# MEI GeoGebra Tasks for GCSE Mathematics

## Using these tasks

These tasks are designed to help students in understanding mathematical relationships better through exploring dynamic constructions. They can be accessed using the computer-based version of GeoGebra or the tablet/smartphone app. Each task is reproducible on a single piece of paper and they are designed to be an activity for a single lesson or a single homework task (approximately).

## Algebra tasks

The algebra tasks have been designed with the following structure –

- **Construction:** Step-by-step guidance of how to construct the objects in GeoGebra. Students will benefit from learning the rigorous steps need to construct objects and this also removes the need to make prepared files available to them. If students become confident with using GeoGebra they can be encouraged to add additional objects to the construction to aid their exploration.

- **Questions for discussion:** This discussion can either be led as a whole class activity or take place in pairs/small groups. The emphasis is on students being able to observe mathematical relationships by changing objects on their screen. They should try to describe what happens, and explain why.

- **Problem:** Students are expected to try the problem with pen and paper first then check it on their software. The purpose is for them to formalise what they have learnt through exploration and discussion and apply this to a "standard" style question. Students could write-up their answers to the discussion questions and their solution to this problem in their notes to help consolidate their learning and provide evidence of what they've achieved. This problem can be supplemented with additional textbook questions at this stage if appropriate.

- **Further Tasks:** Extension activities with less structure for students who have successfully completed the first three sections.

## Geometry tasks

The geometry tasks have been designed as consolidation activities. Students should attempt these once they are familiar with the content.

A step-by-step construction is given to display a particular geometrical property. Once constructed students can verify that this property holds by dragging points on the screen and observing which quantities remain fixed.

They should then attempt the task of constructing a file for one of the other similar properties, though they don't need to do these in the order given. Although they will be checking their understanding of the property, the main challenge is for them to construct the object. This will enable them to practice their problem-solving skills in a geometrical context and encourage them to work rigorously. The teacher guidance contains suggestions for how to help students with the constructions.

# MEI GeoGebra Tasks for GCSE Mathematics

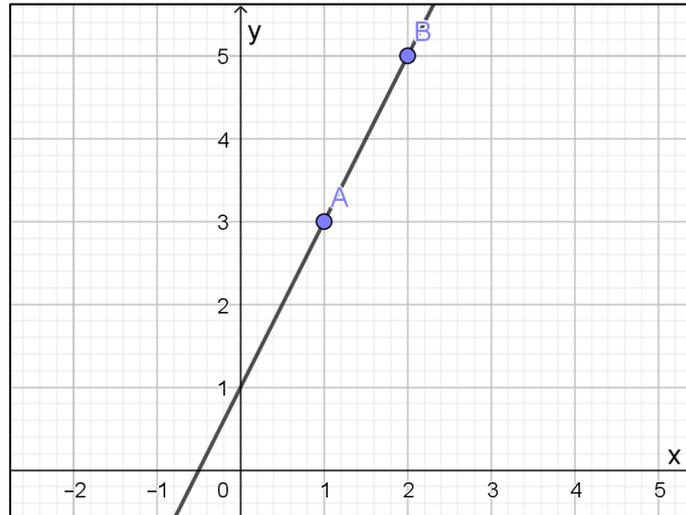## Task 1: Algebra – Equation of a line: $y = mx + c$

1. In the input bar enter: **y=mx+c**
   If prompted select *Create Sliders*.

> The points should move along the line as you drag them

2. Use the **Point** tool to add two points to the line, **A** and **B**.

3. Vary the line with the sliders and the points by dragging them.



## Questions for discussion

- How does changing $c$ affect the line?

> Try setting $m$ and $c$ to values such as 1, 2, 3…
> What do you notice – what stays the same and what changes?

- How does changing $m$ affect the line?

- What is the link between the difference of the $x$-coordinates and the difference of the $y$-coordinates?

> Write down the coordinates of A and B and then subtract the $x$ and $y$ parts separately. Use this to predict a different point on the line.

- How could you find the equation of the line from the two pairs of coordinates?

**Problem** (Try the problem with pen and paper first then check it on your software)

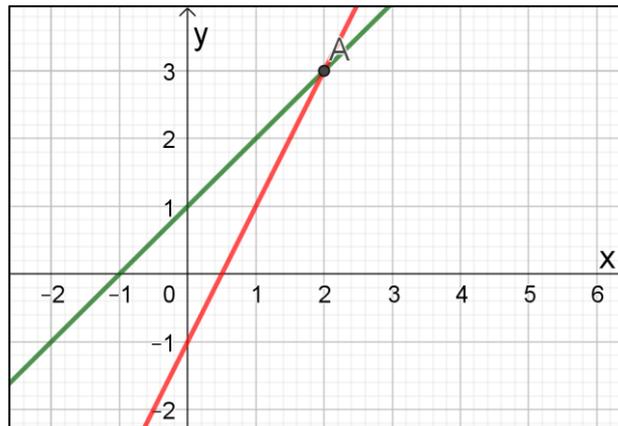Find the equation of the line through the points with coordinates (2,1) and (4,5).

## Further Tasks

- Investigate how to find the equation of a line from the gradient of the line and a point on it, e.g. find the equation of the line with gradient 3 that passes through (1,5).

- Investigate lines with negative gradients.

- Investigate the equations of vertical and horizontal lines.

# MEI GeoGebra Tasks for GCSE Mathematics

# Task 2: Algebra – Intersection of two lines

1. In the input bar enter: **y=mx+c**
   If prompted select *Create Sliders*.

2. In the input bar enter: **y=nx+d**
   If prompted select *Create Sliders*.

3. Vary the line with the sliders so they are different.

4. Use the **Intersect** tool to find the point of intersection of the lines.



## Questions for discussion

- How could you find the point of intersection from the equations of the lines?

  Find the point just using algebra and check it with your graphs. Try this for some different pairs of lines.

- When will the lines not have points of intersection?

  Will your lines ever intersect if you zoom out far enough?

**Problem** (Try the problem with pen and paper first then check it on your software)

Find the coordinates of the point of intersection of the lines $y = 3x - 2$ and $y = x + 4$.

## Further Tasks

- Find an example of a pair of lines that would have an intersection where:
   - The x-value is positive and the y-value is negative.
   - The x-value is negative and the y-value is positive.
   - The x-value is negative and the y-value is negative.

- Investigate different possible equations of lines that pass through a single point, e.g. find some equations of lines that pass through the point (2,3).

# MEI GeoGebra Tasks for GCSE Mathematics

# Task 3: Algebra – Parallel lines

1. In the input bar enter: **y=mx+c**
   If prompted select *Create Sliders*.

2. Use the **Point** tool to create a point, **A**.
   NB this point should not be on the line.

3. Use the **Parallel Line** tool to create a line parallel
   to the original line through the point.

4. Vary the line with the sliders and the point by dragging it .
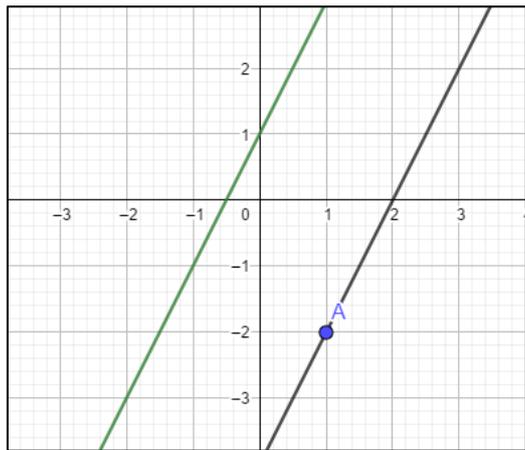
## Questions for discussion

- What is the link between the equations
  of parallel lines?

  Try setting $m$ and $c$ to 1, 2, 3…
  What do you notice – what stays the
  same and what changes?

- How could you find the equation of a
  line if you knew a point on the line and
  the equation of a line parallel to it?

  If you move A to a new point can you
  predict what the equation will be?

**Problem** (Try the problem with pen and paper first then check it on your software)

Find the equation of the line that passes through the point $(2,1)$ and is parallel to $y = 3x + 2$.

## Further Tasks

- Use the **Perpendicular line** tool to investigate the relationship between
  perpendicular lines.

- Investigate the distance between a pair of parallel lines.

# MEI GeoGebra Tasks for GCSE Mathematics

# Task 4 – Algebra: roots of quadratic functions

1. In the Input bar enter: **y=x²+bx+c**
   If prompted select *Create Sliders.*

2. In the Input bar enter: **y=(x+p)(x+q)**
   If prompted select *Create Sliders.*
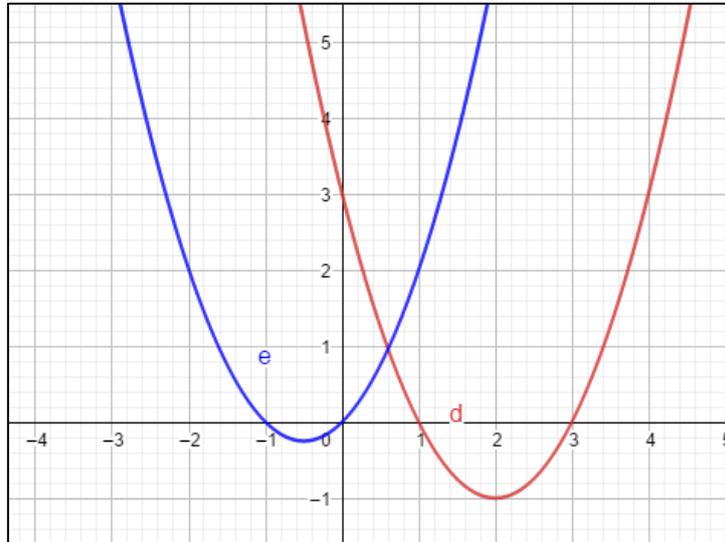


## Questions for discussion

- Why does $y = (x + p)(x + q)$ have roots at $x = -p$ and $x = -q$?

  > What is the $y$-value when $x = -p$ or $x = -q$?

- How can you find values of $b$, $c$, $p$ and $q$ so that the two graphs are the same?

  > Try changing $b$ and $c$ and then predicting what $p$ and $q$ would need to be.

- Are there any values of $b$ and $c$ where $y = x^2 + bx + c$ can't be written as $y = (x + p)(x + q)$?

  > Is it possible for a quadratic graph not to have roots?

**Problem** (Try the problem with pen and paper first then check it on your software)

Solve the quadratic equation $x^2 - x - 6 = 0$ and hence sketch the curve with equation $y = x^2 - x - 6$.

## Further Tasks

- Find a condition on the values of $b$ and $c$ such that the curve $y = x^2 + bx + c$ doesn't cross the $x$-axis.

- Investigate graphs with equations $y = ax^2 + bx + c$.

# Task 5 – Algebra: completed square form

1. In the Input bar enter: **y=x²+bx+c**
   If prompted select *Create Sliders*.

2. In the Input bar enter: **y=(x+p)²+q**
   If prompted select *Create Sliders*.
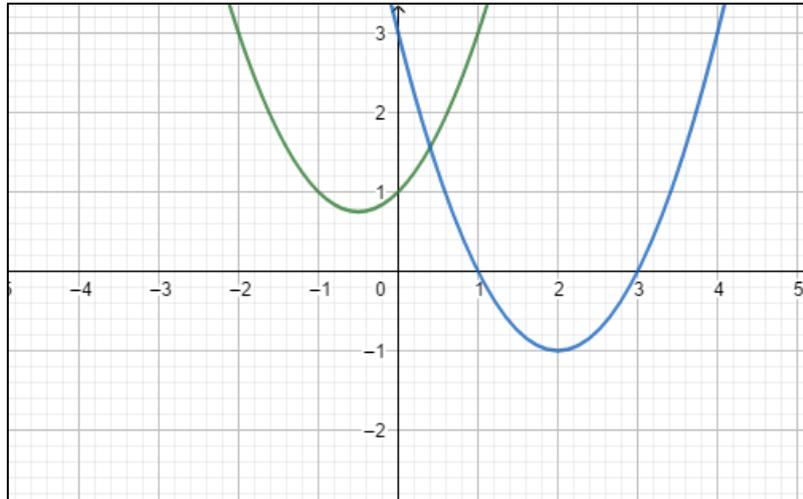


## Questions for discussion

- Where is the minimum point on the graph of $y = (x + p)^2 + q$ (and why)?

Try setting $p$ to 1, 2, 3, … and $q$ to 1, 2, 3, … and finding the minimum point.

- Where is the line of symmetry of the curve $y = (x + p)^2 + q$?

- If you move the graph of $y = x^2 + bx + c$ how could you find values of $p$ and $q$ so that $y = (x + p)^2 + q$ gave the same curve?

If you change $b$ and $c$ can you find values of p and q so the curves are the same?

Try doing this by experimenting with a few different values of $b$ and $c$ and then look for a link between the values.

**Problem** (Try the problem with pen and paper first then check it on your software)

Find the coordinates of the minimum point on the curve with equation $y = x^2 + 6x + 5$.

## Further Tasks

- Investigate how you can use the completed square form to factorise quadratic equations.

- Investigate the minimum points on graphs with equations of the form $y = ax^2 + bx + c$ (where $a \neq 1$).

# Task 6 – Algebra: transformation of functions

1. In the Input bar enter: **f(x)=x³–x**

2. In the Input bar enter: **y=f(x+a)+b**
   If prompted select *Create Sliders*.
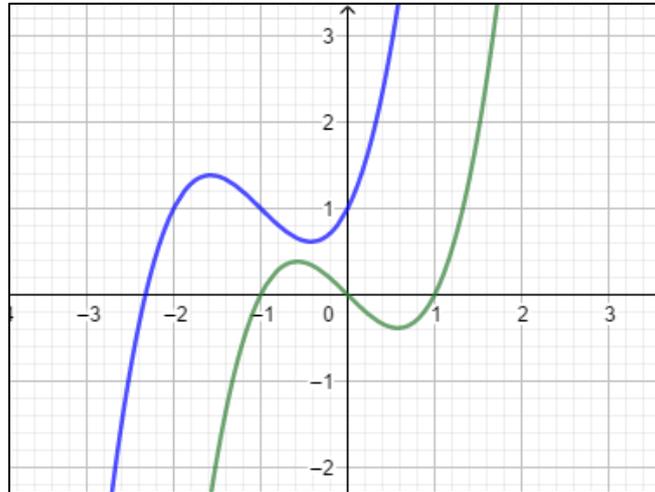


## Questions for discussion

- How does varying $a$ affect the curve, and why?

> Try setting $a$ to 1, 2, 3, … or −1, −2, −3, … and describe how this moves the curve.
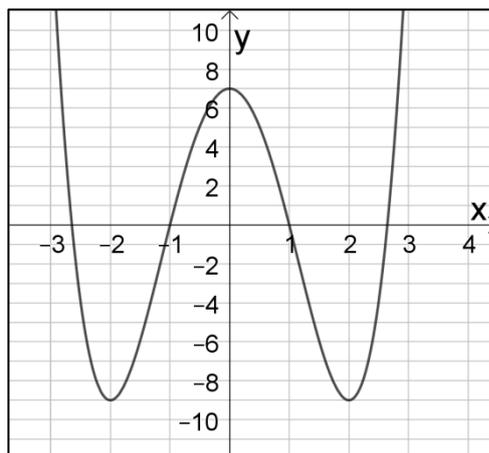
- How does varying $b$ affect the curve, and why?

> Try setting $b$ to 1, 2, 3, … or −1, −2, −3, … and describe how this moves the curve.

## Problem
(Try the problem with pen and paper first then check it on your software)

The graph of $y = f(x)$ for the function $f(x) = x^4 - 8x^2 + 7$ is shown.

Sketch the graph of $y = f(x + 1) + 2$.



## Further Tasks

- Investigate the transformations of the curve $y = f(x)$ obtained by plotting $y = -f(x)$ or $y = f(-x)$.

- Investigate combinations of transformations such as $y = -f(x + a) + b$ or $y = f(-x + a) + b$.

# MEI GeoGebra Tasks for GCSE Mathematics

## Task 7 – Algebra: equations of tangents to circle

1. In the Input bar enter: **$x^2+y^2=25$**

> The point should move along the circle as you drag it.

2. Use the **Point** tool to add a point to the circle, **A**.
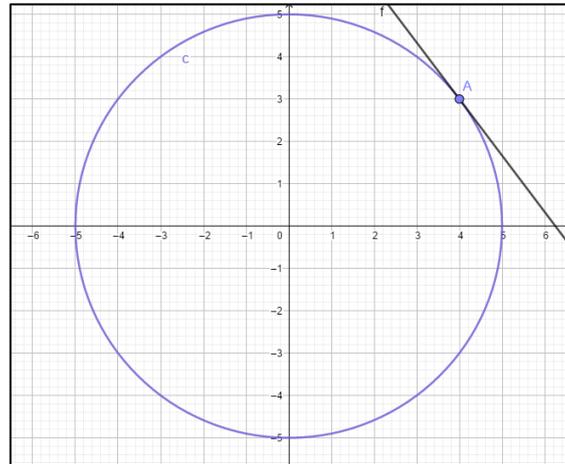
3. Use the **Tangent** tool to add a tangent the curve at A.

> Click once on the point and once on the circle.

### Questions for discussion

- What is the link between the coordinates of the point A and the gradient of the line?

> Move A to some different points where the coordinates are whole numbers and write down the equation of the line.

- How can you find the equation of the line?

> If you know the gradient of a line and a point on it how can you find its equation?

**Problem** (Try the problem with pen and paper first then check it on your software)

Find the equation of the tangent to the circle $x^2 + y^2 = 169$ at the point $(5,12)$.

### Further Tasks

- Investigate pairs of points on a circle where the tangents will be either parallel or perpendicular.

- For a circle centred on the origin investigate the conditions under which a straight line will intersect it 0, 1 or 2 times.

# MEI GeoGebra Tasks for GCSE Mathematics

# Task 8 – Geometry: Transformations

### *To construct a file showing reflection in the line $y = x$:*

1.  Use **Point** to add four points **A**, **B**, **C** and **D**.

    > The polygon tool can be found in the Polygon menu (*More* on the Graphing app).

2.  Use **Polygon** to create the quadrilateral ABCD.

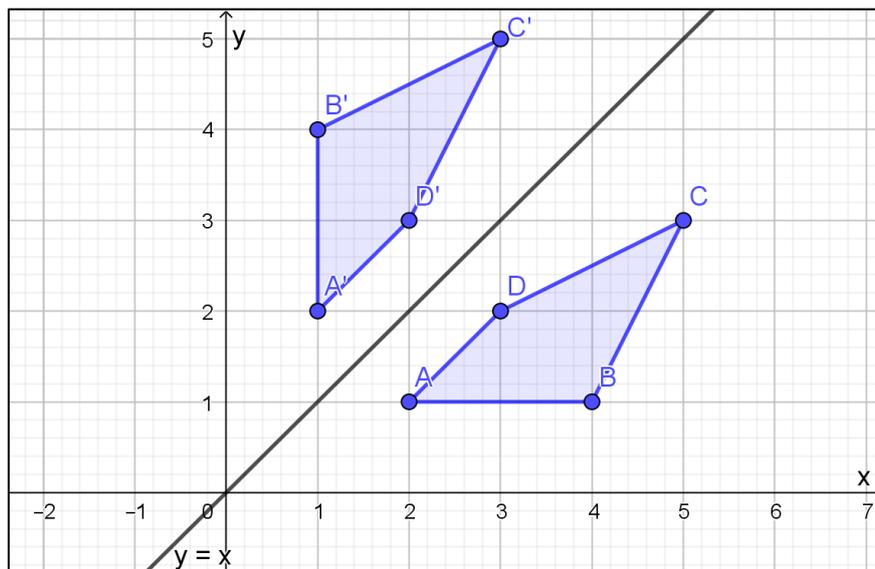    > Click on A, B, C, D then A again to complete it.

3.  In the input bar enter: **y=x**

4.  Use **Reflect about line** to reflect the quadrilateral in the line $y = x$.

    > The Reflect about line tool can be found in the Transform menu (*More* on the Graphing app).
    >
    > Click on the polygon then the line $y = x$ to reflect it.



## Task

Create similar GeoGebra files to demonstrate the following transformations:

*   Reflection about:
    *   The line $y = -x$.
    *   The $x$-axis.
    *   The $y$-axis.

*   Rotation about a point.

    > Construct a file showing rotation by a standard angle, such as 90°, or use a slider to vary the angle.

*   Translation by a vector.

*   Enlargement from a point by a scale factor.

    > Construct a file showing enlargement by a standard scale factor, such as 2, or use a slider to vary the number.

# MEI GeoGebra Tasks for GCSE Mathematics

# Task 9 – Geometry: Properties of quadrilaterals

*To construct a file showing the diagonals of a rectangle are the same length:*

1.  Use the **Point** tool to add two points **A** and **B**.

2.  Use the **Line** tool to construct the line through A and B.

3.  Use the **Perpendicular line** tool to construct the line that is perpendicular to AB and passes through A.

    > Click once on the line AB and once on the point A.

4.  Use the **Point** tool to add a point **C** on the perpendicular line created in step 5.

5.  Use the **Perpendicular line** tool to construct:
    a.  The line the line that is perpendicular to AB and passes through B.
    b.  The line the line that is perpendicular to AC and passes through C.

6.  Use the **Intersect** tool to find the point of intersection of the lines created in step 5. This point should be created as **D**.

7.  Use the **Line segment** tool to find the length of the diagonals AD and BC.
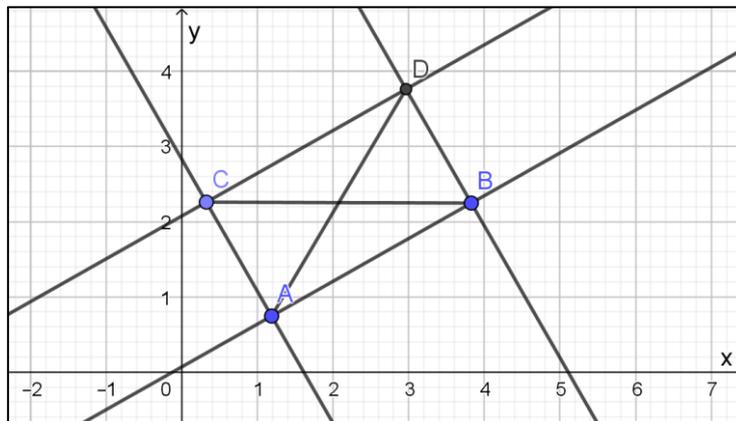
## Task

Create similar GeoGebra files to demonstrate the following transformations:

-   The diagonals of a square are the same length.

    > Use **Regular polygon** from the Polygon tools (*More* in the app)

-   Adjacent angles in a parallelogram sum to 180°.

    > Use the **Parallel line** tool to construct this

-   The diagonals of a rhombus cross at right-angles.

-   The diagonals of a kite cross at right-angles.

    > The **Circle** tool will help you construct lines of the same length

# MEI GeoGebra Tasks for GCSE Mathematics

# Task 10 – Geometry: Circle theorems

## *To construct a file showing that the angle in a semi-circle is a right-angle:*

1. Use **Semi-Circle Through 2 Points** tool to add a semi-circle to the screen with end-points **A** and **B**.

2. Use **Point** to add a new point on the circle, **C**.

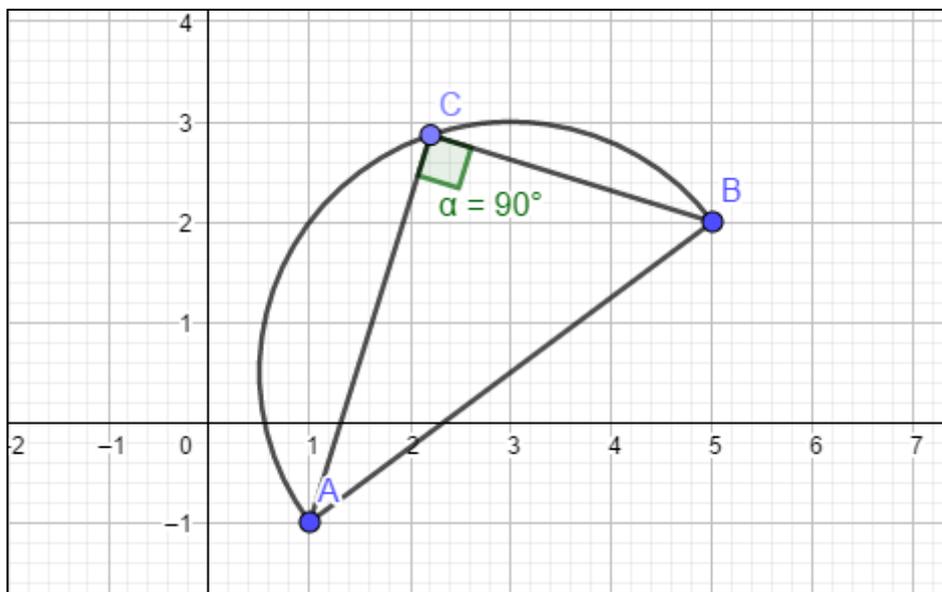   > The point should move along the semicircle as you drag it.

3. Use the **Segment** tool to construct line segments AB, AC and BC.

4. Use the **Angle** tool to measure the angle ACB.

   > With the tool selected click on A, then C, then B.



Check that the angle $ACB = 90°$ whatever the positions of $A$, $B$ and $C$ are.

## Task
Create similar GeoGebra files to demonstrate the following Circle theorems:

- The radius and tangent at a point are perpendicular.

- Angles at the circumference in the same segment are equal.

- The angle at the centre is twice the angle at the circumference.

- The sum of opposite angles in a cyclic quadrilateral is 180°.

# MEI GeoGebra Tasks for GCSE Mathematics

# Teacher guidance

### Task 1: Algebra – Equation of a line: $y = mx + c$
*Foundation/Higher*

This task is designed to help students understand how the equation of a line determines its shape and how the equation can be obtained from two points on the line.

Students should be encouraged to try varying $m$ and $c$ and describe how these affect the shape of the line. Trying some systematic set of values is helpful here, as is trying to predict what the line would look like for other values. One common observation is that the line moves sideways as $c$ varies – a useful suggestion if students say this is to ask them to try and describe any movement in terms of $m$ and $c$ (a horizontal description of the movement is possible but a vertical description is much simpler).

One strategy that students can use to observe the relationship between two points on the line is to write down the coordinates of two points and use this to predict a further point on the line.

Problem solution: $y = 2x - 3$

### Task 2: Algebra – Intersection of two lines
*Foundation/Higher*

This task is aimed at making the link between finding the intersection of two lines graphically and algebraically.

Students should be encouraged to try finding the point of intersection of a pair lines algebraically. Choosing an example so the intersection point isn't a pair of whole numbers is useful to discuss the importance of the algebraic approach.

Problem solution: $(3,7)$

### Task 3: Algebra – Parallel lines
*Foundation/Higher*

This task is designed to support students understanding the line between parallel lines.

Students should be encouraged to vary both $m$ and $c$ and observe the impact. The fact that $c$ doesn't change the equation of the parallel line is worth observing. One useful strategy is to move A to a new point, such as $(1,3)$ and try to predict what the equation of the through it will be. This task can be contrasted with task 1 in that either two points on a line or one point a line along with the gradient are sufficient to find its equation.

Students who have completed the main task can use the same approach to investigating parallel lines.

Problem solution: $y = 3x - 5$

# MEI GeoGebra Tasks for GCSE Mathematics

## Task 4 – Algebra: roots of quadratic functions
*Foundation/Higher*

In this task students investigate the link between the expanded and factorised form of a quadratic equation by considering the graphs of $y = x^2 + bx + c$ and $y = (x + p)(x + q)$.

Students need to know the term *root* to describe where the graph of a function crosses the $x$-axis (i.e. $y = 0$) and this can be explicitly linked to the factorised form: $y = (x + p)(x + q)$. It is important that students understand the difference between graphing $y = x^2 + bx + c$ and solving the equation $x^2 + bx + c = 0$.

Students should be encouraged to try finding examples in both directions: setting $b$ and $c$ and then finding $p$ and $q$ or setting $p$ and $q$ and then find $b$ and $c$. Choosing cases where one of the values is 0 might be helpful.

When finding examples of quadratics that don't factorise it is important to make the distinction between ones that have roots, but don't factorise over the integers, and ones that don't have any roots.

Problem solution: $x = -2$ and $x = 3$

## Task 5 – Algebra: completed square form
*Higher*

This task is designed for students to make the link between the completed square form of a quadratic curve, $y = (x + p)^2 + q$, and the minimum point $(-p, q)$. It is a useful consolidation exercise for students who have learnt completing the square.

Students should be encourage to try different values of $p$ and $q$ and use this to confirm the position of the minimum point. It is useful to think about this in terms of the movement of the point $(0,0)$ on the graph of $y = x^2$ and to relate this to transformations of functions generally (see task 6). Setting $p$ and $q$ to different integer values and writing down the position of the minimum point is a good strategy for students to start with.

To make the link with the completed square form students should seek examples where both of the curves are the same. Students should be encouraged to try finding examples in both directions: setting $b$ and $c$ and then finding $p$ and $q$ or setting $p$ and $q$ and then find $b$ and $c$. Choosing cases where one of the values is 0 might be helpful.

Problem solution: Turning point is at $(-3, -4)$

## Task 6 – Algebra: transformation of functions
*Higher*

In this task students investigate how the graph of $y = f(x + a) + b$ is a translation of the graph of $y = f(x)$ by the vector $\begin{pmatrix} -a \\ b \end{pmatrix}$.

Students should be encourage to change each of the values of $a$ and $b$ separately and move the curve. The roots of the original function are useful reference points to observe how the curve has changed. For the horizontal change it is helpful to consider what values of $x$ are required such that $(x + a)$ is $-1$, 0 or 1.

Problem solution:
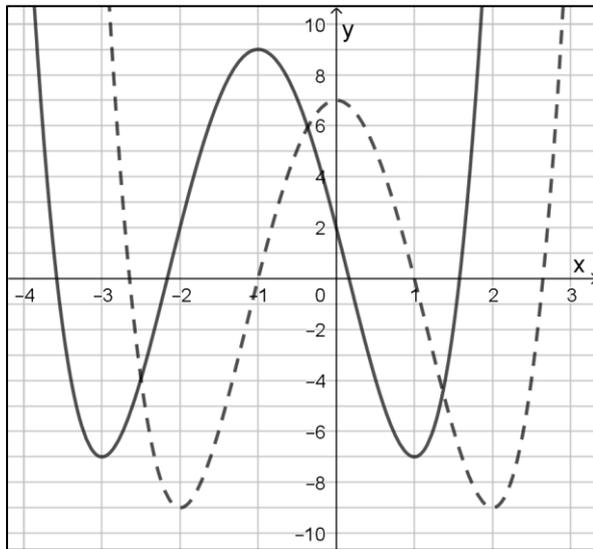


## Task 7 – Algebra: equations of tangents to circle
*Higher*

In this task students investigate the equations of tangents to circles. The circle $x^2 + y^2 = 25$ has been chosen as it has points with non-zero integer coordinates, such as $(3,4)$.

Students should try moving the point around the circle to a point with integer coordinates. It is useful to consider the gradient of the line from the origin to the point and the fact that a perpendicular line will have gradient $-\frac{1}{m}$.

Problem solution: $y = -2.4x + 33.8$

## Task 8 – Geometry: Transformations
*Foundation/Higher*

In this task students construct files demonstrating transformations of a quadrilateral. All of the transformations can be found as tools in the Transform menu.

- Reflections.
  These are similar to the construction given. To reflect in one of the axes click on the axis (or enter the equations $x = 0$ or $y = 0$).

- Rotation about a point.
  A separate point needs to be added for the centre of rotation. Adding a slider with the slider tool gives the option of setting it as an angle ranging from 0° to 360°.

- Translation by a vector.
  To create a vector add two separate points and then use the Vector tool.

- Enlargement from a point by a scale factor.
  A separate point needs to be added for the centre of enlargement. Adding a slider for $k$ allows for the enlargement to be dynamic. It is also helpful to add lines from the centre of enlargement to each of the points.

# MEI GeoGebra Tasks for GCSE Mathematics

## Task 9 – Geometry: Properties of quadrilaterals
*Foundation/Higher*

In this task students construct quadrilaterals so that they can check their properties. The main challenge is construct each type of example so that it is valid when the points are dragged, e.g. to construct a rhombus they need it to have two pairs of parallel sides, all four of which are the same length.

Guidance for constructing the various types of quadrilaterals:

- Square – add two points and use the regular polygon tool.

- Parallelogram – the construction is similar to the rectangle but point C does not need to be on a perpendicular line (it can be a free point), and the parallel line tool can be used for the sides.

- Rhombus – add two points A and B, find the circle through B with centre A and add a point C on this circle, construct lines AB and AC then use the parallel line tool.

- Kite – add two points A and B, construct the line AB, add a point C and then reflect this in the line AB to produce the kite ACBC'.

## Task 10 – Geometry: Circle theorems
*Higher*

In this task students construct circles so that they can check the circle theorems. The main challenge is construct the objects so that the properties hold when the points are dragged.

Guidance for constructing the circle properties (it is useful to create a circle with centre A through point B in each case but then not use B as a point on the circle for the circle property as dragging this changes the circle):

- The radius and tangent at a point are perpendicular.
  Create a circle centre A through point B. Add a point C on the circle. Add the line segment AC for the radius. Add a tangent to the circle through C. Add a point on the tangent D. Use the angle measurement tool to measure the angle DCA (NB to measure inside an angle click on the three points in clockwise order).

- Angles at the circumference in the same segment are equal.
  Create a circle centre A through point B. Add points on the circle C, D, E and F. Add line segments CE, DE, CF and DF. Measure the angles CED and CFD.

- The angle at the centre is twice the angle at the circumference.
  Create a circle centre A through point B. Add points C, D and E on the circle. Add line segments CA, DA, CE and DE. Measure the angles CAD and CED.

- The sum of opposite angles in a cyclic quadrilateral is 180°.
  Create a circle centre A through point B. Add points on the circle C, D, E and F. Add line segments CD, DE, EF and FC. Measure each of the four angles CDE, DEF, EFC and FCD. The angles can be summed in the Algebra panel, e.g. $\mathbf{sum1 = \alpha + \gamma}$.