OCR
Oxford Cambridge and RSA

Wednesday 29 June 2016 – Morning

A2 GCE MATHEMATICS (MEI)

4798/01 Further Pure Mathematics with Technology (FPT)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4798/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator
- Computer with appropriate software

Duration: Up to 2 hours

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** If additional space is required, you should use the lined page(s) at the end of this booklet. The question number(s) must be clearly shown.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

COMPUTING RESOURCES

- Candidates will require access to a computer with a computer algebra system, a spreadsheet, a programming language and graph-plotting software throughout the examination.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 This question concerns the family of curves with parametric equations

$$x = \cos t, \quad y = \sin t - k \tan \frac{t}{2},$$

where k is a positive integer and $-\pi < t < \pi$.

(i) Sketch the curves for the cases $k = 1$, $k = 2$ and $k = 3$ and give the points of intersection with the axes.

Describe two common features of these three curves and one distinct feature for each of the cases $k = 1$ and $k = 2$. **[7]**

(ii) For the case $k = 1$, find, in cartesian form, the points on the curve where the tangent to the curve is parallel to the x-axis. **[4]**

(iii) For the case $k = 2$, confirm the feature of the curve at the point where $t = 0$ by investigating the gradient as $t \to 0$. **[4]**

(iv) For the case $k = 3$, show algebraically that there are no points on the curve where the tangent to the curve is parallel to the x-axis. **[3]**

(v) Sketch the polar curve

$$r = \frac{\cos 2\theta}{\cos \theta}.$$

Show algebraically that the parametric equations

$$x = \cos t, \quad y = \sin t - \tan \frac{t}{2}$$

represent this polar curve.

[6]

2 **(i)** Find, in the form $x + \mathrm{i}y$, the values of $\sinh z$ for $z = \ln 2 + k\,\mathrm{i}$ where $k = -3, -2, -1, 0, 1, 2, 3$.

Sketch the points representing these values on an Argand diagram.

Show that the points in an Argand diagram representing $\sinh(\ln 2 + k\,\mathrm{i})$, where $k \in \mathbb{R}$, lie on an ellipse with equation of the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where a and b are to be determined. **[8]**

(ii) F_1 and F_2 are the points representing the roots of the equation $z^2 + 1 = 0$ on an Argand diagram. The points A and B on the ellipse found in part **(i)** have coordinates $(a, 0)$ and $(0, b)$ respectively.

Show that $F_1A + F_2A = F_1B + F_2B$ where F_1A denotes the distance from F_1 to A. **[3]**

(iii) The function $f(z)$ has derivative $f'(z) = z^2 + 1$. Given that $f\left(\frac{5}{2}\mathrm{i}\right) = 0$, show that the points representing the roots of $f(z) = 0$ on an Argand diagram form an isosceles triangle and that the midpoints of the sides of this triangle lie on the ellipse in part **(i)**. **[8]**

(iv) Find the midpoints of the sides of the triangle formed by the points representing the roots of $z^3+3z+\frac{730\text{i}}{27}=0$ on an Argand diagram. Show that the complex numbers represented by these points can be written in the form $\sinh(\ln 3+k\,\text{i})$ where $k\in\mathbb{R}, -\pi<k<\pi$. **[5]**

3 This question concerns Gaussian integers z of the form $a+b\,\text{i}$, where $a,b\in\mathbb{Z}$.

(i) Create a program that will find all the Gaussian integers z, in the form $a+b\,\text{i}$, that are squares of Gaussian integers where $0\leqslant a\leqslant 20$ and $0\leqslant b\leqslant 20$.

You should write out your program in full and write down the Gaussian integers found. **[8]**

(ii) For the values of z found in part **(i)** for which $\text{Re}(z)=0$ and $\text{Im}(z)>0$, the complex numbers w are defined by $w^2=z$ with $\text{Re}(w)>0$. Sketch the points representing w on an Argand diagram.

Show that $z=2k^2\text{i}$ for these values of z, where $k\in\mathbb{Z}$. **[3]**

(iii) Show that z is a positive real square of a Gaussian integer if, and only if, z is the square of a real integer. **[4]**

(iv) Show that if $a+b\text{i}$ is the square of a Gaussian integer, where a and b are positive integers, then $a^2+b^2=c^2$ for some positive integer c. Show that the converse of this statement is not true. **[5]**

(v) Let z be a Gaussian integer of the form v^2+1 where v is a Gaussian integer. Find, in the form $a+b\,\text{i}$, all the values of z for which $0\leqslant a\leqslant 20$ and $0\leqslant b\leqslant 20$. Indicate clearly the method you have used.

Show that v^2+1, where v is a Gaussian integer and $|v|>2$, is never a Gaussian prime. **[4]**

END OF QUESTION PAPER

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PRINTED ANSWER BOOK

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Duration: up to 2 hours

Candidate forename		Candidate surname	

Centre number						Candidate number				

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DC (NF) 127021/2

Turn over

1 (i)	
1 (i)	**(answer space continued on next page)**

1 (i)	(continued)
1 (ii)	

1 (iii)	

1 (iv)	
1 (v)	**(answer space continued on next page)**

1 (v)	**(continued)**

2 (i)

(answer space continued on next page)

2 (i)	**(continued)**
2 (ii)	

2 (iii)

2 (iv)

3 (i)	

3 (ii)	

3 (iii)

3 (iv)	

3 (v)	

ADDITIONAL ANSWER SPACE

If additional space is required, you should use the following lined page(s). The question number(s) must be clearly shown in the margin(s).

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GCE

Mathematics (MEI)

Unit **4798**: Further Pure Mathematics with Technology

Advanced GCE

Mark Scheme for June 2016

1. **Annotations and abbreviations**

Annotation in scoris	**Meaning**
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

2. Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

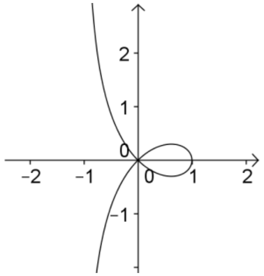
NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

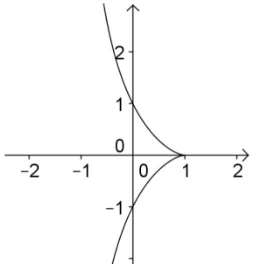
<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td>1</td><td>(i)</td><td></td><td>

$k = 1$



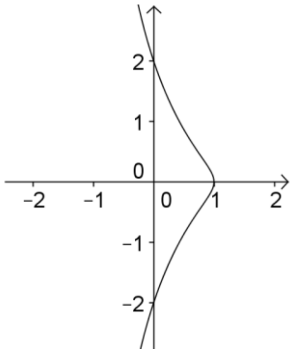
</td><td>G1</td><td>General shape</td></tr>
<tr><td></td><td></td><td></td><td>

$k = 2$



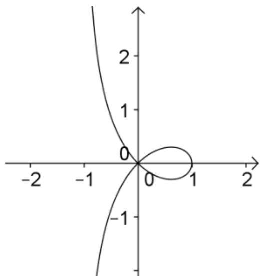
</td><td>G1</td><td>General shape</td></tr>
<tr><td></td><td></td><td></td><td>

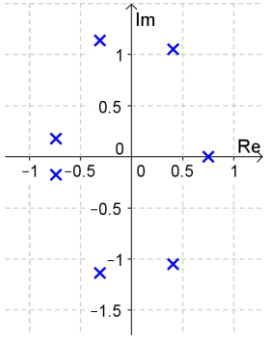
$k = 3$



</td><td>G1</td><td>General shape</td></tr>
<tr><td></td><td></td><td></td><td></td><td>G1</td><td>All curves pass through x-axis at 1. Pass through y-axis at (0,0), (0, ±1), (0, ±2).</td></tr>
<tr><td></td><td></td><td></td><td>All curves have an asymptote at $x = -1$.</td><td>E1</td><td></td></tr>
<tr><td></td><td></td><td></td><td>All curves have symmetry in the x-axis.</td><td>E1</td><td></td></tr>
<tr><td></td><td></td><td></td><td>$k = 1$ has a loop, $k = 2$ has a cusp.</td><td>E1</td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td>[7]</td><td></td></tr>
</table>

Question			Answer	Marks	Guidance
	(ii)		$\frac{dy}{dx}=\frac{dy/dt}{dx/dt}$ $=\frac{\left(\sin^2\frac{t}{2}-(2\cos t-1)\cos^2\frac{t}{2}\right)}{2\sin t\cos^2\frac{t}{2}}$	B1	soi Condone solving $\frac{dy}{dt}=0$ without investigating $\frac{dx}{dt}$
			$\frac{dy}{dx}=0$ when $\sin^2\frac{t}{2}-(2\cos t-1)\cos^2\frac{t}{2}=0$	M1	
			$t=-0.90456$ or $t=0.90456$ (0.61803,0.30028) or (0.618034,–0.30028)	A1 A1 **[4]**	
	(iii)		$\frac{dy}{dx}=\frac{\sin^2\frac{t}{2}-(\cos t-1)\cos^2\frac{t}{2}}{\sin t\cos^2\frac{t}{2}}$	B1	Or any equivalent expression.
			There are two branches either side of $t=0$. Considering the gradient on each branch:	M1	Can be implied by subsequent working. Limit from one direction only scores max B1 M0 A0 B1
			$\lim_{t\to0^+}\left(\frac{dy}{dx}\right)=0$ $\lim_{t\to0^-}\left(\frac{dy}{dx}\right)=0$	A1	
			The curve is defined at $t=0$ so there is a cusp.	B1 **[4]**	Accept point is (1, 0) at t=0.
	(iv)		$\frac{dy}{dx}=\frac{3\sin^2\frac{t}{2}+(3-2\cos t)\cos^2\frac{t}{2}}{2\sin t\cos^2\frac{t}{2}}$	B1	Equivalent expressions may be given.
			$3\sin^2\frac{t}{2}\geq0$, $\cos^2\frac{t}{2}\geq0$ and $(3-2\cos t)>0$ for all t. Therefore numerator >0 .	M1	

Question			Answer	Marks	Guidance
			There are no values of t where $\frac{dy}{dx} = 0$.	E1 **[3]**	
	(v)		 Using $x = r\cos\theta$ and $y = r\sin\theta$: $x = r\cos\theta$ $= \frac{\cos 2\theta}{\cos\theta}\cos\theta$ $= \cos 2\theta$ $= \cos t$ when $t = 2\theta$ $y = r\sin\theta$ $= \frac{\cos 2\theta}{\cos\theta}\sin\theta$ $= \frac{2\cos^2\theta\sin\theta - \sin\theta}{\cos\theta}$ $= \sin 2\theta - \tan\theta$ $= \sin t - \tan\frac{t}{2}$ when $t = 2\theta$	G1 G1 M1 A1 M1 A1 **[6]**	Shape Intersection with axes. Accept alternative argument based on $r^2 = x^2 + y^2$ $= \cos^2 t + \sin^2 t - 2\sin t\tan\frac{t}{2} + \tan^2\frac{t}{2}$ Evidence of use of double angle formula

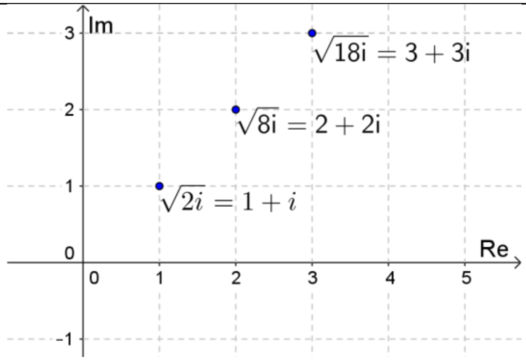
Question			Answer	Marks	Guidance
2	(i)		$w = -0.742 - 0.176\text{i}, -0.312 - 1.137\text{i}, 0.405 - 1.052\text{i}, 0.75\ (+0\text{i})$, $0.405 + 1.052\text{i}, -0.312 + 1.137\text{i}, -0.742 + 0.176\text{i}$	M1 A1	Evidence of at least 2 found correctly.
				G2	All 7 correctly plotted (G1 for at least 2 correct)
			$\sinh(\ln 2 + k\text{i}) = \sinh(\ln 2)\cos k + \text{i}\cosh(\ln 2)\sin k$	M1	Must consider the generalised case
			$= \frac{3}{4}\cos k + \frac{5\text{i}}{4}\sin k$	A1	
			Locus has parametric equation $x = \frac{3}{4}\cos t, y = \frac{5}{4}\sin t$.	A1	soi by equivalent statement about major and minor axes or evidence of linking complex numbers to cartesian equation
			$\frac{x^2}{\left(3/4\right)^2} + \frac{y^2}{\left(5/4\right)^2} = 1$		Accept $\frac{16x^2}{9} + \frac{16y^2}{25} = 1$.
			$a = \frac{3}{4},\ b = \frac{5}{4}$	B1 **[8]**	
	(ii)		$z^2 + 1 = 0 \Rightarrow z = \text{i}, -\text{i}$.	B1	soi
			$F_1 : (0,1), F_2 : (0,-1)$	B1	
			$F_1A + F_2A : \sqrt{1^2 + \left(\frac{3}{4}\right)^2} + \sqrt{1^2 + \left(\frac{3}{4}\right)^2} = \frac{5}{2}$	B1 **[3]**	
			$F_1B + F_2B : \frac{5}{4} + 1 + \frac{5}{4} - 1 = \frac{5}{2}$		

Question		Answer	Marks	Guidance
	(iii)	$f(z)=\dfrac{z^3}{3}+z+c$	B1	Condone absence of c for this mark
		$\dfrac{\left(\frac{5}{2}\mathrm{i}\right)^3}{3}+\frac{5}{2}\mathrm{i}+c=0\Rightarrow c=\frac{65}{24}\mathrm{i}$	M1 A1	
		Solving $\dfrac{z^3}{3}+z+\dfrac{65}{24}\mathrm{i}=0$ $z=\frac{5}{2}\mathrm{i},\frac{3\sqrt{3}}{4}-\frac{5}{4}\mathrm{i},-\frac{3\sqrt{3}}{4}-\frac{5}{4}\mathrm{i}$	M1 A1	
		By symmetry the triangle is isosceles.	E1	Accept $\left\lvert\frac{3\sqrt{3}}{4}-\frac{5}{4}\mathrm{i}-\frac{5}{2}\mathrm{i}\right\rvert=\left\lvert-\frac{3\sqrt{3}}{4}-\frac{5}{4}\mathrm{i}-\frac{5}{2}\mathrm{i}\right\rvert=\frac{3\sqrt{7}}{2}$
		Midpoints are at: $z=-\frac{5}{4}\mathrm{i},\frac{3\sqrt{3}}{8}+\frac{5}{8}\mathrm{i},-\frac{3\sqrt{3}}{8}+\frac{5}{8}\mathrm{i}$	B1	
		Showing $\dfrac{16x^2}{9}+\dfrac{16y^2}{25}=1$ for all three points.	B1 **[8]**	Accept $z=\sinh(\ln 2+k\mathrm{i})$ when $k=-\frac{\pi}{2},\frac{\pi}{6},\frac{5\pi}{6}$
	(iv)	$z=\frac{10}{3}\mathrm{i},\frac{4\sqrt{3}}{3}-\frac{5}{3}\mathrm{i},-\frac{4\sqrt{3}}{3}-\frac{5}{3}\mathrm{i}$	B1	soi
		Midpoints: $\left(0,-\frac{5}{3}\right),\left(\frac{2\sqrt{3}}{3},\frac{5}{6}\right),\left(\frac{-2\sqrt{3}}{3},\frac{5}{6}\right)$	B1	Accept $z=-\frac{5}{3}\mathrm{i},\frac{2\sqrt{3}}{3}+\frac{5}{6}\mathrm{i},\frac{-2\sqrt{3}}{3}+\frac{5}{6}\mathrm{i}$
		$\sinh(\ln 3+k\mathrm{i})=\sinh(\ln 3)\cos k+\mathrm{i}\cosh(\ln 3)\sin k$ $=\frac{4}{3}\cos k+\frac{5\mathrm{i}}{3}\sin k$	M1	

Question			Answer	Marks	Guidance
			Equating real and imaginary parts:	M1	
			$-\frac{5}{3}\mathrm{i} = \sinh(\ln(3) - \frac{\pi}{2}\mathrm{i})$, $\frac{2\sqrt{3}}{3} + \frac{5}{6}\mathrm{i} = \sinh(\ln(3) + \frac{\pi}{6}\mathrm{i})$,		
			$\frac{-2\sqrt{3}}{3} + \frac{5}{6}\mathrm{i} = \sinh(\ln(3) + \frac{5\pi}{6}\mathrm{i})$	A1 **[5]**	

3	**(i)**		Example program:
			
			Define prog1()=
			Prgm
			Local a,b,z
			For a,0,20
			  For b,0,20
			    z:=√(a+b*i)
			    If fPart(real(z))=0 and fPart(imag(z))=0 Then
			      Disp a+b*i
			    EndIf
			  EndFor
			EndFor
			EndPrgm
				M5	If the answers are incorrect allocate method marks as follows:
					M1 Appropriate structure program
					M1 Loop for *a* or equivalent (including 0)
					M1 Loop for *b* or equivalent (including 0)
					M1 Use of Check (If) statement
					M1 Correct If statement
					
					More efficient programs may be possible.

			0,1,4,9,16	A1	
			2i, 8i,18i	A1	
			3+4i, 5+12i, 8+6i, 15+8i, 12+16i	A1 **[8]**	

Question		Answer	Marks	Guidance
	(ii)		G1	Accept –1–i, –2–2i and –3–3i drawn as well or instead.
		$z = (p + q\text{i})^2$ $= p^2 - q^2 + 2pq\text{i}$ This has Re(z) = 0 and Im(z) > 0 when p and q are non-zero and equal: $z = 2k^2\text{i}$	M1 A1 **[3]**	Alternative method also acceptable for second and third marks (M1 A1): $(1+\text{i})^2 = 2\text{i}$ $= 2\times1^2\text{i}$ $(2+2\text{i})^2 = 8\text{i}$ $= 2\times2^2\text{i}$ $(3+3\text{i})^2 = 18\text{i}$ $= 2\times3^2\text{i}$
	(iii)	$z = (p + q\text{i})^2$ $= p^2 - q^2 + 2pq\text{i}$	B1	
		If z is a positive real square of a Gaussian integer then p = 0 or q = 0,	E1	
		but if p=0 then z is negative . Therefore q=0 and z is the square of a real integer.	E1	
		All real integers are Gaussian integers therefore if z is the square of a real integer then z is a positive real square of a Gaussian integer.	E1 **[4]**	

Question			Answer	Marks	Guidance
	(iv)		$z = (p+q\text{i})^2$ $= p^2 - q^2 + 2pq\text{i}$ $a^2+b^2 = (p^2-q^2)^2 + (2pq)^2$ $= p^4 - 2p^2q^2 + q^4 + 4p^2q^2$ $= p^4 + 2p^2q^2 + q^4$ $= (p^2+q^2)^2$ i.e. the square of a positive integer Converse not true: e.g. $\sqrt{4+3\text{i}} = \frac{3\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\text{i}$	M1 A1 E1 M1 A1 **[5]**	 Evidence of attempt to find a counterexample.
	(v)		**Either**: Change the expression to z:=√(a–1+b*i) 0,1,2,5,10,17,1+2i, 1+8i, 1+18i, 4+4i, 6+12i, 9+6i, 16+8i, 13+16i -- **Or**: Adding 1 to the values from (i) gives: 1,2,5,10, 17, 1+2i, 1+8i, 1+18i, 4+4i, 6+12i, 9+6i, 16+8i, 13+16i 0 is also a solution -- $w^2+1 = (w+\text{i})(w-\text{i})$ If $\lvert w \rvert > 2$, w+i and w–i are non-zero Gaussian integers ≠ 1, –1, i or –i .	B1 B1 ---------- B1 B1 ---------- B1 E1 **[4]**	 Attempt to factorise soi. Some (more than 1) numerical cases only: B1E0.

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