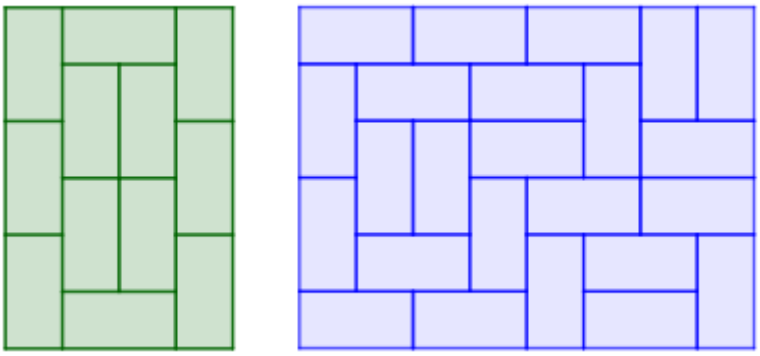
MEI Maths Item of the Month

November 2020
Fault-free tilings

A *fault-free* tiling is an arrangement of tiles in an $m \times n$ grid such that there are no vertical or horizontal lines that can be placed on the grid without crossing one of the tiles.



In the image above a 6×4 and a 6×8 grid have been tiled with 2×1 dominoes. The 6×4 tiling has 2 vertical fault lines and is therefore not fault free. The 6×8 tiling is fault-free.

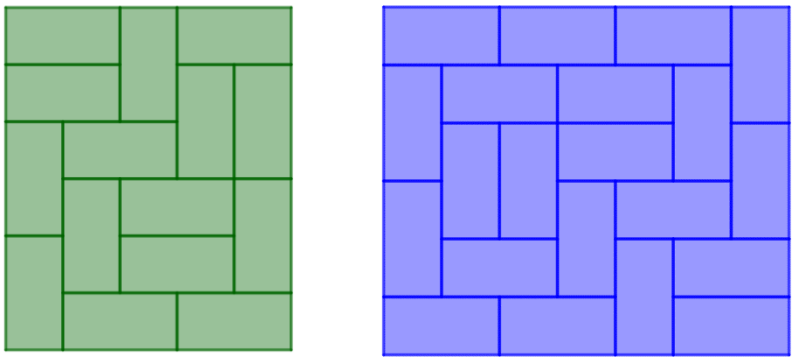
Is it possible, using 2×1 dominoes, to find fault-free tilings of:

- a 6×5 grid
- a 6×6 grid
- a 6×7 grid

Solution

6×5 and 6×7 grids are both possible. A 6×6 grid is not possible.

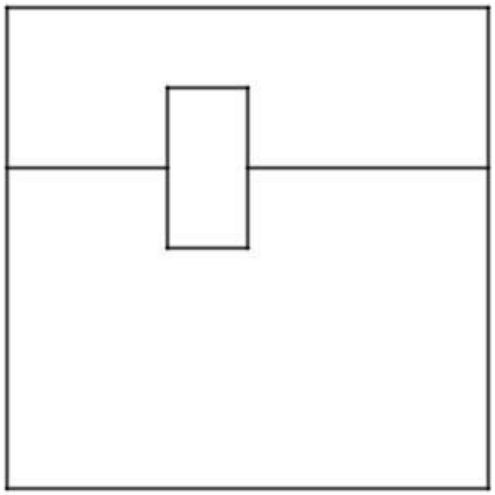
Possible solutions for 6×5 and 6×7 grids are shown below.



To show that a 6×6 grid is not possible, imagine you have achieved a fault-free tiling of a 6 by 6 grid.

Each domino breaks exactly one potential fault line. In a 6 by 6 grid there are 10 (internal) fault lines – five horizontal and five vertical – that need breaking and with only 18 dominoes it is clear that at least one of these lines would be broken by only one domino.

Assume the rest of the tiling looks like this.



Either side of the line there is an odd number of squares but these can't be covered by dominoes. Contradiction! Therefore it is not possible to tile a 6 by 6 grid in a fault-free way.