OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 4757

Further Applications of Advanced Mathematics (FP3)

Monday **12 JUNE 2006** Afternoon 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer any **three** questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is 72.

This question paper consists of 5 printed pages and 3 blank pages.

Option 1: Vectors

1 Four points have coordinates $A(-2, -3, 2)$, $B(-3, 1, 5)$, $C(k, 5, -2)$ and $D(0, 9, k)$.

(i) Find the vector product $\overrightarrow{AB} \times \overrightarrow{CD}$. [4]

(ii) For the case when AB is parallel to CD,

(*A*) state the value of k, [1]

(*B*) find the shortest distance between the parallel lines AB and CD, [6]

(*C*) find, in the form $ax + by + cz + d = 0$, the equation of the plane containing AB and CD. [3]

(iii) When AB is not parallel to CD, find the shortest distance between the lines AB and CD, in terms of k. [4]

(iv) Find the value of k for which the line AB intersects the line CD, and find the coordinates of the point of intersection in this case. [6]

Option 2: Multi-variable calculus

2 A surface has equation $x^2 - 4xy + 3y^2 - 2z^2 - 63 = 0$.

(i) Find a normal vector at the point (x, y, z) on the surface. [4]

(ii) Find the equation of the tangent plane to the surface at the point $Q(17, 4, 1)$. [4]

(iii) The point $(17 + h, 4 + p, 1 - h)$, where h and p are small, is on the surface and is close to Q.

Find an approximate expression for p in terms of h. [4]

(iv) Show that there is no point on the surface where the normal line is parallel to the z-axis. [4]

(v) Find the two values of k for which $5x - 6y + 2z = k$ is a tangent plane to the surface. [8]

Option 3: Differential geometry

3 The curve C has parametric equations $x = 2t^3 - 6t$, $y = 6t^2$.

(i) Find the length of the arc of C for which $0 \leqslant t \leqslant 1$. [6]

(ii) Find the area of the surface generated when the arc of C for which $0 \leqslant t \leqslant 1$ is rotated through 2π radians about the x-axis. [5]

(iii) Show that the equation of the normal to C at the point with parameter t is

$$y = \frac{1}{2}\left(\frac{1}{t} - t\right)x + 2t^2 + t^4 + 3.$$ [4]

(iv) Find the cartesian equation of the envelope of the normals to C. [6]

(v) The point P$(64, a)$ is the centre of curvature corresponding to a point on C. Find a. [3]

Option 4: Groups

4 The group G consists of the 8 complex matrices $\{\mathbf{I}, \mathbf{J}, \mathbf{K}, \mathbf{L}, -\mathbf{I}, -\mathbf{J}, -\mathbf{K}, -\mathbf{L}\}$ under matrix multiplication, where

$$\mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \ \mathbf{J} = \begin{pmatrix} j & 0 \\ 0 & -j \end{pmatrix}, \ \mathbf{K} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \ \mathbf{L} = \begin{pmatrix} 0 & j \\ j & 0 \end{pmatrix}.$$

(i) Copy and complete the following composition table for G. [6]

	I	**J**	**K**	**L**	**–I**	**–J**	**–K**	**–L**
I	**I**	**J**	**K**	**L**	**–I**	**–J**	**–K**	**–L**
J	**J**	**–I**	**L**	**–K**	**–J**	**I**	**–L**	**K**
K	**K**	**–L**	**–I**					
L	**L**	**K**						
–I	**–I**	**–J**						
–J	**–J**	**I**						
–K	**–K**	**L**						
–L	**–L**	**–K**						

(Note that **JK** = **L** and **KJ** = **–L**.)

(ii) State the inverse of each element of G. [3]

(iii) Find the order of each element of G. [3]

(iv) Explain why, if G has a subgroup of order 4, that subgroup must be cyclic. [4]

(v) Find all the proper subgroups of G. [5]

(vi) Show that G is not isomorphic to the group of symmetries of a square. [3]

Option 5: Markov chains

5 A local hockey league has three divisions. Each team in the league plays in a division for a year. In the following year a team might play in the same division again, or it might move up or down one division.

This question is about the progress of one particular team in the league. In 2007 this team will be playing in either Division 1 or Division 2. Because of its present position, the probability that it will be playing in Division 1 is 0.6, and the probability that it will be playing in Division 2 is 0.4.

The following transition probabilities apply to this team from 2007 onwards.

- When the team is playing in Division 1, the probability that it will play in Division 2 in the following year is 0.2.
- When the team is playing in Division 2, the probability that it will play in Division 1 in the following year is 0.1, and the probability that it will play in Division 3 in the following year is 0.3.
- When the team is playing in Division 3, the probability that it will play in Division 2 in the following year is 0.15.

This process is modelled as a Markov chain with three states corresponding to the three divisions.

(i) Write down the transition matrix. [3]

(ii) Determine in which division the team is most likely to be playing in 2014. [6]

(iii) Find the equilibrium probabilities for each division for this team. [3]

In 2015 the rules of the league are changed. A team playing in Division 3 might now be dropped from the league in the following year. Once dropped, a team does not play in the league again.

- The transition probabilities from Divisions 1 and 2 remain the same as before.
- When the team is playing in Division 3, the probability that it will play in Division 2 in the following year is 0.15, and the probability that it will be dropped from the league is 0.1.

The team plays in Division 2 in 2015.

The new situation is modelled as a Markov chain with four states: ‘Division1’, ‘Division 2’, ‘Division 3’ and ‘Out of league’.

(iv) Write down the transition matrix which applies from 2015. [3]

(v) Find the probability that the team is still playing in the league in 2020. [5]

(vi) Find the first year for which the probability that the team is out of the league is greater than 0.5. [4]

RECOGNISING ACHIEVEMENT

ADVANCED GCE UNIT

4757/01

MATHEMATICS (MEI)

Further Applications of Advanced Mathematics (FP3)

THURSDAY 14 JUNE 2007

Afternoon

Time: 1 hour 30 minutes

Additional materials:

Answer booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer any **three** questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

This document consists of **6** printed pages and **2** blank pages.

[Turn over

Option 1: Vectors

1 Three planes P, Q and R have the following equations.

$$\begin{aligned} \text{Plane } P\text{:}\quad & 8x - y - 14z = 20 \\ \text{Plane } Q\text{:}\quad & 6x + 2y - 5z = 26 \\ \text{Plane } R\text{:}\quad & 2x + y - z = 40 \end{aligned}$$

The line of intersection of the planes P and Q is K.

The line of intersection of the planes P and R is L.

(i) Show that K and L are parallel lines, and find the shortest distance between them. [9]

(ii) Show that the shortest distance between the line K and the plane R is $5\sqrt{6}$. [3]

The line M has equation $\mathbf{r} = (\mathbf{i} - 4\mathbf{j}) + \lambda(5\mathbf{i} - 4\mathbf{j} + 3\mathbf{k})$.

(iii) Show that the lines K and M intersect, and find the coordinates of the point of intersection. [7]

(iv) Find the shortest distance between the lines L and M. [5]

Option 2: Multi-variable calculus

2 A surface has equation $z = xy^2 - 4x^2y - 2x^3 + 27x^2 - 36x + 20$.

(i) Find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$. [3]

(ii) Find the coordinates of the four stationary points on the surface, showing that one of them is $(2, 4, 8)$. [8]

(iii) Sketch, on separate diagrams, the sections of the surface defined by $x = 2$ and by $y = 4$. Indicate the point $(2, 4, 8)$ on these sections, and deduce that it is neither a maximum nor a minimum. [6]

(iv) Show that there are just two points on the surface where the normal line is parallel to the vector $36\mathbf{i} + \mathbf{k}$, and find the coordinates of these points. [7]

Option 3: Differential geometry

3 The curve C has equation $y = \frac{1}{2}x^2 - \frac{1}{4}\ln x$, and a is a constant with $a \geqslant 1$.

(i) Show that the length of the arc of C for which $1 \leqslant x \leqslant a$ is $\frac{1}{2}a^2 + \frac{1}{4}\ln a - \frac{1}{2}$. [5]

(ii) Find the area of the surface generated when the arc of C for which $1 \leqslant x \leqslant 4$ is rotated through 2π radians **about the *y*-axis**. [5]

(iii) Show that the radius of curvature of C at the point where $x = a$ is $a\left(a + \frac{1}{4a}\right)^2$. [5]

(iv) Find the centre of curvature corresponding to the point $(1, \frac{1}{2})$ on C. [5]

C is one member of the family of curves defined by $y = px^2 - p^2 \ln x$, where p is a parameter.

(v) Find the envelope of this family of curves. [4]

[Questions 4 and 5 are printed overleaf.]

Option 4: Groups

4 **(i)** Prove that, for a group of order 10, every proper subgroup must be cyclic. [4]

The set $M = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ is a group under the binary operation of multiplication modulo 11.

(ii) Show that M is cyclic. [4]

(iii) List all the proper subgroups of M. [3]

The group P of symmetries of a regular pentagon consists of 10 transformations

$$\{A, B, C, D, E, F, G, H, I, J\}$$

and the binary operation is composition of transformations. The composition table for P is given below.

	A	B	C	D	E	F	G	H	I	J
A	C	J	G	H	A	B	I	F	E	D
B	F	E	H	G	B	A	D	C	J	I
C	G	D	I	F	C	J	E	B	A	H
D	J	C	B	E	D	G	F	I	H	A
E	A	B	C	D	E	F	G	H	I	J
F	H	I	D	C	F	E	J	A	B	G
G	I	H	E	B	G	D	A	J	C	F
H	D	G	J	A	H	I	B	E	F	C
I	E	F	A	J	I	H	C	D	G	B
J	B	A	F	I	J	C	H	G	D	E

One of these transformations is the identity transformation, some are rotations and the rest are reflections.

(iv) Identify which transformation is the identity, which are rotations and which are reflections. [4]

(v) State, giving a reason, whether P is isomorphic to M. [2]

(vi) Find the order of each element of P. [3]

(vii) List all the proper subgroups of P. [4]

Option 5: Markov chains

5 A computer is programmed to generate a sequence of letters. The process is represented by a Markov chain with four states, as follows.

The first letter is A, B, C or D, with probabilities 0.4, 0.3, 0.2 and 0.1 respectively.

After A, the next letter is either C or D, with probabilities 0.8 and 0.2 respectively.

After B, the next letter is either C or D, with probabilities 0.1 and 0.9 respectively.

After C, the next letter is either A or B, with probabilities 0.4 and 0.6 respectively.

After D, the next letter is either A or B, with probabilities 0.3 and 0.7 respectively.

(i) Write down the transition matrix $\mathbf{P}$. [2]

(ii) Use your calculator to find $\mathbf{P}^4$ and $\mathbf{P}^7$. (Give elements correct to 4 decimal places.) [4]

(iii) Find the probability that the 8th letter is C. [2]

(iv) Find the probability that the 12th letter is the same as the 8th letter. [4]

(v) By investigating the behaviour of $\mathbf{P}^n$ when n is large, find the probability that the $(n+1)$th letter is A when

(A) n is a large even number,

(B) n is a large odd number. [4]

The program is now changed. The initial probabilities and the transition probabilities are the same as before, except for the following.

After D, the next letter is A, B or D, with probabilities 0.3, 0.6 and 0.1 respectively.

(vi) Write down the new transition matrix $\mathbf{Q}$. [1]

(vii) Verify that $\mathbf{Q}^n$ approaches a limit as n becomes large, and hence write down the equilibrium probabilities for A, B, C and D. [4]

(viii) When n is large, find the probability that the $(n+1)$th, $(n+2)$th and $(n+3)$th letters are DDD. [3]

OCR RECOGNISING ACHIEVEMENT

ADVANCED GCE **4757/01**

MATHEMATICS (MEI)

Further Applications of Advanced Mathematics (FP3)

FRIDAY 6 JUNE 2008 Afternoon

Time: 1 hour 30 minutes

Additional materials (enclosed): None

Additional materials (required):
Answer Booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer any **three** questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

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- The total number of marks for this paper is **72**.
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This document consists of **4** printed pages.

Option 1: Vectors

1 A tetrahedron ABCD has vertices A $(-3, 5, 2)$, B $(3, 13, 7)$, C $(7, 0, 3)$ and D $(5, 4, 8)$.

(i) Find the vector product $\overrightarrow{AB} \times \overrightarrow{AC}$, and hence find the equation of the plane ABC. [4]

(ii) Find the shortest distance from D to the plane ABC. [3]

(iii) Find the shortest distance between the lines AB and CD. [4]

(iv) Find the volume of the tetrahedron ABCD. [4]

The plane P with equation $3x - 2z + 5 = 0$ contains the point B, and meets the lines AC and AD at E and F respectively.

(v) Find λ and μ such that $\overrightarrow{AE} = \lambda\overrightarrow{AC}$ and $\overrightarrow{AF} = \mu\overrightarrow{AD}$. Deduce that E is between A and C, and that F is between A and D. [5]

(vi) Hence, or otherwise, show that P divides the tetrahedron ABCD into two parts having volumes in the ratio 4 to 17. [4]

Option 2: Multi-variable calculus

2 You are given $g(x, y, z) = 6xz - (x + 2y + 3z)^2$.

(i) Find $\dfrac{\partial g}{\partial x}$, $\dfrac{\partial g}{\partial y}$ and $\dfrac{\partial g}{\partial z}$. [4]

A surface S has equation $g(x, y, z) = 125$.

(ii) Find the equation of the normal line to S at the point P $(7, -7.5, 3)$. [3]

(iii) The point Q is on this normal line and is close to P. At Q, $g(x, y, z) = 125 + h$, where h is small. Find the vector $\mathbf{n}$ such that $\overrightarrow{PQ} = h\mathbf{n}$ approximately. [5]

(iv) Show that there is no point on S at which the normal line is parallel to the z-axis. [4]

(v) Find the two points on S at which the tangent plane is parallel to $x + 5y = 0$. [8]

Option 3: Differential geometry

3 The curve C has parametric equations $x = 8t^3$, $y = 9t^2 - 2t^4$, for $t \geqslant 0$.

(i) Show that $\dot{x}^2 + \dot{y}^2 = (18t + 8t^3)^2$. Find the length of the arc of C for which $0 \leqslant t \leqslant 2$. [6]

(ii) Find the area of the surface generated when the arc of C for which $0 \leqslant t \leqslant 2$ is rotated through 2π radians about the x-axis. [6]

(iii) Show that the curvature at a general point on C is $\dfrac{-6}{t(4t^2 + 9)^2}$. [5]

(iv) Find the coordinates of the centre of curvature corresponding to the point on C where $t = 1$. [7]

Option 4: Groups

4 A binary operation $*$ is defined on real numbers x and y by

$$x * y = 2xy + x + y.$$

You may assume that the operation $*$ is commutative and associative.

(i) Explain briefly the meanings of the terms 'commutative' and 'associative'. [3]

(ii) Show that $x * y = 2\left(x + \frac{1}{2}\right)\left(y + \frac{1}{2}\right) - \frac{1}{2}$. [1]

The set S consists of all real numbers greater than $-\frac{1}{2}$.

(iii) (A) Use the result in part **(ii)** to show that S is closed under the operation $*$.

(B) Show that S, with the operation $*$, is a group. [9]

(iv) Show that S contains no element of order 2. [3]

The group $G = \{0, 1, 2, 4, 5, 6\}$ has binary operation $\circ$ defined by

$x \circ y$ is the remainder when $x * y$ is divided by 7.

(v) Show that $4 \circ 6 = 2$. [2]

The composition table for G is as follows.

$\circ$	0	1	2	4	5	6
0	0	1	2	4	5	6
1	1	4	0	6	2	5
2	2	0	5	1	6	4
4	4	6	1	5	0	2
5	5	2	6	0	4	1
6	6	5	4	2	1	0

(vi) Find the order of each element of G. [3]

(vii) List all the subgroups of G. [3]

[Question 5 is printed overleaf.]

Option 5: Markov chains

This question requires the use of a calculator with the ability to handle matrices.

5 Every day, a security firm transports a large sum of money from one bank to another. There are three possible routes A, B and C. The route to be used is decided just before the journey begins, by a computer programmed as follows.

On the first day, each of the three routes is equally likely to be used.

If route A was used on the previous day, route A, B or C will be used, with probabilities 0.1, 0.4, 0.5 respectively.

If route B was used on the previous day, route A, B or C will be used, with probabilities 0.7, 0.2, 0.1 respectively.

If route C was used on the previous day, route A, B or C will be used, with probabilities 0.1, 0.6, 0.3 respectively.

The situation is modelled as a Markov chain with three states.

(i) Write down the transition matrix **P**. [2]

(ii) Find the probability that route B is used on the 7th day. [4]

(iii) Find the probability that the same route is used on the 7th and 8th days. [3]

(iv) Find the probability that the route used on the 10th day is the same as that used on the 7th day. [4]

(v) Given that $\mathbf{P}^n \to \mathbf{Q}$ as $n \to \infty$, find the matrix **Q** (give the elements to 4 decimal places). Interpret the probabilities which occur in the matrix **Q**. [4]

The computer program is now to be changed, so that the long-run probabilities for routes A, B and C will become 0.4, 0.2 and 0.4 respectively. The transition probabilities after routes A and B remain the same as before.

(vi) Find the new transition probabilities after route C. [4]

(vii) A long time after the change of program, a day is chosen at random. Find the probability that the route used on that day is the same as on the previous day. [3]

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RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) **4757**

Further Applications of Advanced Mathematics (FP3)

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
None

Monday 1 June 2009
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer any **three** questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

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Option 1: Vectors

1 The point A $(-1, 12, 5)$ lies on the plane P with equation $8x - 3y + 10z = 6$. The point B $(6, -2, 9)$ lies on the plane Q with equation $3x - 4y - 2z = 8$. The planes P and Q intersect in the line L.

(i) Find an equation for the line L. **[5]**

(ii) Find the shortest distance between L and the line AB. **[6]**

The lines M and N are both parallel to L, with M passing through A and N passing through B.

(iii) Find the distance between the parallel lines M and N. **[5]**

The point C has coordinates $(k, 0, 2)$, and the line AC intersects the line N at the point D.

(iv) Find the value of k, and the coordinates of D. **[8]**

Option 2: Multi-variable calculus

2 A surface has equation $z = 3x(x+y)^3 - 2x^3 + 24x$.

(i) Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$. **[4]**

(ii) Find the coordinates of the three stationary points on the surface. **[7]**

(iii) Find the equation of the normal line at the point P $(1, -2, 19)$ on the surface. **[3]**

(iv) The point Q $(1 + k, -2 + h, 19 + 3h)$ is on the surface and is close to P. Find an approximate expression for k in terms of h. **[4]**

(v) Show that there is only one point on the surface at which the tangent plane has an equation of the form $27x - z = d$. Find the coordinates of this point and the corresponding value of d. **[6]**

Option 3: Differential geometry

3 A curve has parametric equations $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$, for $0 \leqslant \theta \leqslant \pi$, where a is a positive constant.

(i) Show that the arc length s from the origin to a general point on the curve is given by $s = 4a\sin\frac{1}{2}\theta$. **[6]**

(ii) Find the intrinsic equation of the curve giving s in terms of a and ψ, where $\tan\psi = \frac{\mathrm{d}y}{\mathrm{d}x}$. **[4]**

(iii) Hence, or otherwise, show that the radius of curvature at a point on the curve is $4a\cos\frac{1}{2}\theta$. **[3]**

(iv) Find the coordinates of the centre of curvature corresponding to the point on the curve where $\theta = \frac{2}{3}\pi$. **[6]**

(v) Find the area of the surface generated when the curve is rotated through 2π radians about the x-axis. **[5]**

Option 4: Groups

4 The group $G = \{1, 2, 3, 4, 5, 6\}$ has multiplication modulo 7 as its operation. The group $H = \{1, 5, 7, 11, 13, 17\}$ has multiplication modulo 18 as its operation.

(i) Show that the groups G and H are both cyclic. **[4]**

(ii) List all the proper subgroups of G. **[3]**

(iii) Specify an isomorphism between G and H. **[4]**

The group $S = \{a, b, c, d, e, f\}$ consists of functions with domain $\{1, 2, 3\}$ given by

$$\begin{array}{lll} a(1) = 2 & a(2) = 3 & a(3) = 1 \\ b(1) = 3 & b(2) = 1 & b(3) = 2 \\ c(1) = 1 & c(2) = 3 & c(3) = 2 \\ d(1) = 3 & d(2) = 2 & d(3) = 1 \\ e(1) = 1 & e(2) = 2 & e(3) = 3 \\ f(1) = 2 & f(2) = 1 & f(3) = 3 \end{array}$$

and the group operation is composition of functions.

(iv) Show that $ad = c$ and find da. **[4]**

(v) Give a reason why S is not isomorphic to G. **[1]**

(vi) Find the order of each element of S. **[4]**

(vii) List all the proper subgroups of S. **[4]**

[Question 5 is printed overleaf.]

Option 5: Markov chains

This question requires the use of a calculator with the ability to handle matrices.

5 Each level of a fantasy computer game is set in a single location, Alphaworld, Betaworld, Chiworld or Deltaworld. After completing a level, a player goes on to the next level, which could be set in the same location as the previous level, or in a different location.

In the first version of the game, the initial and transition probabilities are as follows.

Level 1 is set in Alphaworld or Betaworld, with probabilities 0.6, 0.4 respectively.

After a level set in Alphaworld, the next level will be set in Betaworld, Chiworld or Deltaworld, with probabilities 0.7, 0.1, 0.2 respectively.

After a level set in Betaworld, the next level will be set in Alphaworld, Betaworld or Deltaworld, with probabilities 0.1, 0.8, 0.1 respectively.

After a level set in Chiworld, the next level will also be set in Chiworld.

After a level set in Deltaworld, the next level will be set in Alphaworld, Betaworld or Chiworld, with probabilities 0.3, 0.6, 0.1 respectively.

The situation is modelled as a Markov chain with four states.

(i) Write down the transition matrix. **[2]**

(ii) Find the probabilities that level 14 is set in each location. **[3]**

(iii) Find the probability that level 15 is set in the same location as level 14. **[3]**

(iv) Find the level at which the probability of being set in Chiworld first exceeds 0.5. **[3]**

(v) Following a level set in Betaworld, find the expected number of further levels which will be set in Betaworld before changing to a different location. **[3]**

In the second version of the game, the initial probabilities and the transition probabilities after Alphaworld, Betaworld and Deltaworld are all the same as in the first version; but after a level set in Chiworld, the next level will be set in Chiworld or Deltaworld, with probabilities 0.9, 0.1 respectively.

(vi) By considering powers of the new transition matrix, or otherwise, find the equilibrium probabilities for the four locations. **[5]**

In the third version of the game, the initial probabilities and the transition probabilities after Alphaworld, Betaworld and Deltaworld are again all the same as in the first version; but the transition probabilities after Chiworld have changed again. The equilibrium probabilities for Alphaworld, Betaworld, Chiworld and Deltaworld are now 0.11, 0.75, 0.04, 0.1 respectively.

(vii) Find the new transition probabilities after a level set in Chiworld. **[5]**

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ADVANCED GCE

MATHEMATICS (MEI) 4757

Further Applications of Advanced Mathematics (FP3)

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
- Scientific or graphical calculator

Wednesday 9 June 2010
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer any **three** questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **8** pages. Any blank pages are indicated.

Option 1: Vectors

1 Four points have coordinates

$$A\,(3,\,8,\,27),\quad B\,(5,\,9,\,25),\quad C\,(8,\,0,\,1)\quad \text{and}\quad D\,(11,\,p,\,p),$$

where p is a constant.

(i) Find the perpendicular distance from C to the line AB. **[5]**

(ii) Find $\overrightarrow{AB} \times \overrightarrow{CD}$ in terms of p, and show that the shortest distance between the lines AB and CD is

$$\frac{21|p-5|}{\sqrt{17p^2-2p+26}}.$$ **[8]**

(iii) Find, in terms of p, the volume of the tetrahedron ABCD. **[4]**

(iv) State the value of p for which the lines AB and CD intersect, and find the coordinates of the point of intersection in this case. **[7]**

Option 2: Multi-variable calculus

2 In this question, L is the straight line with equation $\mathbf{r} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix}$, and $g(x,\,y,\,z) = (xy + z^2)e^{x-2y}$.

(i) Find $\frac{\partial g}{\partial x}$, $\frac{\partial g}{\partial y}$ and $\frac{\partial g}{\partial z}$. **[4]**

(ii) Show that the normal to the surface $g(x,\,y,\,z) = 3$ at the point $(2,\,1,\,-1)$ is the line L. **[4]**

On the line L, there are two points at which $g(x,\,y,\,z) = 0$.

(iii) Show that one of these points is P (0, 3, 0), and find the coordinates of the other point Q. **[4]**

(iv) Show that, if $x = -2\mu$, $y = 3 + 2\mu$, $z = \mu$, and μ is small, then

$$g(x,\,y,\,z) \approx -6\mu e^{-6}.$$ **[3]**

You are given that h is a small number.

(v) There is a point on L, close to P, at which $g(x,\,y,\,z) = h$. Show that this point is approximately

$$\left(\tfrac{1}{3}e^6h,\ 3 - \tfrac{1}{3}e^6h,\ -\tfrac{1}{6}e^6h\right).$$ **[2]**

(vi) Find the approximate coordinates of the point on L, close to Q, at which $g(x,\,y,\,z) = h$. **[7]**

Option 3: Differential geometry

3 A curve C has equation $y = x^{\frac{1}{2}} - \frac{1}{3}x^{\frac{3}{2}}$, for $x \geqslant 0$.

(i) Show that the arc of C for which $0 \leqslant x \leqslant a$ has length $a^{\frac{1}{2}} + \frac{1}{3}a^{\frac{3}{2}}$. **[5]**

(ii) Find the area of the surface generated when the arc of C for which $0 \leqslant x \leqslant 3$ is rotated through 2π radians about the x-axis. **[5]**

(iii) Find the coordinates of the centre of curvature corresponding to the point $\left(4, -\frac{2}{3}\right)$ on C. **[9]**

The curve C is one member of the family of curves defined by

$$y = p^2 x^{\frac{1}{2}} - \frac{1}{3}p^3 x^{\frac{3}{2}} \quad (\text{for } x \geqslant 0),$$

where p is a parameter (and $p > 0$).

(iv) Find the equation of the envelope of this family of curves. **[5]**

Option 4: Groups

4 The group $F = \{p, q, r, s, t, u\}$ consists of the six functions defined by

$$p(x) = x \qquad q(x) = 1 - x \qquad r(x) = \frac{1}{x} \qquad s(x) = \frac{x-1}{x} \qquad t(x) = \frac{x}{x-1} \qquad u(x) = \frac{1}{1-x},$$

the binary operation being composition of functions.

(i) Show that st = r and find ts. **[4]**

(ii) Copy and complete the following composition table for F. **[3]**

	p	q	r	s	t	u
p	p	q	r	s	t	u
q	q	p	s	r	u	t
r	r	u	p	t	s	q
s	s	t	q	u	r	p
t	t	s	u			
u	u	r	t			

(iii) Give the inverse of each element of F. **[3]**

(iv) List all the subgroups of F. **[4]**

The group M consists of $\left\{1, -1, e^{\frac{\pi}{3}j}, e^{-\frac{\pi}{3}j}, e^{\frac{2\pi}{3}j}, e^{-\frac{2\pi}{3}j}\right\}$ with multiplication of complex numbers as its binary operation.

(v) Find the order of each element of M. **[4]**

The group G consists of the positive integers between 1 and 18 inclusive, under multiplication modulo 19.

(vi) Show that G is a cyclic group which can be generated by the element 2. **[3]**

(vii) Explain why G has no subgroup which is isomorphic to F. **[1]**

(viii) Find a subgroup of G which is isomorphic to M. **[2]**

Option 5: Markov chains

This question requires the use of a calculator with the ability to handle matrices.

5 In this question, give probabilities correct to 4 decimal places.

An electronic control unit on an aircraft is inspected weekly, replaced if necessary, and is labelled A, B, C or D according to whether it is in its first, second, third or fourth week of service.

In Week 1, the unit is labelled A.

At the start of each subsequent week, the following procedure is carried out.

When the unit is labelled A, B or C, it is tested; if it passes the test it is relabelled B, C or D respectively; if it fails the test it is replaced by a new unit which is labelled A.

When the unit is labelled D, it is replaced by a new unit which is labelled A.

The probability that a unit fails the test is 0.16 when it is labelled A, 0.28 when it is labelled B, and 0.43 when it is labelled C.

This situation is modelled as a Markov chain with four states.

(i) Write down the transition matrix. **[2]**

(ii) In Week 10, find the probability that the unit is labelled C. **[3]**

(iii) Find the week (apart from Week 1) in which the probabilities that the unit is labelled A, B, C, D first form a decreasing sequence. Give the values of these probabilities. **[3]**

(iv) Find the probability that the unit is labelled B in Week 8 and is labelled C in Week 16. **[4]**

(v) Following a week in which the unit is labelled D, find the expected number of consecutive weeks in which the unit is labelled A. **[2]**

(vi) Find the equilibrium probabilities that the unit is labelled A, B, C or D. **[4]**

An airline has 145 of these units installed in its aircraft. They are all subjected to the inspection procedure described above, and may be assumed to behave independently.

(vii) In the long run, find how many of these units are expected to be replaced each week. **[2]**

A different manufacturer has now been chosen to make the units. The inspection procedure remains the same as before, but the probabilities that the unit fails the test have changed. The equilibrium probabilities that the unit is labelled A, B, C or D are now found to be 0.4, 0.25, 0.2 and 0.15 respectively.

(viii) Find the new probabilities that the unit fails the test when it is labelled A, B or C. **[4]**

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ADVANCED GCE

MATHEMATICS (MEI) **4757**

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Monday 13 June 2011
Morning

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Option 1: Vectors

1 The points A (2, −1, 3), B (−2, −7, 7) and C (7, 5, 1) are three vertices of a tetrahedron ABCD.

The plane ABD has equation $x + 4y + 7z = 19$.

The plane ACD has equation $2x - y + 2z = 11$.

(i) Find the shortest distance from B to the plane ACD. **[3]**

(ii) Find an equation for the line AD. **[3]**

(iii) Find the shortest distance from C to the line AD. **[6]**

(iv) Find the shortest distance between the lines AD and BC. **[6]**

(v) Given that the tetrahedron ABCD has volume 20, find the coordinates of the two possible positions for the vertex D. **[6]**

Option 2: Multi-variable calculus

2 A surface S has equation $z = 8y^3 - 6x^2y - 15x^2 + 36x$.

(i) Sketch the section of S given by $y = -3$, and sketch the section of S given by $x = -6$. Your sketches should include the coordinates of any stationary points but need not include the coordinates of the points where the sections cross the axes. **[7]**

(ii) From your sketches in part **(i)**, deduce that (−6, −3, −324) is a stationary point on S, and state the nature of this stationary point. **[2]**

(iii) Find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$, and hence find the coordinates of the other three stationary points on S. **[8]**

(iv) Show that there are exactly two values of k for which the plane with equation

$$120x - z = k$$

is a tangent plane to S, and find these values of k. **[7]**

Option 3: Differential geometry

3 **(a)** **(i)** Given that $y = e^{\frac{1}{2}x} + e^{-\frac{1}{2}x}$, show that $1 + \left(\frac{dy}{dx}\right)^2 = \left(\frac{1}{2}e^{\frac{1}{2}x} + \frac{1}{2}e^{-\frac{1}{2}x}\right)^2$. **[3]**

The arc of the curve $y = e^{\frac{1}{2}x} + e^{-\frac{1}{2}x}$ for $0 \leqslant x \leqslant \ln a$ (where $a > 1$) is denoted by C.

(ii) Show that the length of C is $\frac{a-1}{\sqrt{a}}$. **[3]**

(iii) Find the area of the surface formed when C is rotated through 2π radians about the x-axis. **[5]**

(b) An ellipse has parametric equations $x = 2\cos\theta$, $y = \sin\theta$ for $0 \leqslant \theta < 2\pi$.

(i) Show that the normal to the ellipse at the point with parameter θ has equation

$$y = 2x\tan\theta - 3\sin\theta.$$ **[3]**

(ii) Find parametric equations for the evolute of the ellipse, and show that the evolute has cartesian equation

$$(2x)^{\frac{2}{3}} + y^{\frac{2}{3}} = 3^{\frac{2}{3}}.$$ **[6]**

(iii) Using the evolute found in part **(ii)**, or otherwise, find the radius of curvature of the ellipse

(A) at the point (2, 0),

(B) at the point (0, 1). **[4]**

Option 4: Groups

4 **(i)** Show that the set $G = \{1, 3, 4, 5, 9\}$, under the binary operation of multiplication modulo 11, is a group. You may assume associativity. **[6]**

(ii) Explain why any two groups of order 5 must be isomorphic to each other. **[3]**

The set $H = \left\{1, e^{\frac{2}{5}\pi j}, e^{\frac{4}{5}\pi j}, e^{\frac{6}{5}\pi j}, e^{\frac{8}{5}\pi j}\right\}$ is a group under the binary operation of multiplication of complex numbers.

(iii) Specify an isomorphism between the groups G and H. **[3]**

The set K consists of the 25 ordered pairs (x, y), where x and y are elements of G. The set K is a group under the binary operation defined by

$$(x_1, y_1)(x_2, y_2) = (x_1x_2, y_1y_2)$$

where the multiplications are carried out modulo 11; for example, $(3, 5)(4, 4) = (1, 9)$.

(iv) Write down the identity element of K, and find the inverse of the element $(9, 3)$. **[2]**

(v) Explain why $(x, y)^5 = (1, 1)$ for every element (x, y) in K. **[3]**

(vi) Deduce that all the elements of K, except for one, have order 5. State which is the exceptional element. **[3]**

(vii) A subgroup of K has order 5 and contains the element $(9, 3)$. List the elements of this subgroup. **[2]**

(viii) Determine how many subgroups of K there are with order 5. **[2]**

Option 5: Markov chains

This question requires the use of a calculator with the ability to handle matrices.

5 In this question, give probabilities correct to 4 decimal places.

Alpha and Delta are two companies which compete for the ownership of insurance bonds. Boyles and Cayleys are companies which trade in these bonds. When a new bond becomes available, it is first acquired by either Boyles or Cayleys. After a certain amount of trading it is eventually owned by either Alpha or Delta. Change of ownership always takes place overnight, so that on any particular day the bond is owned by one of the four companies. The trading process is modelled as a Markov chain with four states, as follows.

On the first day, the bond is owned by Boyles or Cayleys, with probabilities 0.4, 0.6 respectively.

If the bond is owned by Boyles, then on the next day it could be owned by Alpha, Boyles or Cayleys, with probabilities 0.07, 0.8, 0.13 respectively.

If the bond is owned by Cayleys, then on the next day it could be owned by Boyles, Cayleys or Delta, with probabilities 0.15, 0.75, 0.1 respectively.

If the bond is owned by Alpha or Delta, then no further trading takes place, so on the next day it is owned by the same company.

(i) Write down the transition matrix **P**. **[2]**

(ii) Explain what is meant by an absorbing state of a Markov chain. Identify any absorbing states in this situation. **[2]**

(iii) Find the probability that the bond is owned by Boyles on the 10th day. **[3]**

(iv) Find the probability that on the 14th day the bond is owned by the same company as on the 10th day. **[3]**

(v) Find the day on which the probability that the bond is owned by Alpha or Delta exceeds 0.9 for the first time. **[4]**

(vi) Find the limit of $\mathbf{P}^n$ as n tends to infinity. **[2]**

(vii) Find the probability that the bond is eventually owned by Alpha. **[3]**

The probabilities that Boyles and Cayleys own the bond on the first day are changed (but all the transition probabilities remain the same as before). The bond is now equally likely to be owned by Alpha or Delta at the end of the trading process.

(viii) Find the new probabilities for the ownership of the bond on the first day. **[5]**

OCR RECOGNISING ACHIEVEMENT

Thursday 14 June 2012 – Morning

A2 GCE MATHEMATICS (MEI)

4757 Further Applications of Advanced Mathematics (FP3)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

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- Printed Answer Book 4757
- MEI Examination Formulae and Tables (MF2)

Other materials required:
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Duration: 1 hour 30 minutes

* 4 7 1 5 7 5 0 6 1 2 *

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INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

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Option 1: Vectors

1 A mine contains several underground tunnels beneath a hillside. The hillside is a plane, all the tunnels are straight and the width of the tunnels may be neglected. A coordinate system is chosen with the z-axis pointing vertically upwards and the units are metres. Three points on the hillside have coordinates A(15, –60, 20), B(–75, 100, 40) and C(18, 138, 35.6).

(i) Find the vector product $\overrightarrow{AB} \times \overrightarrow{AC}$ and hence show that the equation of the hillside is $2x - 2y + 25z = 650$. **[5]**

The tunnel T_A begins at A and goes in the direction of the vector $15\mathbf{i} + 14\mathbf{j} - 2\mathbf{k}$; the tunnel T_C begins at C and goes in the direction of the vector $8\mathbf{i} + 7\mathbf{j} - 2\mathbf{k}$. Both these tunnels extend a long way into the ground.

(ii) Find the least possible length of a tunnel which connects B to a point in T_A. **[6]**

(iii) Find the least possible length of a tunnel which connects a point in T_A to a point in T_C. **[6]**

(iv) A tunnel starts at B, passes through the point $(18, 138, p)$ vertically below C, and intersects T_A at the point Q. Find the value of p and the coordinates of Q. **[7]**

Option 2: Multi-variable calculus

2 You are given that $\mathrm{g}(x, y, z) = x^2 + 2y^2 - z^2 + 2xz + 2yz + 4z - 3$.

(i) Find $\dfrac{\partial \mathrm{g}}{\partial x}$, $\dfrac{\partial \mathrm{g}}{\partial y}$ and $\dfrac{\partial \mathrm{g}}{\partial z}$. **[3]**

The surface S has equation $\mathrm{g}(x, y, z) = 0$, and P(–2, –1, 1) is a point on S.

(ii) Find an equation for the normal line to the surface S at the point P. **[3]**

(iii) A point Q lies on this normal line and is close to P. At Q, $\mathrm{g}(x, y, z) = h$, where h is small. Find the constant c such that $\mathrm{PQ} \approx c|h|$. **[5]**

(iv) Show that there is no point on S at which the normal line is parallel to the z-axis. **[5]**

(v) Given that $x + y + z = k$ is a tangent plane to the surface S, find the two possible values of k. **[8]**

Option 3: Differential geometry

3 A curve has parametric equations

$$x = a(1 - \cos^3\theta), \quad y = a\sin^3\theta, \quad \text{for } 0 \leqslant \theta \leqslant \frac{\pi}{3},$$

where a is a positive constant.

The arc length from the origin to a general point on the curve is denoted by s, and ψ is the acute angle defined by $\tan\psi = \frac{\mathrm{d}y}{\mathrm{d}x}$.

(i) Express s and ψ in terms of θ, and hence show that the intrinsic equation of the curve is

$$s = \frac{3}{2}a\sin^2\psi.$$ **[9]**

(ii) For the point on the curve given by $\theta = \frac{\pi}{6}$, find the radius of curvature and the coordinates of the centre of curvature. **[9]**

(iii) Find the area of the curved surface generated when the curve is rotated through 2π radians about the y-axis. **[6]**

Option 4: Groups

4 **(i)** Show that the set $P = \{1, 5, 7, 11\}$, under the binary operation of multiplication modulo 12, is a group. You may assume associativity. **[4]**

A group Q has identity element e. The result of applying the binary operation of Q to elements x and y is written xy, and the inverse of x is written x^{-1}.

(ii) Verify that the inverse of xy is $y^{-1}x^{-1}$. **[2]**

Three elements a, b and c of Q all have order 2, and $ab = c$.

(iii) By considering the inverse of c, or otherwise, show that $ba = c$. **[2]**

(iv) Show that $bc = a$ and $ac = b$. Find cb and ca. **[4]**

(v) Complete the composition table for $R = \{e, a, b, c\}$. Hence show that R is a subgroup of Q and that R is isomorphic to P. **[4]**

The group T of symmetries of a square contains four reflections A, B, C, D, the identity transformation E and three rotations F, G, H. The binary operation is composition of transformations. The composition table for T is given below.

	A	B	C	D	E	F	G	H
A	E	G	H	F	A	D	B	C
B	G	E	F	H	B	C	A	D
C	F	H	E	G	C	A	D	B
D	H	F	G	E	D	B	C	A
E	A	B	C	D	E	F	G	H
F	C	D	B	A	F	G	H	E
G	B	A	D	C	G	H	E	F
H	D	C	A	B	H	E	F	G

(vi) Find the order of each element of T. **[3]**

(vii) List all the proper subgroups of T. **[5]**

Option 5: Markov chains

This question requires the use of a calculator with the ability to handle matrices.

5 In this question, give probabilities correct to 4 decimal places.

A 'random walk' is modelled as a Markov chain with five states A, B, C, D, E representing the possible positions, from left to right, of an object. At each 'step' the object moves as follows.

- If the object is at A, it moves one place to the right (to B).
- If the object is at E, it moves one place to the left (to D).
- Otherwise, the probability that the object moves one place to the left is 0.4, and the probability that it moves one place to the right is 0.6.

Steps occur at intervals of one minute, and the time taken to move may be neglected. The object starts at A, so after the first step (one minute later) the object is at B.

(i) Which of the five states are reflecting barriers? **[1]**

(ii) Write down the transition matrix $\mathbf{P}$. **[2]**

(iii) State the possible positions of the object after 10 steps, and give the probabilities that the object is in each of these positions. **[4]**

(iv) Find the probability that after 15 steps the object is in the same position as it was after 13 steps. **[3]**

(v) Find the number of steps after which the probability that the object is at D exceeds 0.69 for the first time. **[3]**

(vi) Find the limits of $\mathbf{P}^{2n}$ and $\mathbf{P}^{2n+1}$ as the positive integer n tends to infinity. **[4]**

(vii) For the interval of 100 minutes between the 200th step and the 300th step, find the expected length of time for which the object is at each of the five positions. **[3]**

(viii) At a certain instant, the object arrives at D. Find the expected number of successive occasions that the object moves to E (and then back to D). Hence find the expected time after this instant when the object first moves to C. **[4]**

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4757/01 Further Applications of Advanced Mathematics (FP3)

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Option 1: Vectors

1 Three points have coordinates $A(3, 2, 10)$, $B(11, 0, -3)$, $C(5, 18, 0)$, and L is the straight line through A with equation

$$\frac{x-3}{-1} = \frac{y-2}{4} = \frac{z-10}{1}.$$

(i) Find the shortest distance between the lines L and BC. **[5]**

(ii) Find the shortest distance from A to the line BC. **[6]**

A straight line passes through B and the point $P(5, 18, k)$, and intersects the line L.

(iii) Find k, and the point of intersection of the lines BP and L. **[7]**

The point D is on the line L, and AD has length 12.

(iv) Find the volume of the tetrahedron ABCD. **[6]**

Option 2: Multi-variable calculus

2 A surface has equation $z = 2(x^3 + y^3) + 3(x^2 + y^2) + 12xy$.

(i) For a point on the surface at which $\frac{\partial z}{\partial x} = \frac{\partial z}{\partial y}$, show that either $y = x$ or $y = 1 - x$. **[5]**

(ii) Show that there are exactly two stationary points on the surface, and find their coordinates. **[7]**

(iii) The point $P(\frac{1}{2}, \frac{1}{2}, 5)$ is on the surface, and $Q(\frac{1}{2} + h, \frac{1}{2} + h, 5 + w)$ is a point on the surface close to P. Find an approximate expression for h in terms of w. **[5]**

(iv) Find the four points on the surface at which the normal line is parallel to the vector $24\mathbf{i} + 24\mathbf{j} - \mathbf{k}$. **[7]**

Option 3: Differential geometry

3 **(a)** Find the length of the arc of the polar curve $r = a(1 + \cos\theta)$ for which $0 \leqslant \theta \leqslant \frac{1}{2}\pi$. **[6]**

(b) A curve C has cartesian equation $y = \frac{x^3}{6} + \frac{1}{2x}$.

(i) The arc of C for which $1 \leqslant x \leqslant 2$ is rotated through 2π radians about the x-axis to form a surface of revolution. Find the area of this surface. **[8]**

For the point on C at which $x = 2$,

(ii) show that the radius of curvature is $\frac{289}{64}$, **[5]**

(iii) find the coordinates of the centre of curvature. **[5]**

Option 4: Groups

4 **(a)** The composition table for a group G of order 8 is given below.

	a	b	c	d	e	f	g	h
a	c	e	b	f	a	h	d	g
b	e	c	a	g	b	d	h	f
c	b	a	e	h	c	g	f	d
d	f	g	h	a	d	c	e	b
e	a	b	c	d	e	f	g	h
f	h	d	g	c	f	b	a	e
g	d	h	f	e	g	a	b	c
h	g	f	d	b	h	e	c	a

(i) State which is the identity element, and give the inverse of each element of G. **[3]**

(ii) Show that G is cyclic. **[4]**

(iii) Specify an isomorphism between G and the group H consisting of $\{0, 2, 4, 6, 8, 10, 12, 14\}$ under addition modulo 16. **[3]**

(iv) Show that G is not isomorphic to the group of symmetries of a square. **[2]**

(b) The set S consists of the functions $\mathrm{f}_n(x) = \dfrac{x}{1 + nx}$, for all integers n, and the binary operation is composition of functions.

(i) Show that $\mathrm{f}_m \mathrm{f}_n = \mathrm{f}_{m+n}$. **[2]**

(ii) Hence show that the binary operation is associative. **[2]**

(iii) Prove that S is a group. **[6]**

(iv) Describe one subgroup of S which contains more than one element, but which is not the whole of S. **[2]**

Option 5: Markov chains

This question requires the use of a calculator with the ability to handle matrices.

5 In this question, give probabilities correct to 4 decimal places.

A contestant in a game-show starts with one, two or three 'lives', and then performs a series of tasks. After each task, the number of lives either decreases by one, or remains the same, or increases by one. The game ends when the number of lives becomes either four or zero. If the number of lives is four, the contestant wins a prize; if the number of lives is zero, the contestant loses and leaves with nothing.

At the start, the number of lives is decided at random, so that the contestant is equally likely to start with one, two or three lives. The tasks do not involve any skill, and after every task:

- the probability that the number of lives decreases by one is 0.5,
- the probability that the number of lives remains the same is 0.05,
- the probability that the number of lives increases by one is 0.45.

This is modelled as a Markov chain with five states corresponding to the possible numbers of lives. The states corresponding to zero lives and four lives are absorbing states.

(i) Write down the transition matrix **P**. **[3]**

(ii) Show that, after 8 tasks, the probability that the contestant has three lives is 0.0207, correct to 4 decimal places. **[2]**

(iii) Find the probability that, after 10 tasks, the game has not yet ended. **[3]**

(iv) Find the probability that the game ends after exactly 10 tasks. **[3]**

(v) Find the smallest value of N for which the probability that the game has not yet ended after N tasks is less than 0.01. **[3]**

(vi) Find the limit of $\mathbf{P}^n$ as n tends to infinity. **[2]**

(vii) Find the probability that the contestant wins a prize. **[3]**

The beginning of the game is now changed, so that the probabilities of starting with one, two or three lives can be adjusted.

(viii) State the maximum possible probability that the contestant wins a prize, and how this can be achieved. **[2]**

(ix) Given that the probability of starting with one life is 0.1, and the probability of winning a prize is 0.6, find the probabilities of starting with two lives and starting with three lives. **[3]**

Friday 6 June 2014 – Afternoon

A2 GCE MATHEMATICS (MEI)

4757/01 Further Applications of Advanced Mathematics (FP3)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4757/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

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INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

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Option 1: Vectors

1 Three points have coordinates $A(-3, 12, -7)$, $B(-2, 6, 9)$, $C(6, 0, -10)$. The plane P passes through the points A, B and C.

(i) Find the vector product $\overrightarrow{AB} \times \overrightarrow{AC}$. Hence or otherwise find an equation for the plane P in the form $ax + by + cz = d$. **[5]**

The plane Q has equation $6x + 3y + 2z = 32$. The perpendicular from A to the plane Q meets Q at the point D. The planes P and Q intersect in the line L.

(ii) Find the distance AD. **[3]**

(iii) Find an equation for the line L. **[5]**

(iv) Find the shortest distance from A to the line L. **[6]**

(v) Find the volume of the tetrahedron ABCD. **[5]**

Option 2: Multi-variable calculus

2 A surface S has equation $g(x, y, z) = 0$, where $g(x, y, z) = x^2 + 3y^2 + 2z^2 + 2yz + 6xz - 4xy - 24$. $P(2, 6, -2)$ is a point on the surface S.

(i) Find $\frac{\partial g}{\partial x}$, $\frac{\partial g}{\partial y}$ and $\frac{\partial g}{\partial z}$. **[3]**

(ii) Find the equation of the normal line to the surface S at the point P. **[3]**

(iii) The point Q is on this normal line and close to P. At Q, $g(x, y, z) = h$, where h is small. Find, in terms of h, the approximate perpendicular distance from Q to the surface S. **[4]**

(iv) Find the coordinates of the two points on the surface at which the normal line is parallel to the y-axis. **[6]**

(v) Given that $10x - y + 2z = 6$ is the equation of a tangent plane to the surface S, find the coordinates of the point of contact. **[8]**

Option 3: Differential geometry

3 **(a)** A curve has intrinsic equation $s = 2\ln\left(\dfrac{\pi}{\pi - 3\psi}\right)$ for $0 \leqslant \psi < \frac{1}{3}\pi$, where s is the arc length measured from a fixed point P and $\tan\psi = \dfrac{dy}{dx}$. P is in the third quadrant. The curve passes through the origin O, at which point $\psi = \frac{1}{6}\pi$. Q is the point on the curve at which $\psi = \frac{3}{10}\pi$.

(i) Express ψ in terms of s, and sketch the curve, indicating the points O, P and Q. **[4]**

(ii) Find the arc length OQ. **[3]**

(iii) Find the radius of curvature at the point O. **[3]**

(iv) Find the coordinates of the centre of curvature corresponding to the point O. **[3]**

(b) **(i)** Find the surface area of revolution formed when the curve $y = \frac{1}{3}\sqrt{x}(x-3)$ for $1 \leqslant x \leqslant 4$ is rotated through 2π radians about the y-axis. **[7]**

(ii) The curve in part **(b)(i)** is one member of the family $y = \frac{1}{9}\lambda\sqrt{x}(x-\lambda)$, where λ is a positive parameter. Find the equation of the envelope of this family of curves. **[4]**

Option 4: Groups

4 The twelve distinct elements of an abelian multiplicative group G are

$$e, a, a^2, a^3, a^4, a^5, b, ab, a^2b, a^3b, a^4b, a^5b$$

where e is the identity element, $a^6 = e$ and $b^2 = e$.

(i) Show that the element a^2b has order 6. **[3]**

(ii) Show that $\{e, a^3, b, a^3b\}$ is a subgroup of G. **[3]**

(iii) List all the cyclic subgroups of G. **[6]**

You are given that the set

$$H = \{1, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 49, 53, 59, 61, 67, 71, 73, 77, 79, 83, 89\}$$

with binary operation multiplication modulo 90 is a group.

(iv) Determine the order of each of the elements 11, 17 and 19. **[4]**

(v) Give a cyclic subgroup of H with order 4. **[2]**

(vi) By identifying possible values for the elements a and b above, or otherwise, give one example of each of the following:

(A) a non-cyclic subgroup of H with order 12, **[3]**

(B) a non-cyclic subgroup of H with order 4. **[3]**

Option 5: Markov chains

This question requires the use of a calculator with the ability to handle matrices.

5 In this question, give probabilities correct to 4 decimal places.

The speeds of vehicles are measured on a busy stretch of road and are categorised as A (not more than 30 mph), B (more than 30 mph but not more than 40 mph) or C (more than 40 mph).

- Following a vehicle in category A, the probabilities that the next vehicle is in categories A, B, C are 0.9, 0.07, 0.03 respectively.
- Following a vehicle in category B, the probabilities that the next vehicle is in categories A, B, C are 0.3, 0.6, 0.1 respectively.
- Following a vehicle in category C, the probabilities that the next vehicle is in categories A, B, C are 0.1, 0.7, 0.2 respectively.

This is modelled as a Markov chain with three states corresponding to the categories A, B, C. The speed of the first vehicle is measured as 28 mph.

(i) Write down the transition matrix **P**. **[2]**

(ii) Find the probabilities that the 10th vehicle is in each of the three categories. **[3]**

(iii) Find the probability that the 12th and 13th vehicles are in the same category. **[4]**

(iv) Find the smallest value of n for which the probability that the nth and $(n+1)$th vehicles are in the same category is less than 0.8, and give the value of this probability. **[4]**

(v) Find the expected number of vehicles (including the first vehicle) in category A before a vehicle in a different category. **[2]**

(vi) Find the limit of $\mathbf{P}^n$ as n tends to infinity, and hence write down the equilibrium probabilities for the three categories. **[3]**

(vii) Find the probability that, after many vehicles have passed by, the next three vehicles are all in category A. **[2]**

On a new stretch of road, the same categories are used but some of the transition probabilities are different.

- Following a vehicle in category A, the probability that the next vehicle is in category B is equal to the probability that it is in category C.
- Following a vehicle in category B, the probability that the next vehicle is in category A is equal to the probability that it is in category C.
- Following a vehicle in category C, the probabilities that the next vehicle is in categories A, B, C are 0.1, 0.7, 0.2 respectively.

In the long run, the proportions of vehicles in categories A, B, C are 50%, 40%, 10% respectively.

(viii) Find the transition matrix for the new stretch of road. **[4]**

END OF QUESTION PAPER

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OCR
Oxford Cambridge and RSA

Wednesday 3 June 2015 – Morning

A2 GCE MATHEMATICS (MEI)

4757/01 Further Applications of Advanced Mathematics (FP3)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

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- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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Option 1: Vectors

1 The point A has coordinates (2, 5, 4) and the line BC has equation

$$\mathbf{r} = \begin{pmatrix} 8 \\ 25 \\ 43 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 15 \\ 25 \end{pmatrix}.$$

You are given that AB = AC = 15.

(i) Show that the coordinates of one of the points B and C are (4, 10, 18). Find the coordinates of the other point. These points are B and C respectively. **[6]**

(ii) Find the equation of the plane ABC in cartesian form. **[4]**

(iii) Show that the plane containing the line BC and perpendicular to the plane ABC has equation $5y - 3z + 4 = 0$. **[4]**

The point D has coordinates (1, 1, 3).

(iv) Show that $|\overrightarrow{BC} \times \overrightarrow{AD}| = \sqrt{7667}$ and hence find the shortest distance between the lines BC and AD. **[7]**

(v) Find the volume of the tetrahedron ABCD. **[3]**

Option 2: Multi-variable calculus

2 A surface has equation $z = 3x^2 - 12xy + 2y^3 + 60$.

(i) Show that the point A $(8, 4, -4)$ is a stationary point on the surface. Find the coordinates of the other stationary point, B, on this surface. **[5]**

(ii) A point P with coordinates $(8+h, 4+k, p)$ lies on the surface.

(A) Show that $p = -4 + 3(h-2k)^2 + 2k^2(6+k)$. **[3]**

(B) Deduce that the stationary point A is a local minimum. **[3]**

(C) By considering sections of the surface near to B in each of the planes $x = 0$ and $y = 0$, investigate the nature of the stationary point B. **[4]**

(iii) The point Q with coordinates (1, 1, 53) lies on the surface.

Show that the equation of the tangent plane at Q is

$$6x + 6y + z = 65.$$ **[4]**

(iv) The tangent plane at the point R has equation $6x + 6y + z = \lambda$ where $\lambda \neq 65$.

Find the coordinates of R. **[5]**

Option 3: Differential geometry

3 Fig. 3 shows an ellipse with parametric equations $x = a\cos\theta$, $y = b\sin\theta$, for $0 \leqslant \theta \leqslant 2\pi$, where $0 < b \leqslant a$.
The curve meets the positive x-axis at A and the positive y-axis at B.

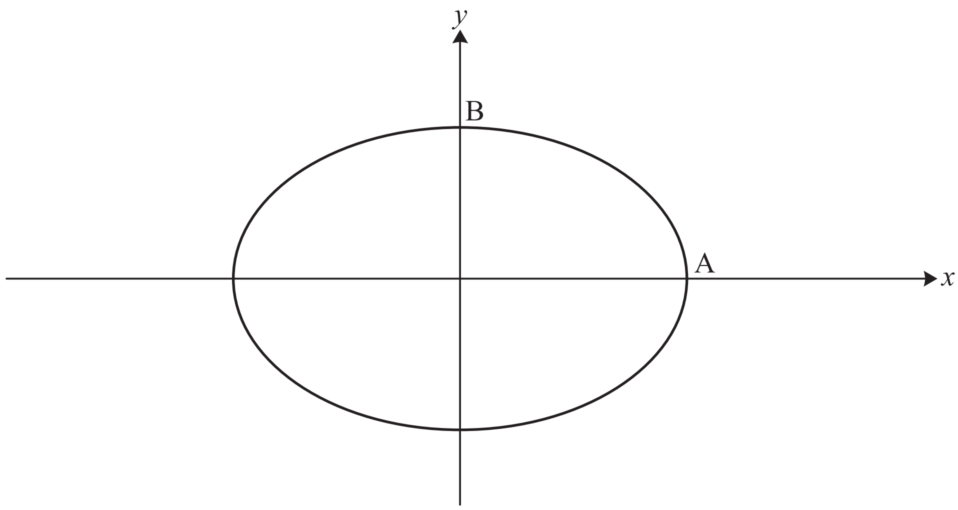

Fig. 3

(i) Show that the radius of curvature at A is $\dfrac{b^2}{a}$ and find the corresponding centre of curvature. **[7]**

(ii) Write down the radius of curvature and the centre of curvature at B. **[2]**

(iii) Find the relationship between a and b if the radius of curvature at B is equal to the radius of curvature at A. What does this mean geometrically? **[1]**

(iv) Show that the arc length from A to B can be expressed as

$$b\int_0^{\frac{\pi}{2}} \sqrt{1+\lambda^2\sin^2\theta}\,d\theta,$$

where λ^2 is to be determined in terms of a and b.

Evaluate this integral in the case $a = b$ and comment on your answer. **[7]**

(v) Find the cartesian equation of the evolute of the ellipse. **[7]**

Option 4: Groups

4 M is the set of all 2×2 matrices m(a,b) where a and b are rational numbers and

$$\mathrm{m}(a,b) = \begin{pmatrix} a & b \\ 0 & \frac{1}{a} \end{pmatrix}, a \neq 0.$$

(i) Show that under matrix multiplication M is a group. You may assume associativity of matrix multiplication. **[7]**

(ii) Determine whether the group is commutative. **[3]**

The set N_k consists of all 2×2 matrices m(k,b) where k is a fixed positive integer and b can take any integer value.

(iii) Prove that N_k is closed under matrix multiplication if and only if $k = 1$. **[4]**

Now consider the set P consisting of the matrices m(1,0), m(1,1), m(1,2) and m(1,3). The elements of P are combined using matrix multiplication but with arithmetic carried out modulo 4.

(iv) Show that $(\mathrm{m}(1,1))^2 = \mathrm{m}(1,2)$. **[2]**

(v) Construct the group combination table for P. **[4]**

The group R consists of the set $\{e, a, b, c\}$ combined under the operation *. The identity element is e, and elements a, b and c are such that

$a*a = b*b = c*c$ and $a*c = c*a = b$.

(vi) Determine whether R is isomorphic to P. **[4]**

Option 5: Markov chains

This question requires the use of a calculator with the ability to handle matrices.

5 An inspector has three factories, A, B, C, to check. He spends each day in one of the factories. He chooses the factory to visit on a particular day according to the following rules.

- If he is in A one day, then the next day he will never choose A but he is equally likely to choose B or C.
- If he is in B one day, then the next day he is equally likely to choose A, B or C.
- If he is in C one day, then the next day he will never choose A but he is equally likely to choose B or C.

(i) Write down the transition matrix, **P**. **[2]**

(ii) On Day 1 the inspector chooses A.

(*A*) Find the probability that he will choose A on Day 4. **[3]**

(*B*) Find the probability that the factory he chooses on Day 7 is the same factory that he chose on Day 2. **[4]**

(iii) Find the equilibrium probabilities and explain what they mean. **[4]**

The inspector is not satisfied with the number of times he visits A so he changes the rules as follows.

- If he is in A one day, then the next day he will choose A, B, C, with probabilities 0.8, 0.1, 0.1, respectively.
- If he is in B or C one day, then the probabilities for choosing the factory the next day remain as before.

(iv) Write down the new transition matrix, **Q**, and find the new equilibrium probabilities. **[3]**

(v) On a particular day, the inspector visits factory A. Find the expected number of consecutive further days on which he will visit factory A. **[3]**

Still not satisfied, the inspector changes the rules as follows.

- If he is in A one day, then the next day he will choose A, B, C, with probabilities 1, 0, 0, respectively.
- If he is in B or C one day, then the probabilities for choosing the factory the next day remain as before.

The new transition matrix is **R**.

(vi) On Day 15 he visits C. Find the first subsequent day for which the probability that he visits B is less than 0.1. **[3]**

(vii) Show that in this situation there is an absorbing state, explaining what this means. **[2]**

END OF QUESTION PAPER

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OCR
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Wednesday 8 June 2016 – Morning

A2 GCE MATHEMATICS (MEI)

4757/01 Further Applications of Advanced Mathematics (FP3)

QUESTION PAPER

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Option 1: Vectors

1 Positions in space around an aerodrome are modelled by a coordinate system with a point on the runway as the origin, O. The x-axis is east, the y-axis is north and the z-axis is vertically upwards. Units of distance are kilometres. Units of time are hours.

At time $t = 0$, an aeroplane, P, is at $(3, 4, 8)$ and is travelling in a direction $\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$ at a constant speed of $900\,\text{km}\,\text{h}^{-1}$.

(i) Find the least distance of the path of P from the point O. **[4]**

At time $t = 0$, a second aeroplane, Q, is at (80, 40, 10). It is travelling in a straight line towards the point O. Its speed is constant at $270\,\text{km}\,\text{h}^{-1}$.

(ii) Show that the shortest distance between the paths of the two aeroplanes is 2.24 km correct to three significant figures. **[6]**

(iii) By finding the points on the paths where the shortest distance occurs and the times at which the aeroplanes are at these points, show that in fact the aeroplanes are never this close. **[7]**

(iv) A third aeroplane, R, is at position (29, 19, 5.5) at time $t = 0$ and is travelling at $285\,\text{km}\,\text{h}^{-1}$ in a direction $\begin{pmatrix} 18 \\ 6 \\ 1 \end{pmatrix}$. Given that Q is in the process of landing and cannot change course, show that R needs to be instructed to alter course or change speed. **[7]**

Option 2: Multi-variable calculus

2 A surface, S, has equation $z = 3x^2 + 6xy + y^3$.

(i) Find the equation of the section where $y = 1$ in the form $z = \mathrm{f}(x)$. Sketch this section.

Find in three-dimensional vector form the equation of the line of symmetry of this section. **[5]**

(ii) Show that there are two stationary points on S, at O (0, 0, 0) and at P (−2, 2, −4). **[4]**

(iii) Given that the point $(-2 + h, 2 + k, \lambda)$ lies on the surface, show that

$$\lambda = -4 + 3(h + k)^2 + k^2 (k + 3).$$

By considering small values of h and k, deduce that there is a local minimum at P. **[5]**

(iv) By considering small values of x and y, show that the stationary point at O is neither a maximum nor a minimum. **[3]**

(v) Given that $18x + 18y - z = d$ is a tangent plane to S, find the two possible values of d. **[7]**

Option 3: Differential geometry

3 Fig. 3 shows the curve with parametric equations $x = t - 3t^3$, $y = 1 + 3t^2$.

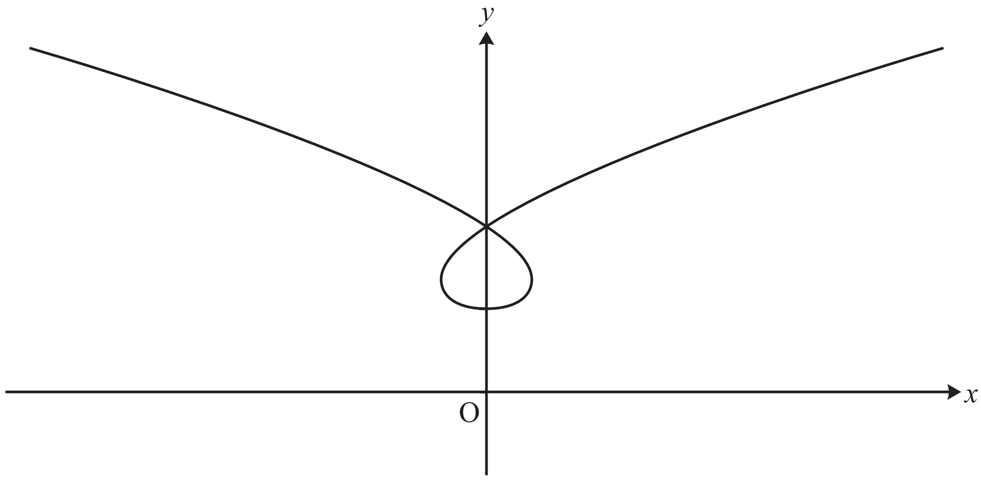

Fig. 3

(i) Show that the values of t where the curve cuts the y-axis are $t = 0, \pm\frac{1}{\sqrt{3}}$. Write down the corresponding values of y. **[2]**

(ii) Find the radius and centre of curvature when $t = \frac{1}{\sqrt{3}}$. **[11]**

The arc of the curve given by $0 \leqslant t \leqslant \frac{1}{\sqrt{3}}$ is denoted by C.

(iii) Find the length of C. **[5]**

(iv) Show that the area of the curved surface generated when C is rotated about the y-axis through 2π radians is $\frac{\pi}{3}$. **[6]**

Option 4: Groups

4 **(a)** The elements of the set $P = \{1, 3, 9, 11\}$ are combined under the binary operation, $*$, defined as multiplication modulo 16.

(i) Demonstrate associativity for the elements 3, 9, 11 in that order.

Assuming associativity holds in general, show that P forms a group under the binary operation $*$. **[6]**

(ii) Write down the order of each element. **[2]**

(iii) Write down all subgroups of P. **[1]**

(iv) Show that the group in part **(i)** is cyclic. **[1]**

(b) Now consider a group of order 4 containing the identity element e and the two distinct elements, a and b, where $a^2 = b^2 = e$. Construct the composition table. Show that the group is non-cyclic. **[4]**

(c) Now consider the four matrices **I**, **X**, **Y** and **Z** where

$$\mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \mathbf{X} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \mathbf{Y} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \mathbf{Z} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}.$$

The group G consists of the set $\{\mathbf{I}, \mathbf{X}, \mathbf{Y}, \mathbf{Z}\}$ with binary operation matrix multiplication. Determine which of the groups in parts **(a)** and **(b)** is isomorphic to G, and specify the isomorphism. **[6]**

(d) The distinct elements $\{p, q, r, s\}$ are combined under the binary operation $\circ$. You are given that $p \circ q = r$ and $q \circ p = s$.

By reference to the group axioms, prove that $\{p, q, r, s\}$ is not a group under $\circ$. **[4]**

Option 5: Markov chains

This question requires the use of a calculator with the ability to handle matrices.

5 Each day that Adam is at work he carries out one of three tasks A, B or C. Each task takes a whole day. Adam chooses the task to carry out on each day according to the following set of three rules.

1. If, on any given day, he has worked on task A then the next day he will choose task A with probability 0.75, and tasks B and C with equal probability.
2. If, on any given day, he has worked on task B then the next day he will choose task B or task C with equal probability but will never choose task A.
3. If, on any given day, he has worked on task C then the next day he will choose task A with probability p and tasks B and C with equal probability.

(i) Write down the transition matrix. **[3]**

(ii) Over a long period Adam carries out the tasks A, B and C with equal frequency. Find the value of p. **[4]**

(iii) On day 1 Adam chooses task A. Find the probability that he also chooses task A on day 5. **[3]**

Adam decides to change rule 3 as follows.

If, on any given day, he has worked on task C then the next day he will choose tasks A, B, C with probabilities 0.4, 0.3, 0.3 respectively.

(iv) On day 1 Adam chooses task A. Find the probability that he chooses the same task on day 7 as he did on day 4. **[5]**

(v) On a particular day, Adam chooses task A. Find the expected number of consecutive further days on which he will choose A. **[3]**

Adam changes all three rules again as follows.

- If he works on A one day then on the next day he chooses C.
- If he works on B one day then on the next day he chooses A or C each with probability 0.5.
- If he works on C one day then on the next day he chooses A or B each with probability 0.5.

(vi) Find the long term probabilities for each task. **[6]**

END OF QUESTION PAPER

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