

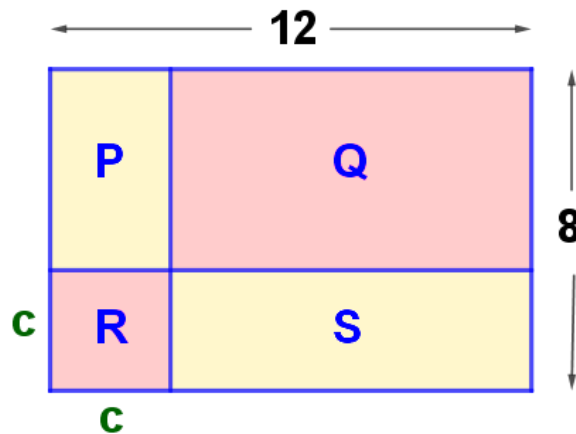
MEI Maths Item of the Month

October 2020

Ritangle 2020

A rectangle has sides of length 12 and 8 units. A square of side c is drawn in one corner, creating the rectangular areas P , Q , R and S as in the diagram.

What is the minimum value that $\frac{Q+R}{P+S}$ can take?



Solution

The areas are:

$$P = (8 - c)c$$

$$Q = (12 - c)(8 - c)$$

$$R = c^2$$

$$S = (12 - c)c$$

$$\begin{aligned}\frac{Q + R}{P + S} &= \frac{96 - 20c + 2c^2}{20c - c^2} \\ &= \frac{96}{20c - c^2} - 1\end{aligned}$$

The minimum value of $\frac{96}{20c - c^2}$ occurs at the maximum value of $20c - c^2$.

$20c - c^2 = -2((c - 5)^2 - 25)$ which has a maximum of 50.

Therefore the maximum value of $\frac{Q+R}{P+S}$ is $\frac{96}{50} - 1 = \frac{23}{25}$.