

Monday 25 June 2012 – Afternoon

A2 GCE MATHEMATICS (MEI)

4777 Numerical Computation

Candidates answer on the Answer Booklet.

OCR supplied materials:

- 8 page Answer Booklet (sent with general stationery)
- MEI Examination Formulae and Tables (MF2)
- Graph paper

Other materials required:

- Scientific or graphical calculator
- Computer with appropriate software and printing facilities

Duration: 2 hours 30 minutes



INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the Answer Booklet. Please write clearly and in capital letters.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer any **three** questions.
- Additional sheets, including computer print-outs, should be fastened securely to the Answer Booklet.
- Do **not** write in the bar codes.

COMPUTING RESOURCES

- Candidates will require access to a computer with a spreadsheet program and suitable printing facilities throughout the examination.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- In each of the questions you are required to write spreadsheet routines to carry out various numerical analysis processes.
- You will not receive credit for using any numerical analysis functions which are provided within the spreadsheet. For example, many spreadsheets provide a solver routine; you will not receive credit for using this routine when asked to write your own procedure for solving an equation.
You may use the following built-in mathematical functions: square root, sin, cos, tan, arcsin, arccos, arctan, ln, exp.
- For each question you attempt, you should submit print-outs showing the spreadsheet routine you have written and the output it generates. It will be necessary to print out the formulae in the cells as well as the values in the cells.
You are not expected to print out and submit everything your routine produces, but you are required to submit sufficient evidence to convince the examiner that a correct procedure has been used.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

- 1 (i) x_0, x_1, x_2 are three terms in a first order iteration converging to α . Given that the error in x_0 is ε and the error in x_1 is $k\varepsilon$ (where ε is small), what can you say about the error in x_2 ?

Show that α may be estimated as $x_0 - \frac{(\Delta x_0)^2}{\Delta^2 x_0}$. [6]

The equation $x = \cos(bx)$, where x is in radians and $0 \leq b \leq 3$, has a root α which depends upon b .

- (ii) Use a spreadsheet to show that the iteration $x_{r+1} = \cos(bx_r)$, with $x_0 = 1$,

(A) converges slowly when $b = 1$,

(B) diverges when $b = 2$.

Show that the formula obtained in part (i) may be used to give more rapid convergence when $b = 1$. What does the use of this formula achieve when $b = 2$? [10]

- (iii) Obtain a graph of α against b for $0 \leq b \leq 3$. Find, correct to 4 decimal places, the value of b for which α is closest to 0.5. [Hint: you may find it convenient to use starting values other than $x_0 = 1$ for some values of b .] [8]

- 2 The Gaussian 3-point integration formula has the form

$$\int_{-h}^h f(x) dx = af(-\alpha) + bf(0) + af(\alpha).$$

- (i) Obtain the three equations that determine a , b and α . Verify that these equations are satisfied by

$$\alpha = \sqrt{\frac{3}{5}}h, \quad a = \frac{5}{9}h, \quad b = \frac{8}{9}h. \quad [8]$$

- (ii) Taking $h = \frac{\pi}{4}$ initially, use the Gaussian 3-point rule to estimate the value of

$$\int_0^{\frac{\pi}{2}} (\sin x + 2\cos x)^{\frac{1}{2}} dx.$$

Repeat the process, halving h as necessary, in order to establish the value of the integral correct to 6 decimal places. [12]

- (iii) Determine, correct to 3 decimal places, the value of k such that

$$\int_0^{\frac{\pi}{2}} (\sin x + 2\cos x)^k dx = 2. \quad [4]$$

3 The second order differential equation

$$\frac{d^2y}{dx^2} - 2y \frac{dy}{dx} = x^2$$

with initial conditions $x = 1, y = 1, \frac{dy}{dx} = -1$, is to be solved using finite difference methods.

(i) Show that, in the usual notation,

$$y_{r+1}(1 - hy_r) = h^2x_r^2 + 2y_r - y_{r-1} - hy_r y_{r-1}$$

and

$$y_1 = 1 - h - \frac{1}{2}h^2. \quad [8]$$

(ii) Obtain a solution from $x = 1$ to $x = 3$ with $h = 0.1$. Use your spreadsheet to produce a graph of this solution. [8]

(iii) Halving h as necessary, find the values of y at $x = 2$ and at $x = 3$, each correct to 3 significant figures.

Show that this method of solution is second order. [8]

4 (i) Describe the conditions for convergence of the Gauss-Jacobi and Gauss-Seidel methods for the solution of a system of linear equations. [3]

(ii) A system of linear equations is represented by the following augmented matrix.

$$\left(\begin{array}{cccc|c} k & 1 & 2 & 0 & 1 \\ 1 & 3 & 0 & 2 & 0 \\ 2 & 0 & 3 & 1 & 0 \\ 0 & 2 & 1 & 3 & 0 \end{array} \right)$$

Investigate the convergence of the Gauss-Jacobi method applied to this system of equations in the cases $k = 1, k = 3, k = 5$.

Relate your results to your answer to part (i). [12]

(iii) Modify your routine from part (ii) to find the inverse of the coefficient matrix in the case $k = 5$. [9]

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Mathematics (MEI)

Advanced GCE

Unit 4777: Numerical Computation

Mark Scheme for June 2012

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of candidates of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, OCR Nationals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

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1. Annotations and abbreviations

Annotation in scoris	Meaning
✓ and ✕	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

2. Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

- a. Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

- b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

- c. The following types of marks are available.

M

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B

Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- d. When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e. The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

- f. Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g. Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

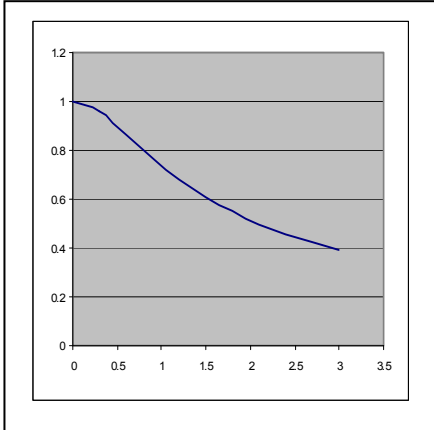
If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

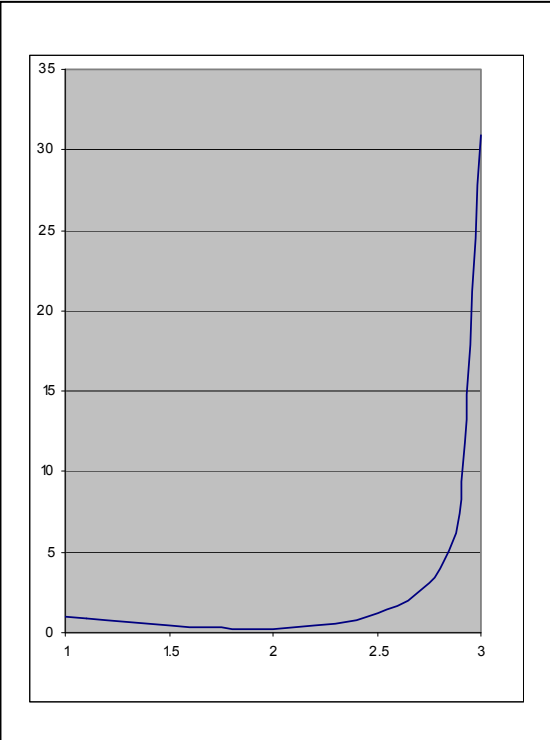
h. For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question		Answer	Marks	Guidance																																																																			
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1	(ii)	<table style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th style="text-align: left;">b</th> <th style="text-align: left;">r</th> <th style="text-align: left;">x_r</th> <th style="text-align: left;">b</th> <th style="text-align: left;">r</th> <th style="text-align: left;">x_r</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>0</td> <td>1</td> <td>2</td> <td>0</td> <td>1</td> </tr> <tr> <td></td> <td>1</td> <td>0.540302</td> <td></td> <td>1</td> <td>-0.41615</td> </tr> <tr> <td></td> <td>2</td> <td>0.857553</td> <td></td> <td>2</td> <td>0.673181</td> </tr> <tr> <td></td> <td>3</td> <td>0.65429</td> <td></td> <td>3</td> <td>0.222554</td> </tr> <tr> <td></td> <td>4</td> <td>0.79348</td> <td></td> <td>4</td> <td>0.902564</td> </tr> <tr> <td></td> <td>-----</td> <td>-----</td> <td></td> <td>-----</td> <td>-----</td> </tr> <tr> <td></td> <td>30</td> <td>0.739087</td> <td></td> <td>30</td> <td>0.803826</td> </tr> <tr> <td></td> <td>31</td> <td>0.739084</td> <td></td> <td>31</td> <td>-0.03685</td> </tr> <tr> <td></td> <td>32</td> <td>0.739086</td> <td>converging</td> <td>32</td> <td>0.997286</td> </tr> <tr> <td></td> <td>33</td> <td>0.739085</td> <td>slowly</td> <td>33</td> <td>-0.4112</td> <td>diverging</td> </tr> </tbody> </table>	b	r	x_r	b	r	x_r	1	0	1	2	0	1		1	0.540302		1	-0.41615		2	0.857553		2	0.673181		3	0.65429		3	0.222554		4	0.79348		4	0.902564		-----	-----		-----	-----		30	0.739087		30	0.803826		31	0.739084		31	-0.03685		32	0.739086	converging	32	0.997286		33	0.739085	slowly	33	-0.4112	diverging	M1A1 A1A1	setup results
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<p>1 (iii)</p>	<table style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th style="text-align: left;">eg b</th> <th style="text-align: left;">α</th> </tr> </thead> <tbody> <tr><td>0</td><td>1</td></tr> <tr><td>0.3</td><td>0.958907</td></tr> <tr><td>0.6</td><td>0.867554</td></tr> <tr><td>0.9</td><td>0.769576</td></tr> <tr><td>1.2</td><td>0.682741</td></tr> <tr><td>1.5</td><td>0.609904</td></tr> <tr><td>1.8</td><td>0.549477</td></tr> <tr><td>2.1</td><td>0.499139</td></tr> <tr><td>2.4</td><td>0.456823</td></tr> <tr><td>2.7</td><td>0.420881</td></tr> <tr><td>3</td><td>0.39004</td></tr> </tbody> </table> <div style="text-align: center; margin: 10px 0;">  </div> <table style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th style="text-align: left;">b</th> <th style="text-align: left;">α</th> </tr> </thead> <tbody> <tr><td>2.1</td><td>0.499139</td></tr> <tr><td>2.0</td><td>0.514933</td></tr> <tr><td>2.095</td><td>0.499907</td></tr> <tr><td>2.094</td><td>0.500061</td></tr> <tr><td>2.0944</td><td>0.499999</td></tr> <tr><td>2.0943</td><td>0.500015</td></tr> </tbody> </table> <p style="margin-left: 20px;">best is $b = 2.0944$</p>	eg b	α	0	1	0.3	0.958907	0.6	0.867554	0.9	0.769576	1.2	0.682741	1.5	0.609904	1.8	0.549477	2.1	0.499139	2.4	0.456823	2.7	0.420881	3	0.39004	b	α	2.1	0.499139	2.0	0.514933	2.095	0.499907	2.094	0.500061	2.0944	0.499999	2.0943	0.500015	<p>A1A1 G2 M2 A2 [8]</p>	<p>values graph trial result</p>																																																																																																																																				
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Question		Answer						Marks	Guidance		
2	(iii)	eg									
		k	L	R	m	h	x_1	x_2			
		0.381	0	0.785398	0.392699	0.392699	0.088516	0.696882			
		function values			1.357491		1.321988	1.344658			
		weights			0.349066		0.218166	0.218166			
		integral			0.473854		0.288413	0.293359	1.055626		
			L	R	m	h	x_1	x_2			
			0.785398	1.570796	1.178097	0.392699	0.873914	1.482281			
		function values			1.221099		1.314677	1.062641			
		weights			0.349066		0.218166	0.218166			
		integral			0.426244		0.286818	0.231832	0.944894		
									2.00052		
		k	0.380	0.381	<i>(NB: these changes in the integral are greater than those arising from halving h again.)</i>						
		integral	1.999235	2.00052							
										M2	
										M1A1	
										[4]	
										Modification to include k	

Question	Answer	Marks	Guidance																																																																		
3 (i)	Use central difference formulae for 2nd and 1st derivatives to obtain first given result Hence obtain $y_1(1-h) = h^2 + 2 - (1+h)y_{-1}$ State or show that $y_1 - y_{-1} = -2h$ or equivalent Eliminate y_{-1} ; convincing algebra to given result for y_1	M1A1A1 M1A1 B1 M1A1 [8]																																																																			
3 (ii)	<table style="display: inline-table; vertical-align: top; margin-right: 20px;"> <thead> <tr> <th>h</th> <th>x</th> <th>y</th> </tr> </thead> <tbody> <tr><td>0.1</td><td>1</td><td>1</td></tr> <tr><td></td><td>1.1</td><td>0.895</td></tr> <tr><td></td><td>1.2</td><td>0.782647</td></tr> <tr><td></td><td>1.3</td><td>0.666837</td></tr> <tr><td></td><td>1.4</td><td>0.552585</td></tr> <tr><td></td><td>1.5</td><td>0.445715</td></tr> <tr><td></td><td>1.6</td><td>0.352423</td></tr> <tr><td></td><td>1.7</td><td>0.27885</td></tr> <tr><td></td><td>1.8</td><td>0.230785</td></tr> <tr><td></td><td>1.9</td><td>0.213615</td></tr> <tr><td></td><td>2</td><td>0.232584</td></tr> <tr><td></td><td>2.1</td><td>0.293408</td></tr> <tr><td></td><td>2.2</td><td>0.403342</td></tr> <tr><td></td><td>2.3</td><td>0.572952</td></tr> <tr><td></td><td>2.4</td><td>0.819293</td></tr> <tr><td></td><td>2.5</td><td>1.172342</td></tr> <tr><td></td><td>2.6</td><td>1.689964</td></tr> <tr><td></td><td>2.7</td><td>2.499465</td></tr> <tr><td></td><td>2.8</td><td>3.945671</td></tr> <tr><td></td><td>2.9</td><td>7.406389</td></tr> <tr><td></td><td>3</td><td>30.95641</td></tr> </tbody> </table> <div style="display: inline-block; border: 1px solid black; padding: 5px; text-align: center;">  </div>	h	x	y	0.1	1	1		1.1	0.895		1.2	0.782647		1.3	0.666837		1.4	0.552585		1.5	0.445715		1.6	0.352423		1.7	0.27885		1.8	0.230785		1.9	0.213615		2	0.232584		2.1	0.293408		2.2	0.403342		2.3	0.572952		2.4	0.819293		2.5	1.172342		2.6	1.689964		2.7	2.499465		2.8	3.945671		2.9	7.406389		3	30.95641	M3 A3 G2 [8]	setup values
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Question		Answer						Marks	Guidance		
3	(iii)	<i>h</i>	<i>y</i> (2)	diffs	ratios	<i>y</i> (3)	diffs	ratios			
		0.1	0.232584			30.95641					
		0.05	0.234604	0.00202		37.62817	6.671755				
		0.025	0.235106	0.000502	0.248632	39.83126	2.203093	0.330212	M1A1A1	<i>y</i> (2)	
		0.0125	0.235232	0.000125	0.249662	40.42700	0.595738	0.27041			
		0.00625	0.235 to 3 sf			40.57898	0.151988	0.255125			
		0.003125				40.61718	0.038192	0.251283	M1A1A1	<i>y</i> (3)	
						40.6 to 3 sf					
		ratios about 0.25 (in each case), so second order method							M1E1		
									[8]		

Question	Answer	Marks	Guidance																																																																																																													
4 (i)	A sufficient, but not necessary, condition for G-J and G-S to converge is strict diagonal dominance: the diagonal elements in each row are, in magnitude, greater than or equal to the sum of the magnitudes of the other elements, and at least one inequality is strict.	E1 E1 E1 [3]																																																																																																														
4 (ii)	<table style="width:100%; border:none;"> <tr> <td style="text-align:center;">$k=1$</td> <td style="text-align:center;">$k=3$</td> <td></td> <td></td> </tr> <tr> <td style="text-align:center;">x_1</td> <td style="text-align:center;">x_2</td> <td style="text-align:center;">x_3</td> <td style="text-align:center;">x_4</td> </tr> <tr> <td style="text-align:center;">0</td> <td style="text-align:center;">0</td> <td style="text-align:center;">0</td> <td style="text-align:center;">0</td> </tr> <tr> <td style="text-align:center;">1</td> <td style="text-align:center;">0</td> <td style="text-align:center;">0</td> <td style="text-align:center;">0</td> </tr> <tr> <td style="text-align:center;">1</td> <td style="text-align:center;">-0.33333</td> <td style="text-align:center;">-0.66667</td> <td style="text-align:center;">0</td> </tr> <tr> <td style="text-align:center;">2.66667</td> <td style="text-align:center;">-0.33333</td> <td style="text-align:center;">-0.66667</td> <td style="text-align:center;">0.444444</td> </tr> <tr> <td style="text-align:center;">2.66667</td> <td style="text-align:center;">-1.18519</td> <td style="text-align:center;">-1.92593</td> <td style="text-align:center;">0.444444</td> </tr> <tr> <td style="text-align:center;">6.037037</td> <td style="text-align:center;">-1.18519</td> <td style="text-align:center;">-1.92593</td> <td style="text-align:center;">1.432099</td> </tr> <tr> <td style="text-align:center;">6.037037</td> <td style="text-align:center;">-2.96708</td> <td style="text-align:center;">-4.50206</td> <td style="text-align:center;">1.432099</td> </tr> <tr> <td style="text-align:center;">12.97119</td> <td style="text-align:center;">-2.96708</td> <td style="text-align:center;">-4.50206</td> <td style="text-align:center;">3.478738</td> </tr> <tr> <td style="text-align:center;">12.97119</td> <td style="text-align:center;">-6.64289</td> <td style="text-align:center;">-9.80704</td> <td style="text-align:center;">3.478738</td> </tr> </table> <p>differences increasing so diverging</p> <table style="width:100%; 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border:none;"> <tr> <td style="text-align:center;">0.5</td> <td style="text-align:center;">-0.5</td> <td style="text-align:center;">-0.5</td> <td style="text-align:center;">0.499999</td> <td></td> </tr> <tr> <td style="text-align:center;">0.5</td> <td style="text-align:center;">-0.5</td> <td style="text-align:center;">-0.5</td> <td style="text-align:center;">0.499999</td> <td></td> </tr> <tr> <td style="text-align:center;">0.5</td> <td style="text-align:center;">-0.5</td> <td style="text-align:center;">-0.5</td> <td style="text-align:center;">0.5</td> <td></td> </tr> <tr> <td style="text-align:center;">0.5</td> <td style="text-align:center;">-0.5</td> <td style="text-align:center;">-0.5</td> <td style="text-align:center;">0.5</td> <td></td> </tr> </table>	$k=1$	$k=3$			x_1	x_2	x_3	x_4	0	0	0	0	1	0	0	0	1	-0.33333	-0.66667	0	2.66667	-0.33333	-0.66667	0.444444	2.66667	-1.18519	-1.92593	0.444444	6.037037	-1.18519	-1.92593	1.432099	6.037037	-2.96708	-4.50206	1.432099	12.97119	-2.96708	-4.50206	3.478738	12.97119	-6.64289	-9.80704	3.478738	x_1	x_2	x_3	x_4		0	0	0	0		0.2	0	0	0		0.2	-0.06667	-0.13333	0		0.266667	-0.06667	-0.13333	0.088889		0.266667	-0.14815	-0.20741	0.088889		0.312593	-0.14815	-0.20741	0.167901		0.312593	-0.21613	-0.26436	0.167901		0.348971	-0.21613	-0.26436	0.232209		0.5	-0.5	-0.5	0.499999		0.5	-0.5	-0.5	0.499999		0.5	-0.5	-0.5	0.5		0.5	-0.5	-0.5	0.5		M3 A1E1 A1E1 E1 E1 E1 A1 [12]	Setup $k=1$ $k=3$ $k=5$
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Question	Answer	Marks	Guidance
4 (iii)	First column of inverse matrix is $(0.5, -0.5, -0.5, 0.5)^T$ Modify routine to solve for the other three unit vectors Obtain solutions Write inverse matrix: $\begin{matrix} 0.5 & -0.5 & -0.5 & 0.5 \\ -0.5 & 1.166667 & 0.666667 & -1 \\ -0.5 & 0.666667 & 0.916667 & -0.75 \\ 0.5 & -1 & -0.75 & 1.25 \end{matrix}$	B1 M1,1,1 A1,1,1 M1A1 [9]	

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4777 Numerical Computation

There were fewer than 10 candidates for this unit, so no report is published.