is on students being able to observe mathematical relationships by changing objects on their screen. They should try to describe what happens, and explain why.

- **Task:** Students should construct the geometrical objects in Desmos using the properties they’ve observed in the construction and discussion.

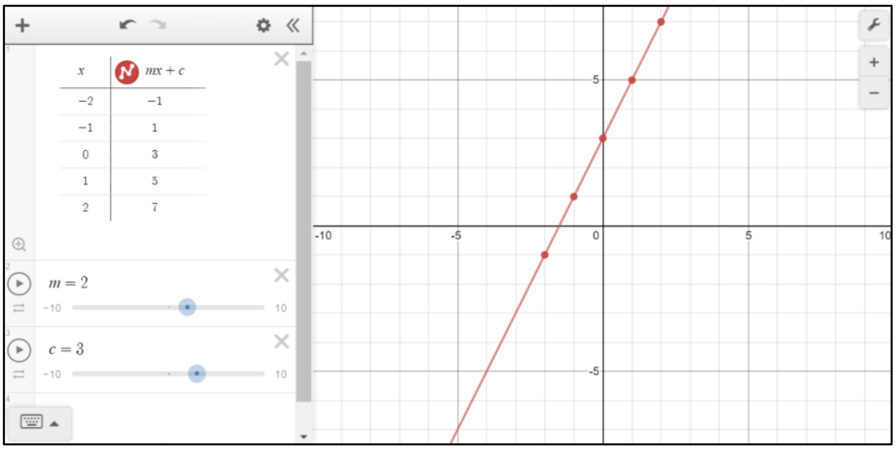
Task 1: Algebra – Equation of a line: $y = mx + c$

1. Add the equation: $\boldsymbol{y = mx + c}$

   In the *add sliders* option click **all**.

2. Add a table for $y = mx + c$ by clicking the cog icon in the list of equations:

   then selecting the table icon for $y = mx + c$:



Questions for discussion

- How does changing c affect the line?
- How does changing m affect the line?
- Choose any two rows from the table. What is the link between the difference of the x-coordinates and the difference of the y-coordinates?
- How could you use this pattern to find the equation of the line from two sets of coordinates?

Try setting m and c to values such as 1, 2, 3… (click in the box and enter 1 for the step value).
Change one value at a time.
What do you notice – what stays the same and what changes?

Write down the coordinates of 2 points and then subtract the x and y parts separately. Use this to predict a different point on the line.

Problem *(Try the problem with pen and paper first then check it on Desmos)*

Find the equation of the line through the points with coordinates $(2,1)$ and $(4,5)$.

Further Tasks

- Investigate the equations of straight lines with a negative gradient – where can you see the negative in the equation?
- Investigate the equations of vertical and horizontal lines – how do their equations look different?
- How do you find the equation of a line when you are told its gradient and a point on the line? e.g. find the equation of the line with gradient 3 that passes through $(1,5)$.

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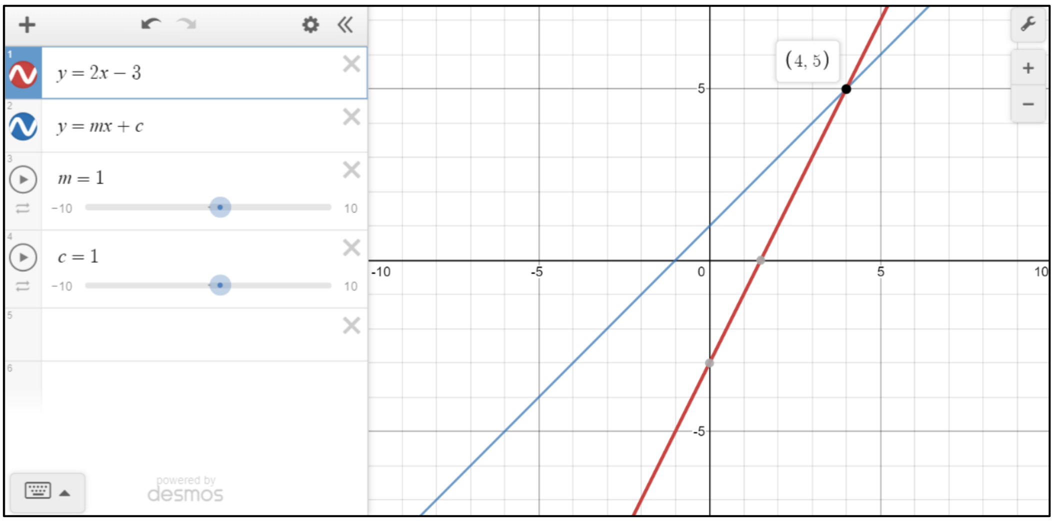
Task 2: Algebra – Intersection of two lines

1. Add the equation: $\boldsymbol{y = 2x - 3}$

2. Add the equation: $\boldsymbol{y = mx + c}$

   In the *add sliders* option click **all**.

3. Click on the point of intersection to view its coordinates.



Questions for discussion

- How does changing c affect the point of intersection?
- How does changing m affect the point of intersection?
- How could you find the points of intersection from the equations of the lines? Hint: the point of intersection is a solution to both equations
- When will the lines not have points of intersection?

Try this for some different pairs of lines.

Will your lines ever intersect if you zoom out far enough?

Problem *(Try the problem with pen and paper first then check it on Desmos)*

Find the coordinates of the point of intersection of the lines $y = 3x - 2$ and $y = x + 4$.

Further Tasks

- Find an example of a pair of lines that would have an intersection where:
  - The x-value is positive and the y-value is negative.
  - The x-value is negative and the y-value is positive.
  - The x-value is negative and the y-value is negative.

- Investigate different possible equations of lines that pass through a single point, e.g. find some equations of lines that pass through the point $(2,3)$.

Task 3: Algebra – Parallel lines

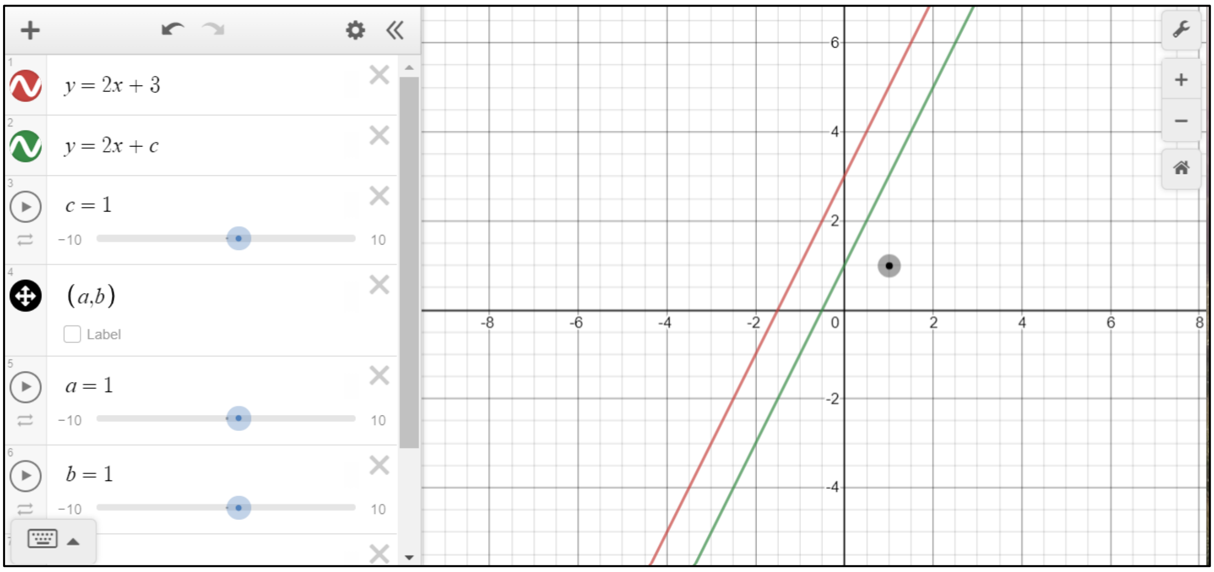
1. Add the equation: $\boldsymbol{y = 2x + 3}$

2. Add the equation: $\boldsymbol{y = 2x + c}$

   In the *add sliders* option click $\boldsymbol{c}$.

3. Add the point: $(\boldsymbol{a}, \boldsymbol{b})$

   In the *add sliders* option click **all**.



Questions for discussion

- What value of c will make the line parallel to $y = 2x + 3$ pass through the point $(1,1)$?

- Move the point so that a and b are different. How can you find the equation of the line parallel to $y = 2x + 3$ that passes through the point? (Think about what information you know about this new line.)

Problem *(Try the problem with pen and paper first then check it on Desmos)*

Find the equation of the line that passes through the point $(2,1)$ and is parallel to $y = 3x + 2$.

Further Tasks

- Investigate lines parallel to the x-axis and lines parallel to the y-axis. What do you notice about these equations?

- Investigate the distance between a pair of parallel lines. How can you find the distance if you know 1 point on each line? Can you write a general formula to find this distance in every case?

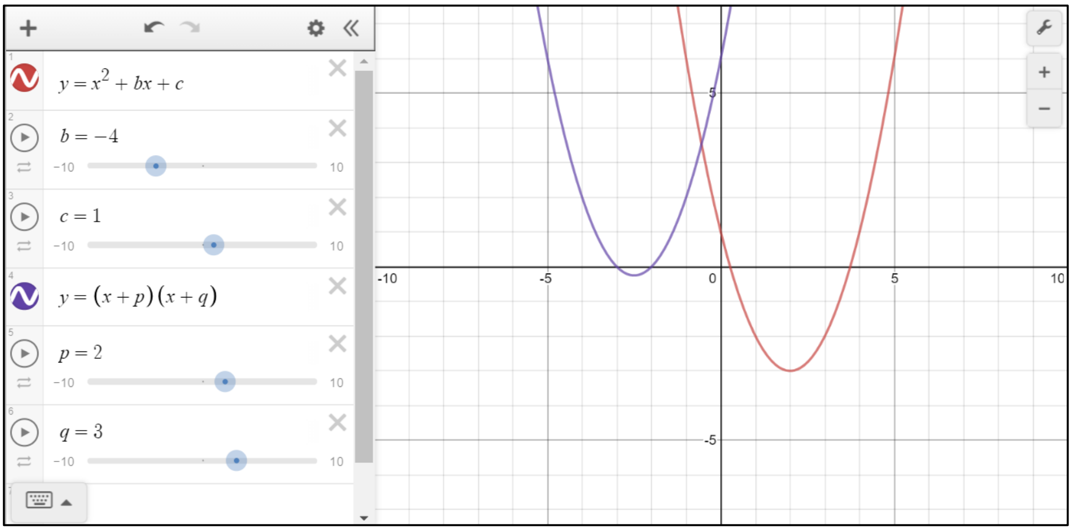
Task 4 – Algebra: roots of quadratic functions

1. Add the equation: $\boldsymbol{y = x^2 + bx + c}$

   In the *add sliders* option click **all**.

2. Add the equation: $\boldsymbol{y = (x + p)(x + q)}$

   In the *add sliders* option click **all**.



Questions for discussion

- Why does $y = (x + p)(x + q)$ cut the x-axis at $x = -p$ and $x = -q$?
- How can you find values of b, c, p and q so that the two graphs are the same?
- Are there any values of b and c where $y = x^2 + bx + c$ can't be written as $y = (x + p)(x + q)$?

What is the y-value when $x = -p$ or $x = -q$?

Try changing b and c and then predicting what p and q would need to be.

Is it possible for a quadratic graph not to have roots?

Problem *(Try the problem with pen and paper first then check it on Desmos)*

Solve the quadratic equation $x^2 - x - 6 = 0$ and hence sketch the curve with equation $y = x^2 - x - 6$.

Further Tasks

- Find a condition on the values of b and c such that the curve $y = x^2 + bx + c$ doesn't cross the x-axis.
- Investigate graphs with equations $y = ax^2 + bx + c$. How does changing the value of a, b, c affect the shape of the graph? How does it affect the position of the graph?

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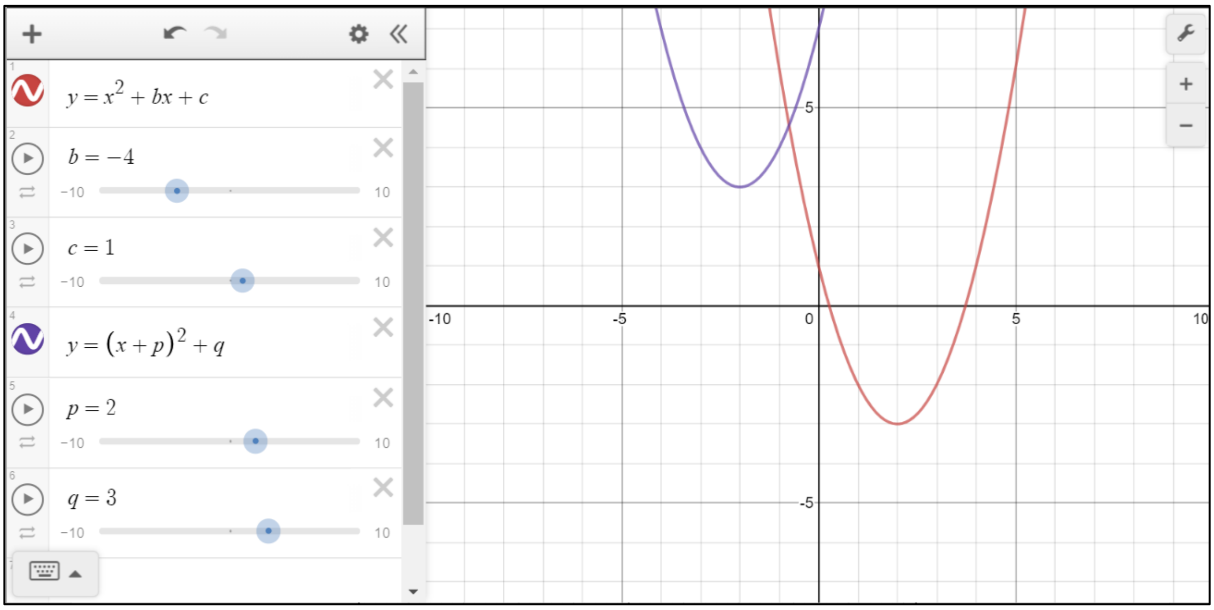
Task 5 – Algebra: completed square form

1. Add the equation: $\boldsymbol{y = x^2 + bx + c}$

   In the *add sliders* option click **all**.

2. Add the equation: $\boldsymbol{y = (x + p)^2 + q}$

   In the *add sliders* option click **all**.



Questions for discussion

- Where is the turning point on the graph of $y = (x + p)^2 + q$ (and why)?
- Where is the line of symmetry of the curve $y = (x + p)^2 + q$ in terms of p and q?
- Can you change the values of b & c and p & q to give the same curve?
- If you move the graph of $y = x^2 + bx + c$ how could you find values of p and q so that $y = (x + p)^2 + q$ gave the same curve?

Problem *(Try the problem with pen and paper first then check it on Desmos)*

Find the coordinates of the turning point on the curve with equation $y = x^2 + 6x + 5$.

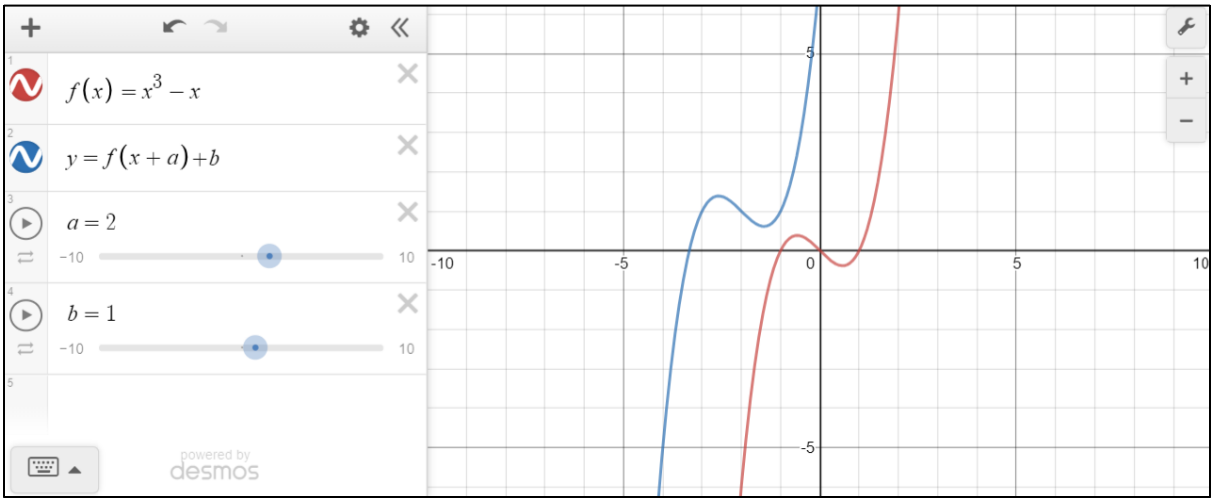
Further Tasks

- Investigate how you can use the completed square form to factorise quadratic equations.
- Investigate the minimum points on graphs with equations of the form $y = ax^2 + bx + c$ (where $a \neq 1$). How is this linked to the completed square format?

Task 6 – Algebra: transformation of functions

1. Add the equation: $\boldsymbol{f(x) = x^3 - x}$
2. Add the equation: $\boldsymbol{y = f(x + a) + b}$

   In the *add sliders* option click **all**.

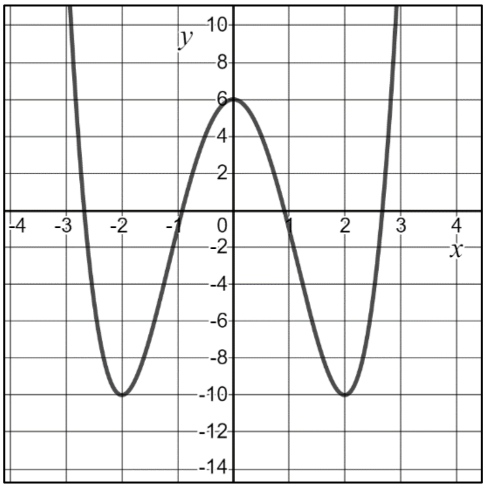


Questions for discussion

- How does changing the value of a affect the curve, and why?
- How does changing the value of b affect the curve, and why?

Problem

(Try the problem with pen and paper first then check it on Desmos)



The graph of $y = \mathrm{f}(x)$ for the function $\mathrm{f}(x) = x^4 - 8x^2 + 6$ is shown.

Sketch the graph of $y = \mathrm{f}(x + 1) + 2$.
Predict what this would look like before your plot the graph

Further Tasks

- Investigate the transformations of the curve $y = \mathrm{f}(x)$ obtained by plotting $y = -\mathrm{f}(x)$ or $y = \mathrm{f}(-x)$.
- Investigate combinations of transformations such as $y = -\mathrm{f}(x + a) + b$ or $y = \mathrm{f}(-x + a) + b$.

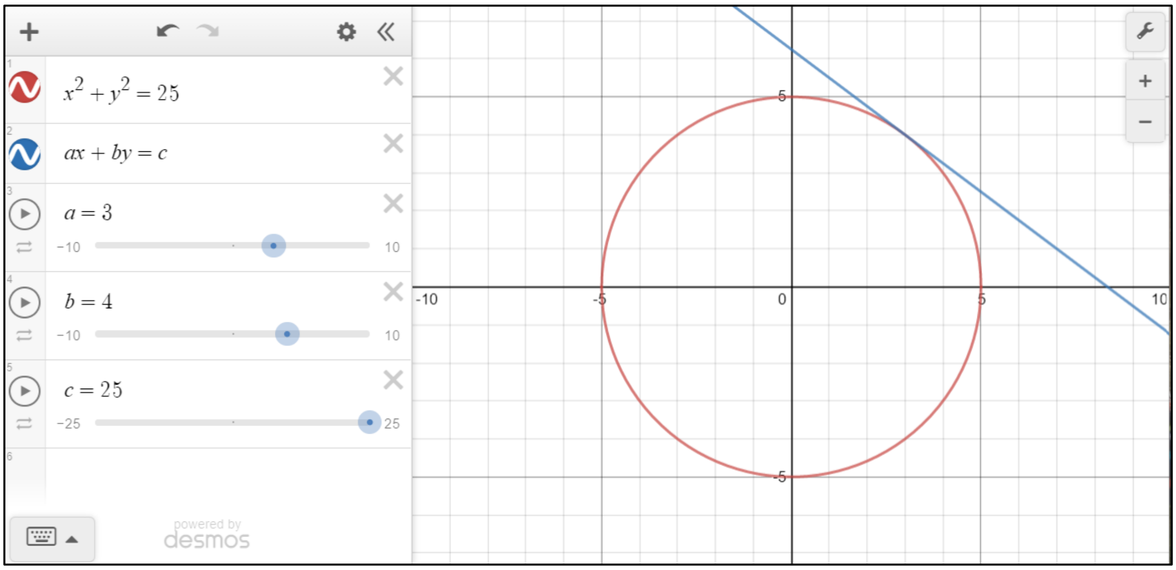
Task 7 – Algebra: equations of tangents to circle

1. Add the equation: $\boldsymbol{x^2 + y^2 = 25}$
2. Add *the* equation: $\boldsymbol{ax + by = c}$

   In the *add sliders* option click **all**.

You can edit the **-10** and **10** at either end of the slider to change the maximum and minimum.

3. Change the slider for $\boldsymbol{c}$ so that its minimum is -30 and its maximum is 30?
4. Click the line – you will see dots appear at each intersection point.
5. Vary the sliders to display the line $\boldsymbol{3x + 4y = 25}$



Questions for discussion

- How can you tell that the line $3x + 4y = 5$ is a tangent to the circle $x^2 + y^2 = 25$? Hint: what do you know about a tangent to a circle and the radius of the circle?
- What other values of a, b and c will give a line that is a tangent to the circle?

Problem *(Try the problem with pen and paper first then check it on Desmos)*

Find the equation of the tangent to the circle $x^2 + y^2 = 169$ at the point $(5, 12)$.

Further Tasks

- What is the equation of the radius of the circle in terms of a & b? What do you notice about the intersection points of these two lines?
- Investigate pairs of points on a circle where the two tangents will be either parallel or perpendicular lines. What do you notice?
- For a circle centred on the origin investigate the conditions under which a straight line will intersect it 0, 1 or 2 times.

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Task 8 – Geometry: Transformations

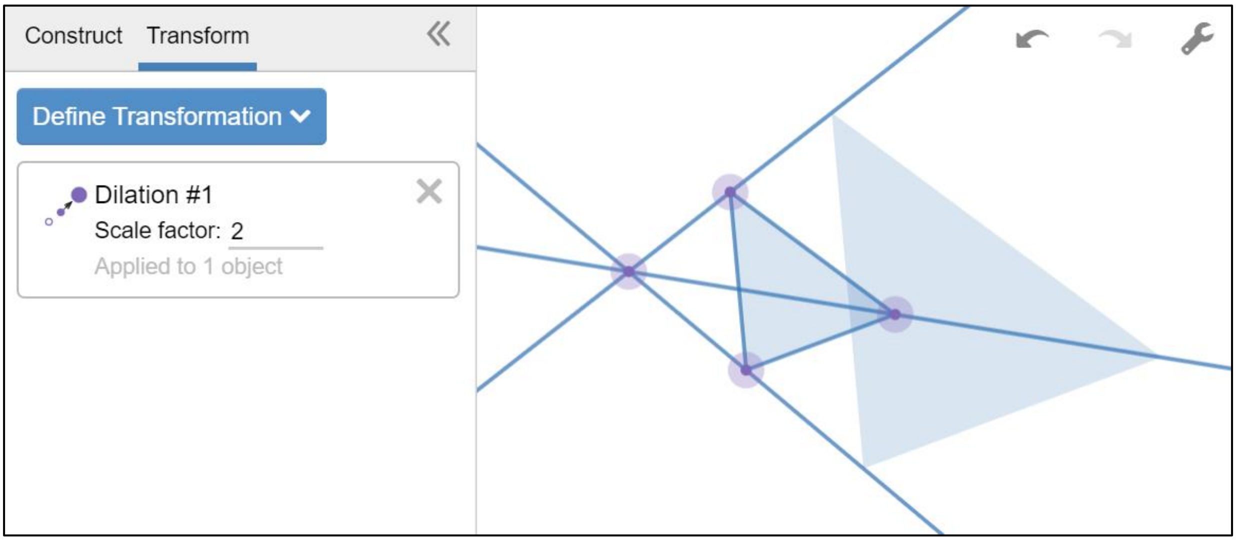
This task uses Desmos Geometry: www.desmos.com/geometry

Constructing the Enlargement from a point by a scale factor

1. Add a **Point** (this will be the centre of the enlargement).
2. Add a **Polygon** (click on each new point then the first point again to finish it).
3. Use the **Line** tool to lines that pass through each vertex (i.e. corner) of the polygon and the original point.
4. Select **Transform** > **Define Transformation** > **Dilation**.

   Dilation is the American term for enlargement.

   Choose the original point, click the polygon, choose your scale factor and then **Apply**.



Questions for discussion

- What happens to the enlarged shape (the image) as you move the centre of enlargement closer to the original shape?
- What happens to the enlarged shape (the image) if you make the scale factor a negative number?
- What scale factors make the image smaller than the original shape?

Task

Create similar files to demonstrate the following transformations:

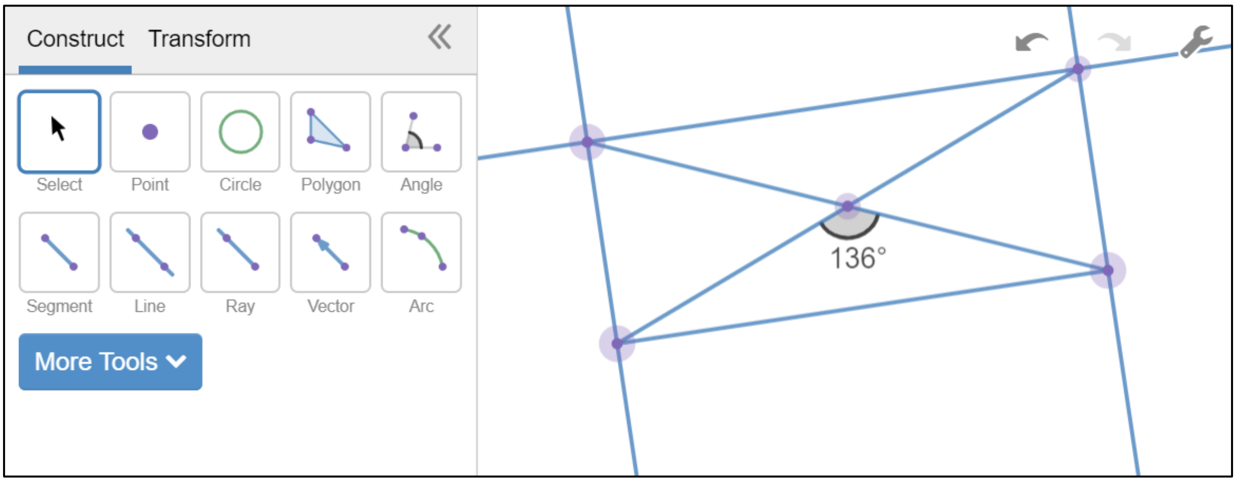
- Rotation about a point.
- Reflection about a line.
- Translation.

Task 9 – Geometry: Angles in quadrilaterals

This task uses Desmos Geometry: www.desmos.com/geometry

How to construct a rectangle and measure the sizes of angles:

1. Add a line **Segment**.
2. Use **More Tools** > **Perpendicular Line** to construct perpendicular lines through each of the ends of the line segment.
3. Add a **Point** on one of the perpendicular lines.
4. Use **More Tools** > **Perpendicular Line** to construct a perpendicular line through the new point (this should complete the rectangle.
5. Add a **Point** at the fourth corner of the rectangle.
6. Use the **Segment** tool to construct the diagonals.
7. Add a **Point** at the intersection of the diagonals
8. Measure the **Angle** between the diagonals at the point of intersection.



Question for discussion

- Without measuring any other angles, how can you find the sizes of the other angles in the diagram from just this angle?

Use Desmos to check your answers

Task

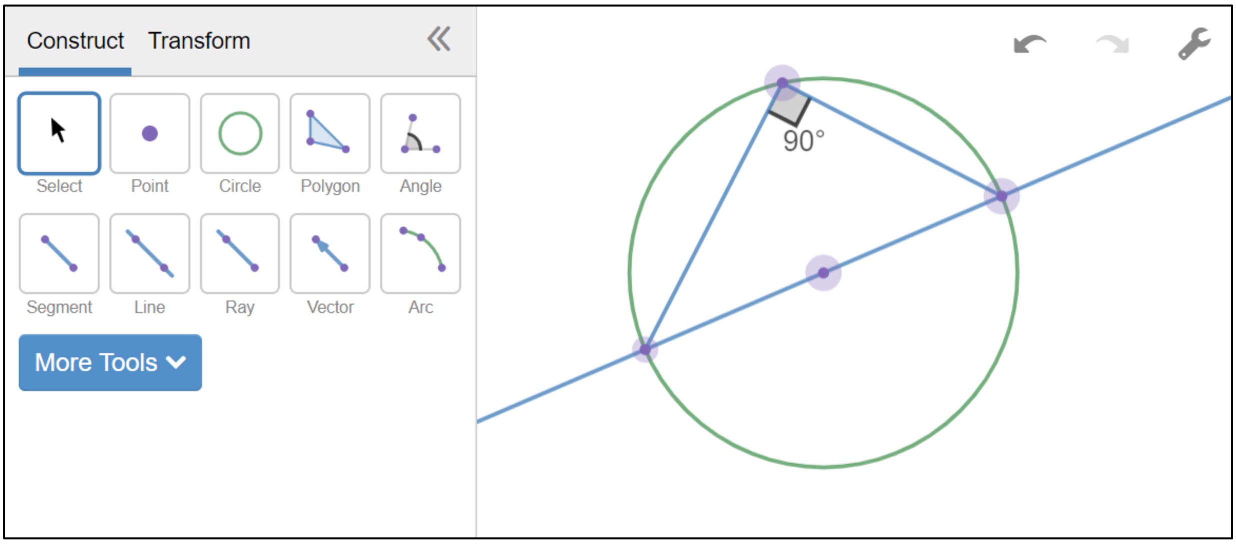
- Create a parallelogram using a similar method.
- Investigate the link between the sizes of angles in a parallelogram.

Task 10 – Geometry: Circle theorems

This task uses Desmos Geometry: www.desmos.com/geometry

Constructing the angle in a semi-circle is a right-angle:

1. Add a **Circle**.
2. Add a **Line** through the centre and the point on the radius of the circle (click the centre, then the circle)
3. Add a **Point** on the circle (not at the intersection points).
4. Use **Segment** to add in two line segments to create the sides of the triangle.
5. Measure the **Angle** at the circumference by clicking on the three points in the triangle (the angle you want to measure must be the 2nd point you click)



Questions for discussion

- Is the angle always 90° wherever you drag the points to?
- What happens if you draw a line that does not pass through the centre?

Task

Create similar files to demonstrate the following Circle theorems:

- Angles at the circumference in the same segment are equal.
- The line perpendicular to the radius on a circle is a tangent.
- The angle at the centre is twice the angle at the circumference.
- The sum of opposite angles in a cyclic quadrilateral is 180°

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Teacher guidance

Task 1: Algebra – Equation of a line: $y = mx + c$

Foundation/Higher

This task is designed to help students understand how the equation of a line determines its shape and how the equation can be obtained from two points on the line.

Students should be encouraged to try varying m and c and describe how these affect the shape of the line. Trying some systematic set of values is helpful here, as is trying to predict what the line would look like for other values. One common observation is that the line moves sideways as c varies – a useful suggestion if students say this is to ask them to try and describe any movement in terms of m and c (a horizontal description of the movement is possible but a vertical description is much simpler).

One strategy that students can use to observe the relationship between two points on the line is to write down the coordinates of two points and use this to predict a further point on the line.

Problem solution: $y = 2x - 3$

Task 2: Algebra – Intersection of two lines

Foundation/Higher

This task is aimed at making the link between finding the intersection of two lines graphically and algebraically.

Students should be encouraged to try finding the point of intersection of a pair of lines algebraically. Choosing an example so the intersection point isn't a pair of whole numbers is useful to discuss the importance of the algebraic approach.

Displaying the table of values can help students make the connection to the numerical values of x and y being the same on both lines.

Problem solution: $(3,7)$

Task 3: Algebra – Parallel lines

Foundation/Higher

This task is designed to support students understanding the line between parallel lines.

Students should be encouraged to vary both m and c and observe the impact. The fact that c doesn't change the equation of the parallel line is worth observing. One useful strategy is to move A to a new point, such as $(1,3)$ and try to predict what the equation of the through it will be. This task can be contrasted with task 1 in that either two points on a line or one point a line along with the gradient are sufficient to find its equation.

A useful question to ask students is: "What do you need to know to find the equation of a straight line?".

Students who have completed the main task can use the same approach to investigating parallel lines.

Problem solution: $y = 3x - 5$

Task 4 – Algebra: roots of quadratic functions

Foundation/Higher

In this task students investigate the link between the expanded and factorised form of a quadratic equation by considering the graphs of $y = x^2 + bx + c$ and $y = (x + p)(x + q)$.

Students should know the term *root* to describe where the graph of a function crosses the x-axis (i.e. $y = 0$) and this can be explicitly linked to the factorised form: $y = (x + p)(x + q)$. It is important that students understand the difference between graphing $y = x^2 + bx + c$ and solving the equation $x^2 + bx + c = 0$.

Students should be encouraged to try finding examples in both directions: setting b and c and then finding p and q or setting p and q and then find b and c. Choosing cases where one of the values is 0 might be helpful.

When finding examples of quadratics that don't factorise it is important to make the distinction between ones that have roots, but don't factorise over the integers, and ones that don't have any roots.

Problem solution: $x = -2$ and $x = 3$

Task 5 – Algebra: completed square form

Higher

In this task students investigate the link between the expanded and completed square form of a quadratic equation by considering the graphs of $y = x^2 + bx + c$ and $y = (x + p)^2 + q$.

Students should be encouraged to consider quadratics where b (the coefficient of x) is an odd number.

The completed square form and its graph can be related to the general transformation of a functions in task 6.

Problem solution: Turning point is at $(-3, -4)$

Task 6 – Algebra: transformation of functions

Higher

The purpose of this task is to demonstrate the effect of transformations. Get students to discuss if the changes they see are what they expect.

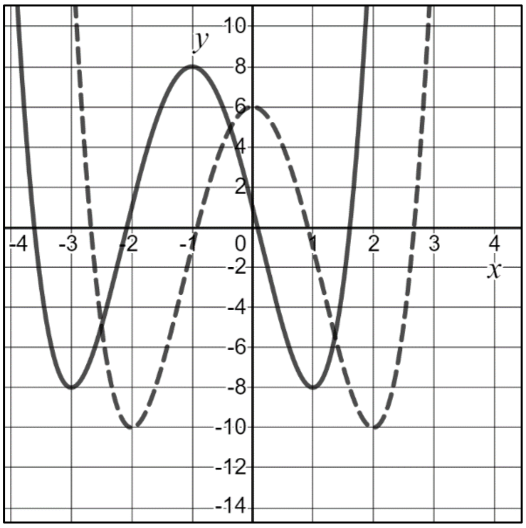
Students should be encouraged to compare the effects of changing a and b.
Hopefully they will notice that changes in b relate to vertical translations whereas changes in a relate to horizontal translations.

Highlight the link between the value of b and the y intercept – would they be the same if f(x) had a constant?

They are likely to 'see' that if a=3 the graph moves 3 to the left – students should be encouraged to try to explain why it doesn't move to the right.

For the further tasks ask pupils to describe each of the reflections.
Key question: Can these transformations be applied in any order?

Problem solution:



Task 7 – Algebra: equations of tangents to circle

Higher

Students should be encouraged to consider the number of intersections with the circle. They can get Desmos to display dots at intersection points by clicking the line.

Ask them to consider simple solutions to the problem first e.g. horizontal and vertical lines

As an extension and to link to the circle theorem, you can ask them to find the equation of the radius - it is perpendicular to the straight line and passes through the origin so can be represented in terms of the variables a & b. This shows dynamically that the only time the radius and line intersect at the circumference is when the line is a tangent.

When considering two different tangents, ask students to consider whether they intersect and if they do what they notice about the distance from the circle to each intersection point.

Problem solution: $y = -2.4x + 33.8$

Task 8 – Geometry: Transformations

Foundation/Higher

Encourage pupils not to create uniform shapes as it is more difficult to see the effect of the enlargement.

Use the dynamic software to make sure pupils make the link between the proximity of the Centre of Enlargement to the shape and the distance from the shape to the image. They often think that moving the Centre of Enlargement closer will ‘push’ the image further away.

When investigating rotations encourage discussion around the need for a direction of rotation - in which case is it not needed? Is there more than one way to describe the same rotation?

Task 9 – Geometry: Angles in quadrilaterals
Foundation/Higher

This activity allows pupils to consider how much information they can get from one angle and to consider key properties of 2D shapes.

Encourage students to make links to what the know about angles between parallel lines as well as what they know about angle sums.

Task 10 – Geometry: Circle theorems
Higher

The aim of this task is to get students creating dynamic files that demonstrate all the circle theorems.

Encourage students to 'break' the theorems – e.g. what happens when the line does not pass through the centre? Therefore what do we know about the line if the angle is $90°$?
They need to be able to use the information from a circle theorem from both 'directions': angle in a semi-circle is a right-angle ⇔ if angle at circumference is $90°$ the line is a diameter.

For the second theorem encourage investigation into angles that aren't in the same segment – how does that link to the fact that the angles in a cyclic quadrilateral add to 180°?

For the angle at the centre – what happens when you move the point from the centre?