OCR RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) 4764

Mechanics 4

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
- Scientific or graphical calculator

Tuesday 15 June 2010
Morning

Duration: 1 hour 30 minutes

4764

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

Section A (24 marks)

1 At time t a rocket has mass m and is moving vertically upwards with velocity v. The propulsion system ejects matter at a constant speed u relative to the rocket. The only additional force acting on the rocket is its weight.

(i) Derive the differential equation $m\frac{dv}{dt} + u\frac{dm}{dt} = -mg$. **[4]**

The rocket has initial mass m_0 of which 75% is fuel. It is launched from rest. Matter is ejected at a constant mass rate k.

(ii) Assuming that the acceleration due to gravity is constant, find the speed of the rocket at the instant when all the fuel is burnt. **[8]**

2 A particle of mass m kg moves horizontally in a straight line with speed $v\,\text{m s}^{-1}$ at time t s. The total resistance force on the particle is of magnitude $mkv^{\frac{3}{2}}$ N where k is a positive constant. There are no other horizontal forces present. Initially $v = 25$ and the particle is at a point O.

(i) Show that $v = 4\left(kt + \frac{2}{5}\right)^{-2}$. **[7]**

(ii) Find the displacement from O of the particle at time t. **[3]**

(iii) Describe the motion of the particle as t increases. **[2]**

Section B (48 marks)

3 A uniform rod AB of mass m and length $4a$ is hinged at a fixed point C, where AC = a, and can rotate freely in a vertical plane. A light elastic string of natural length $2a$ and modulus λ is attached at one end to B and at the other end to a small light ring which slides on a fixed smooth horizontal rail which is in the same vertical plane as the rod. The rail is a vertical distance $2a$ above C. The string is always vertical. This system is shown in Fig. 3 with the rod inclined at θ to the horizontal.

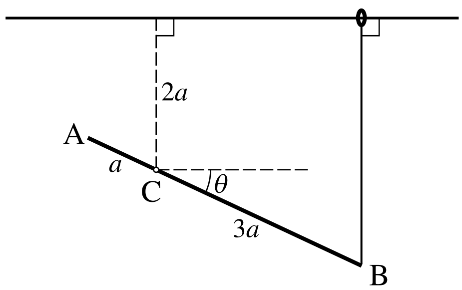

Fig. 3

(i) Find an expression for V, the potential energy of the system relative to C, and show that $\frac{dV}{d\theta} = a\cos\theta\left(\frac{9}{2}\lambda\sin\theta - mg\right)$. **[6]**

(ii) Determine the positions of equilibrium and the nature of their stability in the cases

(A) $\lambda > \frac{2}{9}mg$, **[10]**

(B) $\lambda < \frac{2}{9}mg$, **[4]**

(C) $\lambda = \frac{2}{9}mg$. **[4]**

4 Fig. 4.1 shows a uniform cone of mass M, base radius a and height $2a$.

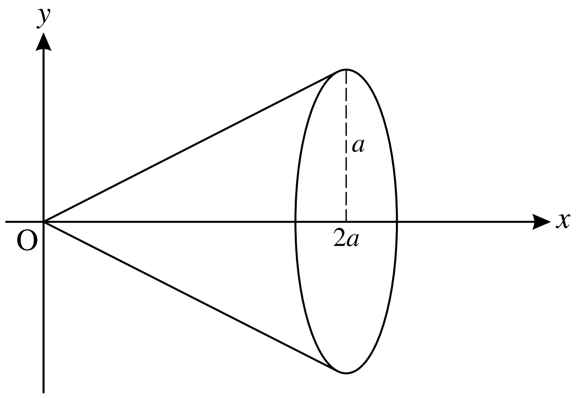

Fig. 4.1

(i) Show, by integration, that the moment of inertia of the cone about its axis of symmetry is $\frac{3}{10}Ma^2$. [You may assume the standard formula for the moment of inertia of a uniform circular disc about its axis of symmetry and the formula $V = \frac{1}{3}\pi r^2 h$ for the volume of a cone.] **[8]**

A frustum is made by taking a uniform cone of base radius 0.1 m and height 0.2 m and removing a cone of height 0.1 m and base radius 0.05 m as shown in Fig. 4.2. The mass of the frustum is 2.8 kg.

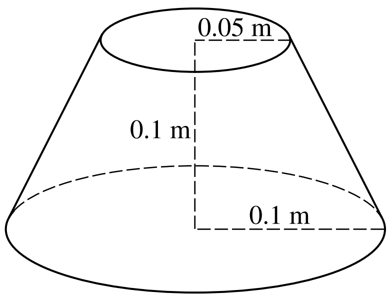

Fig. 4.2

The frustum can rotate freely about its axis of symmetry which is fixed and vertical.

(ii) Show that the moment of inertia of the frustum about its axis of symmetry is 0.0093 kg m^2. **[4]**

The frustum is accelerated from rest for t seconds by a couple of magnitude 0.05 N m about its axis of symmetry, until it is rotating at 10 rad s^{-1}.

(iii) Calculate t. **[4]**

(iv) Find the position of G, the centre of mass of the frustum. **[3]**

The frustum (rotating at 10 rad s^{-1}) then receives an impulse tangential to the circumference of the larger circular face. This reduces its angular speed to 5 rad s^{-1}.

(v) To reduce its angular speed further, a parallel impulse of the same magnitude is now applied tangentially in the horizontal plane through G at the curved surface of the frustum. Calculate the resulting angular speed. **[5]**

THERE ARE NO QUESTIONS PRINTED ON THIS PAGE.

GCE

Mathematics

Advanced GCE **4764**

Mechanics 4

Mark Scheme for June 2010

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All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the Report on the Examination.

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1(i)	$(m - \lvert\delta m\rvert)(v + \delta v) + \lvert\delta m\rvert(v - u) - mv = -mg\delta t$	M1	Impulse = change in momentum	
		A1	Accept sign errors in δm	
	$m\delta v - u\lvert\delta m\rvert - \lvert\delta m\rvert\delta v = -mg\delta t$			
	$m\frac{\delta v}{\delta t} + u\frac{\delta m}{\delta t} + \delta m\frac{\delta v}{\delta t} = -mg$	M1	Form DE	
	$\Rightarrow m\frac{\mathrm{d}v}{\mathrm{d}t} + u\frac{\mathrm{d}m}{\mathrm{d}t} = -mg$	E1	Complete argument (including signs)	
				4
(ii)	$\frac{\mathrm{d}m}{\mathrm{d}t} = -k \Rightarrow m = m_0 - kt$	M1		
	So $(m_0 - kt)\frac{\mathrm{d}v}{\mathrm{d}t} - uk = -(m_0 - kt)g$	A1		
	$\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{uk}{m_0 - kt} - g$			
	$v = \int\left(\frac{uk}{m_0 - kt} - g\right)\mathrm{d}t$	M1	Integrate	
	$= -u\ln(m_0 - kt) - gt + c$	A1		
	$t = 0, v = 0 \Rightarrow 0 = -u\ln m_0 + c$	M1	Use condition	
	$v = -u\ln\left(1 - \frac{k}{m_0}t\right) - gt$	A1		
	Fuel burnt when $m_0 - kt = 0.25m_0$	M1		
	$v = -u\ln 0.25 - \frac{0.75m_0 g}{k}$	A1		
				8

2(i)	$m\frac{dv}{dt} = -mkv^{\frac{3}{2}}$	M1	N2L	
		A1		
	$\int -v^{-\frac{3}{2}}dv = \int k\,dt$	M1	Separate and integrate	
	$2v^{-\frac{1}{2}} = kt + c$	A1		
	$t = 0, v = 25 \Rightarrow c = \frac{2}{5}$	M1	Use condition	
	$2v^{-\frac{1}{2}} = kt + \frac{2}{5}$	M1	Rearrange	
	$v = 4\left(kt + \frac{2}{5}\right)^{-2}$	E1		
				7
(ii)	$x = \int 4\left(kt + \frac{2}{5}\right)^{-2} dt$			
	$= -\frac{4}{k}\left(kt + \frac{2}{5}\right)^{-1} + A$	M1	Integrate	
	$t = 0, x = 0 \Rightarrow A = \frac{10}{k}$	M1	Use condition	
	$x = \frac{1}{k}\left(10 - \frac{4}{kt + \frac{2}{5}}\right)$	A1		
				3
(iii)	The speed decreases, tending to zero	B1		
	The displacement tends to $\frac{10}{k}$	B1	Cv $(10/k)$	
				2

3(i)	$V = -mga\sin\theta + \frac{\lambda}{2(2a)}(3a\sin\theta)^2$	M1	GPE term	
		M1	EPE term	
		A1		
	$\frac{dV}{d\theta} = -mga\cos\theta + \frac{\lambda}{4a}\cdot 9a^2 \cdot 2\sin\theta\cdot\cos\theta$	M1	Differentiate	
		A1		
	$= a\cos\theta\left(\frac{9}{2}\lambda\sin\theta - mg\right)$	E1		
				6
(ii)	$\frac{dV}{d\theta} = 0 \Leftrightarrow \cos\theta = 0$ or $\sin\theta = \frac{2mg}{9\lambda}$	M1	Solve $\frac{dV}{d\theta} = 0$	
(A)	$\lambda > \frac{2}{9}mg$			
	$\theta = \frac{\pi}{2}$	A1		
	and $\theta = \sin^{-1}\frac{2mg}{9\lambda}$	A1		
	$\frac{d^2V}{d\theta^2} = -a\sin\theta\left(\frac{9}{2}\lambda\sin\theta - mg\right) + a\cos\theta\left(\frac{9}{2}\lambda\cos\theta\right)$	M1	Second derivative (or other valid method)	
		A1	Any correct form	
	$= a\left(\frac{9}{2}\lambda(1-2\sin^2\theta) + mg\sin\theta\right)$			
	$V''\left(\frac{\pi}{2}\right) = a\left(-\frac{9}{2}\lambda + mg\right) < 0$	M1	Substitute $\theta = \frac{\pi}{2}$	
	$\Rightarrow$ unstable	A1	Deduce unstable	
	$V''\left(\sin^{-1}\left(\frac{2mg}{9\lambda}\right)\right) = a\left(\frac{9}{2}\lambda\left(1 - 2\left(\frac{2mg}{9\lambda}\right)^2\right) + \frac{2(mg)^2}{9\lambda}\right)$	M1	Substitute other value	

	$= \frac{9}{2}\lambda a\left(1 - \left(\frac{2mg}{9\lambda}\right)^2\right)$			
	$\lambda > \frac{2}{9}mg \Rightarrow \left(\frac{2mg}{9\lambda}\right)^2 < 1 \Rightarrow V'' > 0$	M1	Consider second derivative	
	$\Rightarrow$ stable	A1	Complete argument	
				10
(B)	$\lambda < \frac{2}{9}mg \Rightarrow$	M1	Consider solutions	
	$\theta = \frac{\pi}{2}$ only	A1		
	$V''\left(\frac{\pi}{2}\right) = a\left(-\frac{9}{2}\lambda + mg\right) > 0$	M1	Consider second derivative	
	$\Rightarrow$ stable	A1	Complete argument	
				4
(C)	$\lambda = \frac{2}{9}mg$ gives $\theta = \frac{1}{2}\pi$ only (from both factors)	M1	Consider solutions	
		A1		
	$V''\left(\frac{\pi}{2}\right) = 0$			
	$V'\left(\frac{\pi}{2} - \epsilon\right) = (+)(-) = (-)$			
	$V'\left(\frac{\pi}{2} + \epsilon\right) = (-)(-) = (+)$	M1	Valid method	
	Hence stable	A1	Complete argument	
				4

4(i)	Mass of slice $\approx \rho\pi y^2 \delta x$	M1		
	So $I_{\text{slice}} \approx \frac{1}{2}(\rho\pi y^2 \delta x)y^2$	M1		
	$= \frac{1}{32}\rho\pi x^4 \delta x$	A1		
	So $I_{\text{cone}} \approx \int_0^{2a} \frac{1}{32}\rho\pi x^4 \mathbf{d}x$	M1		
	$= \left[\frac{1}{160}\rho\pi x^5\right]_0^{2a}$	A1	ft	
	$= \frac{1}{5}\pi\rho a^5$	A1		
	$\rho = \frac{M}{\frac{2}{3}\pi a^3}$	M1		
	$\Rightarrow I_{\text{cone}} = \frac{3}{10}Ma^2$	E1		
				8
(ii)	Mass of small cone $= \left(\frac{1}{2}\right)^3 M = \frac{1}{8}M$			
	Mass of frustum $= \frac{7}{8}M$	B1		
	$I_{\text{large cone}} = I_{\text{small cone}} + I$	M1		
	$\frac{3}{10}Ma^2 = \frac{3}{10}\left(\frac{1}{8}M\right)\left(\frac{1}{2}a\right)^2 + I$	M1	Moment of inertia of small cone	
	$\Rightarrow I = \frac{93}{320}Ma^2$			
	$\frac{7}{8}M = 2.8, a = 0.1 \Rightarrow I = 0.0093$	E1		
				4

(iii)	$C = I\ddot{\theta} \Rightarrow \ddot{\theta} = \frac{0.05}{0.0093}$	M1 A1		
	$t = \frac{10}{\ddot{\theta}} = 1.86$	M1 A1		
				4
(iv)	Centre of mass: $\frac{7}{8}M\bar{x} + \frac{1}{8}M \cdot \frac{3a}{4} = M \cdot \frac{3a}{2}$	M1 A1		
	OG $= \bar{x} = \frac{45a}{28} = \frac{4.5}{28} \approx 0.1607$	A1	Any distance which locates G	
	i.e. G is $\frac{1.7}{28} \approx 0.0607$ m from the small circular face			
				3
(v)	$0.1J = I(10 - 5)$	M1	Moment of impulse = ang. momentum	
	$J = 0.465$	A1		
	Radius at G is $\frac{1}{2}\bar{x}$	B1		
	$\left(\frac{4.5}{56}\right)J = I(5 - \omega)$	M1	Moment of impulse = ang. momentum	
	$\Rightarrow \omega = \frac{55}{56} \approx 0.98$	A1		
				5

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4764 Mechanics 4

General comments

The standard of work seen was, in general, very high. Many candidates found the work on equilibrium difficult.

Comments on individual questions

1) *(Variable mass – Rockets)*

1) (i) Generally well answered, though the majority of candidates failed to account properly for the signs of their δm or $|\delta m|$ terms.

1) (ii) The technique of separating variables and integrating was well understood and executed by most candidates, though a common error in some candidates was to treat *m* as a constant.

2) *(Variable force)*

2) (i) Again, the process of separation of variables was well understood. This was well answered, with only small errors in signs or manipulation.

2) (ii) Most candidates knew that they had to replace *v* with $\frac{dx}{dt}$ and integrate again. Errors were mostly in signs and manipulation as in part (i).

2) (iii) Candidates who had answered parts (i) and (ii) usually gave good answers here, though many discussed only the velocity *or* displacement. The mark for the velocity required candidates to state that the velocity of the particle was tending to zero, not just "slowing down".

3) *(Equilibrium)*

3) (i) This was almost universally done correctly. Some candidates found the GPE relative to the horizontal rail, which gave the correct value for *V′*, but not for *V*.

3) (ii) This question caused many candidates problems. Though almost all knew how to find the positions of equilibrium and their nature, they were not able to follow through the process, particularly with regards to the position where $\theta = \sin^{-1}\frac{2mg}{9\lambda}$.

(A) The first part was usually well answered by most candidates, with both values of θ found and the correct expression for the second derivative. Some candidates included extra values for θ. Most candidates knew that they had to find the sign of the second derivative for their value of θ. In general the work for $\theta = \pi/2$ was good, but many struggled to make any progress with $\theta = \sin^{-1}\frac{2mg}{9\lambda}$, often because they did not rewrite their expression for *V*`` in terms of sinθ.

(B) Almost all candidates that got this far through the question realised that the condition resulted in only one position of equilibrium and then successfully determined the nature of that position.

(C) Again many candidates found that only one position of equilibrium existed and that $V'' = 0$ at that point. However, very few realised that this resulted in an ambiguity that could be resolved; most labelled this position as a point of inflection and were unsure how to proceed.

4) *(Rotation - Moments of inertia and dynamics of a rotating body)*

4) (i) This part was not well answered in general. Many candidates used an expression of the form $\rho\pi r^2\delta r$ and those that gave the correct expression for δm in terms of *y* and δx often failed to change variables accurately, or used $I_{\text{slice}} = \frac{1}{2}Mx^2$. As a result, most candidates did not get the required expression for the moment of inertia of the cone; many then attempted to reverse engineer an extra constant multiple rather than find their error.
Some candidates added unnecessary work by deriving the mass of the cone from first principles instead of using the formula provided.

4) (ii) This part was usually done correctly. Many candidates did far more work than was necessary by finding the density of the cone and the volumes of each solid to find their masses instead of using proportionality.

4) (iii) Almost universally correct.

4) (iv) The concept was often well understood, but many of the candidates did not find the centres of mass of the cones relative to the same point

4) (v) This part was not answered well, or at all, by many candidates. The essential concept that moment of impulse is equal to the change in angular momentum was often quoted, but not used correctly.