OCR

Tuesday 24 June 2014 – Morning

A2 GCE MATHEMATICS (MEI)

4777/01 Numerical Computation

Candidates answer on the Answer Booklet.

OCR supplied materials:
- 12 page Answer Booklet (OCR12) (sent with general stationery)
- MEI Examination Formulae and Tables (MF2)
- Graph paper

Other materials required:
- Scientific or graphical calculator
- Computer with appropriate software and printing facilities

Duration: 2 hours 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the Answer Booklet. Please write clearly and in capital letters.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer any **three** questions.
- Additional sheets, including computer print-outs, should be fastened securely to the Answer Booklet.
- Do **not** write in the bar codes.

COMPUTING RESOURCES

- Candidates will require access to a computer with a spreadsheet program and suitable printing facilities throughout the examination.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- In each of the questions you are required to write spreadsheet routines to carry out various numerical analysis processes.
- You will not receive credit for using any numerical analysis functions which are provided within the spreadsheet. For example, many spreadsheets provide a solver routine; you will not receive credit for using this routine when asked to write your own procedure for solving an equation.
  You may use the following built-in mathematical functions: square root, sin, cos, tan, arcsin, arccos, arctan, ln, exp.
- For each question you attempt, you should submit print-outs showing the spreadsheet routine you have written and the output it generates. It will be necessary to print out the formulae in the cells as well as the values in the cells.
  You are not expected to print out and submit everything your routine produces, but you are required to submit sufficient evidence to convince the examiner that a correct procedure has been used.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 **(i)** The equation $f(x) = 0$ has a root α. The equation is rearranged to $x = g(x)$.

State the condition for the iteration $x_{r+1} = g(x_r)$ to converge to α. (You may assume that a suitable starting value is used.)

Obtain the relaxed iteration

$$x_{r+1} = (1-\lambda)x_r + \lambda g(x_r).$$

Find, in terms of α, the best value of λ. How would a value of λ be chosen in practice? **[5]**

(ii) Show graphically that the equation

$$kx = \exp\left(x + \frac{1}{x}\right), \qquad (*)$$

where $k = 10$, has two positive roots.

Show also that (*) with $k = 10$ has no negative roots.

Show that the iteration

$$x_{r+1} = \frac{1}{10}\exp\left(x_r + \frac{1}{x_r}\right)$$

converges to one of the roots but diverges from the other.

Use the method of relaxation to find the other root correct to 4 decimal places. **[14]**

(iii) Obtain, correct to 4 decimal places, both roots of (*) in the case $k = 20$. **[5]**

2 In the table, the values of x are exact and the values of y are subject to experimental error.

x	1	2	3	4	5
y	7.24	12.15	13.84	12.25	7.07

It is thought that the relationship between y and x can be modelled as a quadratic function with $y = 0$ when $x = 0$. That is, $y = bx + cx^2$ for some constants b and c. These constants are to be estimated using least squares.

(i) Show, using partial differentiation, that one of the normal equations is

$$\Sigma xy = b\Sigma x^2 + c\Sigma x^3.$$

Obtain the other normal equation. **[6]**

(ii) Using a spreadsheet and the given data,

- find values for b and c,
- draw a graph of the data points and the fitted curve,
- find the sum of the residuals and the sum of the squares of the residuals. **[14]**

(iii) State why the sum of the residuals is not zero, and explain how this relates to the assumptions about the relationship between y and x.

Suppose the model is modified so that y is still a quadratic function of x, but without y being zero when x is zero. State, with a reason, what effect this change will have on the sum of squares of the residuals. (You are not required to do any further calculations.) **[4]**

3 **(i)** The trapezium rule, using n strips of equal width h, is used to find an estimate T_n of the integral

$$I = \int_a^b f(x)\,dx.$$

The global error in T_n is of the form

$$A_2 h^2 + A_4 h^4 + A_6 h^6 + \ldots,$$

where the coefficients $A_2, A_4, A_6, \ldots$ are independent of n and h.

Show that $T_n^* = \frac{1}{3}(4T_{2n} - T_n)$ is an estimate of I with global error of order h^4.

Write down, without proof, an expression, T_n^{**}, in terms of T_{2n}^* and T_n^*, that will be an estimate of I with global error of order h^6. **[6]**

(ii) Set up a spreadsheet that uses Romberg's method to find the value of the integral

$$I = \int_0^{\frac{\pi}{2}} \sqrt{k + \sin x}\,dx$$

for $k = 1$. Your method should show the values of T, T^* and T^{**}.

Show that, when $k = 1$, $I = 2$ (to at least 7 decimal places).

Show by means of ratios of differences that T, T^* and T^{**} have errors consistent with your working in part **(i)**. **[12]**

(iii) Modify your spreadsheet to obtain the values of I for $k = 0.8$, 0.4, 0.2 and 0.

Comment on the accuracy of your estimates and the ratios of differences as k gets smaller. **[6]**

Question 4 begins on page 4.

4 In this question, the Gauss-Seidel iterative method is to be used to solve the matrix equation

$$\mathbf{Mx} = \mathbf{c}, \qquad (*)$$

$$\text{where } \mathbf{M} = \begin{pmatrix} a & 1 & 0 & -1 \\ 2 & b & 2 & 0 \\ 0 & 2 & b & 2 \\ 1 & 0 & -1 & a \end{pmatrix}, \ \mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} \text{ and } \mathbf{c} = \begin{pmatrix} 4 \\ 3 \\ 2 \\ 1 \end{pmatrix}.$$

(i) State the conditions on a and b for **M** to be diagonally dominant.

State the conditions on a and b for **M** to be strictly diagonally dominant.

Explain the relevance of these conditions for the convergence of the Gauss-Seidel method. **[6]**

(ii) For the case $a = 2$ and $b = 4$, use the Gauss-Seidel method on a spreadsheet to obtain a numerical solution for (*). Deduce the exact solution as fractions and show that your deduction is correct. **[10]**

(iii) Show, by means of examples, that larger values of a lead to faster convergence. Show similarly that larger values of b also lead to faster convergence. **[4]**

(iv) For the case $a = 2$, find correct to 1 decimal place the largest value of b for which the Gauss-Seidel method diverges. **[2]**

(v) For the case $b = a^2$, find correct to 1 decimal place the largest value of a for which the Gauss-Seidel method diverges. **[2]**

END OF QUESTION PAPER

GCE

Mathematics (MEI)

Unit **4777**: Numerical Computation

Advanced GCE

Mark Scheme for June 2014

1. Annotations and abbreviations

Annotation in scoris	**Meaning**
BP	Blank Page – this annotation **must** be used on all blank pages within an answer booklet (structured or unstructured) and on each page of an additional object where there is no candidate response.
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

2. Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.
Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

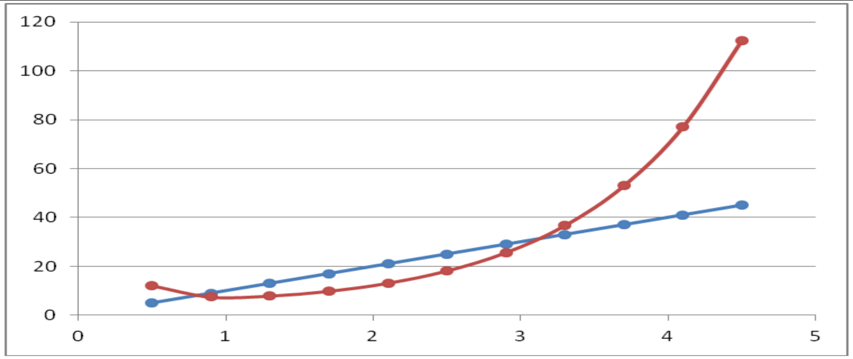
If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question		Answer	Marks	Guidance
1	(i)	$-1 < g'(\alpha) < 1$ Correct algebra to obtain given expression Derivative of RHS is $(1-\lambda) + \lambda\, g'(x)$ Set this to zero at $x = \alpha$ to obtain $\lambda = 1 / (1 - g'(\alpha))$ In practice use $\lambda = 1 / (1 - g'(x_0))$	B1 M1 B1 B1 E1 **[5]**	 Accept λ being re-set at each it'n
1	(ii)	 For $x < 0$, LHS < 0 and RHS > 0 so no negative roots Evidence of convergence to smaller root , divergence from larger. Eg: 1 3 3.2 0.738906 2.803162 3.353199 0.81032 2.35683 3.852941 0.772453 1.613722 6.109826 0.790132 0.933161 53.03287 0.781264 0.742452 1.1E+22 0.785571 0.807959 #NUM! 0.783445 0.773415 #NUM! 0.784486 0.78962 #NUM! 0.783974 0.781505 #NUM! 0.784226 0.78545 #NUM! Converging (to 0.784143)	G2 G1 E1 M1A1 A1A1	Curve Line cao Setting up the iteration and using it from one starting point For using it for two more starting points

Question			Answer	Marks	Guidance	
			Set up and use the relaxed iteration. Eg: k: 3 10 3.131955 $g'(x_0)$: 3.117202 2.491699 3.120372 lambda: 3.119725 –0.67037609 3.119859 3.119831 3.119837 3.119836 3.119836 3.119836	M1A1 M1A1 M1A1 **[14]**	Use $g'(x_0)$ to calculate λ Set up relaxed iteration Use it to obtain correct root	
1	**(iii)**		k = 20 needs relaxation for both roots: k: 0.5 k: 4 20 0.53859578 20 4.216399 $g'(x_0)$ 0.54212038 $g'(x_0)$ 4.181296 –1.82737 0.54274725 3.286191 4.1920499 lambda: 0.54286338 lambda: 4.1889768 0.353685 0.54288504 –0.43741 4.1898748 0.54288908 4.189614 0.54288984 4.1896899 0.54288998 4.1896678 0.54289001 4.1896742 0.54289001 4.1896724 0.5429 (4 dp) 4.1897	M1A1 A1 A1 A1 **[5]**	Set up, g', lambda Smaller root Set up, g', lambda Larger root	

Question		Answer	Marks	Guidance
2	(i)	$Q = \Sigma (y - bx^2 - cx^3)^2$	M1	
		Partial derivative of Q wrt b	M1A1	
		Set to zero and re-arrange convincingly to given expression	M1	
		Partial derivative of Q wrt c and set to zero	M1	
		Obtain $\Sigma x^2 y = b\,\Sigma x^3 + c\,\Sigma x^4$	A1	
			[6]	
2	(ii)	(see working below)	M1A1A1	Form sums
			M1	Formulate equations
			M1A1A1	Solve
			M1A1A1	Find residuals, sum, sum of squares

x	y	xy	x^2y	x^2	x^3	x^4
1	7.24	7.24	7.24	1	1	1
2	12.15	24.3	48.6	4	8	16
3	13.84	41.52	124.56	9	27	81
4	12.25	49	196	16	64	256
5	7.07	35.35	176.75	25	125	625
		157.41	553.15	55	225	979

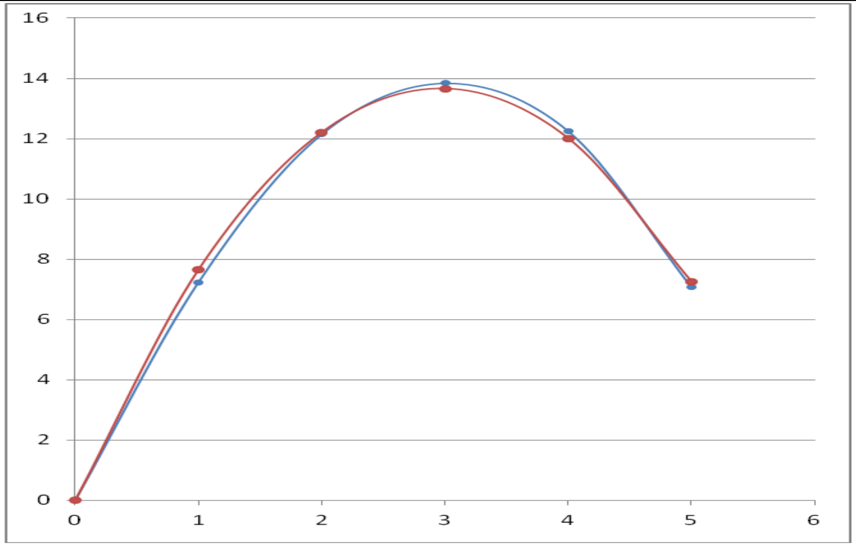
$$55\,b + 225\,c = 157.41$$
$$225\,b + 979\,c = 553.15$$

$$-14.3111\,c = 22.19556$$

c = –1.55093

b = 9.20672

x	y	y-fitted	residual	resid^2
1	7.24	7.655789	0.415789	0.17288
2	12.15	12.20971	0.059714	0.003566
3	13.84	13.66178	–0.17822	0.031764
4	12.25	12.01198	–0.23802	0.056656
5	7.07	7.260311	0.190311	0.036218
			0.249565	**0.301084**

Question			Answer	Marks	Guidance	
2	(ii) cont			G2 G2 **[14]**	Original data (higher at 3) Fitted curve (lower at 3)	
2	(iii)		Sum of residuals is not zero because there is no constant term. There is no constant term in order to give a curve through origin. The sum of squares of residuals would reduce because the fit would be better with one additional parameter.	E1 E1 E1 E1 **[4]**		
3	(i)		$T_n - I = A_2h^2 + A_4h^4 + A_6h^6 + \dots$ $T_{2n} - I = A_2(h/2)^2 + A_4(h/2)^4 + A_6(h/2)^6 + \dots$ $4(T_{2n} - I) - (T_n - I) = b_4h^4 + b_6h^6 + \dots$ $4T_{2n} - T_n - 3I = b_4h^4 + b_6h^6 + \dots$ $(4T_{2n} - T_n)/3 - I = B_4h^4 + B_6h^6 + \dots$ ($T_n^* = (4T_{2n} - T_n)/3$ has error of order h^4 as given) $T_n^{**} = (16T_{2n}^* - T_n^*)/15$ has error of order h^6	M1 M1A1 M1A1 B1 **[6]**		

Question		Answer	Marks	Guidance
3	(ii)	see table below	M1A1	Correct table of values used
			M1A1	T values
			M1A1	T*
			M1A1	T**

k:	*h*	*x*	f(*x*)	*T*	*T**	*T***
1	1.570796	0.000000	1.000000			
		1.570796	1.414214	1.896119		
	0.785398	0.785398	1.306563	1.974232	2.000269	
	0.392699	0.392699	1.175876			
		1.178097	1.387040	1.993570	2.000017	2.000000
	0.196350	0.196350	1.093202			
		0.589049	1.247225			
		0.981748	1.353318			
		1.374447	1.407404	1.998393	2.000001	2.000000
	0.098175	0.098175	1.047863			
		0.294524	1.135907			
		0.490874	1.213011			
		0.687223	1.278434			
		0.883573	1.331544			
		1.079922	1.371831			
		1.276272	1.398907			
		1.472622	1.412510	1.999598	2.000000	2.000000

Rates of convergence:

T	**diffs**	**ratios**
1.896119		
1.974232	0.078113	
1.993570	0.019339	0.247575
1.998393	0.004823	0.249397
1.999598	0.001205	0.249849
1.999900	0.000301	0.249962
		approx ¼

M1A1

Question			Answer	Marks	Guidance	
3	(ii) cont		**T*** **diifs** **ratios** 2.000269 2.000017 –0.000253 2.000001 –0.000016 0.061595 2.000000 –0.000001 0.062274 2.000000 0.000000 0.062444 approx 1/16 **T**** **diifs** **ratios** 2.000000 2.000000 0.000000 2.000000 0.000000 0.015385 2.000000 0.000000 0.015565 approx 1/64	A1 A1 **[12]**		
3	(iii)		Obtain values of *I* as follows *k* *I* 1 2 0.8 1.870992 0.6 1.731752 0.4 1.579036 0.2 1.40692 0 1.196? Comment that as *k* gets close to zero convergence is slower / results are less accurate. Comment that ratios of differences drift away from their theoretical values.	M1 A1A1 A1 E1 E1 **[6]**	Spreadsheet handles variable *k*. Some, all values of *I* To appropriate accuracy for working shown	

Question			Answer	Marks	Guidance	
4	(i)		Diagonal dominance: $a \geq 2$ and $b \geq 4$	B1B1	B1 inequalities, B1 'and' soi	
			Strict diagonal dominance: as above but with at least one '>'	B1B1	B1 for >, B1 for correct statement	
			Diagonal dominance is not sufficient, strict diagonal dominance is a sufficient but not necessary condition for G-S to converge	B1B1		
				[6]		
4	(ii)		G-S starting, e.g.			
			a = 2, *b* = 4; x_1, x_2, x_3, x_4: 0, 0, 0, 0			
			2, –0.25, 0.625, –0.1875	M1A1	G-S starts off correctly ... (NB: not G-J)	
			2.03125, –0.57813, 0.882813, –0.07422			
			2.251953, –0.81738, 0.945801, –0.15308			
			2.332153, –0.88898, 1.021027, –0.15556	M1A1	... continues correctly ...	
			2.366707, –0.94387, 1.049715, –0.1585			
			and converging to			
			2.416667, –1, 1.083333, –0.16667	M1A1	... converges correctly	
			Conjecture solution is: 29/12, –1, 13/12, –1/6	M1A1		
			Demonstrate correct by substitution	M1A1		
				[10]		
4	(iii)		Examples of fast convergence with *a* >>2, *b* = 4	M1A1		
			Examples of fast convergence with *a* = 2 and *b* >> 4	M1A1		
				[4]		
4	(iv)		Evidence of sensible trial and improvement	M1		
			Conclude *b* = 2.5 (ie *b* = 2.6 G-S converges)	A1		
				[2]		
4	(v)		Evidence of sensible trial and improvement	M1		
			Conclude *a* = 1.6 (ie, *a* = 1.7 G-S converges)	A1		
				[2]		

The G-S table in (ii):

a	*b*	x_1	x_2	x_3	x_4
2	4	0	0	0	0
		2	–0.25	0.625	–0.1875
		2.03125	–0.57813	0.882813	–0.07422
		2.251953	–0.81738	0.945801	–0.15308
		2.332153	–0.88898	1.021027	–0.15556
		2.366707	–0.94387	1.049715	–0.1585

Unit level raw mark and UMS grade boundaries June 2014 series

AS GCE / Advanced GCE / AS GCE Double Award / Advanced GCE Double Award

GCE Mathematics (MEI)

		Max Mark	a	b	c	d	e	u
4751/01 (C1) MEI Introduction to Advanced Mathematics	Raw	72	61	56	51	46	42	0
	UMS	100	80	70	60	50	40	0
4752/01 (C2) MEI Concepts for Advanced Mathematics	Raw	72	57	51	45	39	33	0
	UMS	100	80	70	60	50	40	0
4753/01 (C3) MEI Methods for Advanced Mathematics with Coursework: Written Paper	Raw	72	58	52	47	42	36	0
4753/02 (C3) MEI Methods for Advanced Mathematics with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4753/82 (C3) MEI Methods for Advanced Mathematics with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
4753 (C3) MEI Methods for Advanced Mathematics with Coursework	UMS	100	80	70	60	50	40	0
4754/01 (C4) MEI Applications of Advanced Mathematics	Raw	90	68	61	54	47	41	0
	UMS	100	80	70	60	50	40	0
4755/01 (FP1) MEI Further Concepts for Advanced Mathematics	Raw	72	63	57	51	45	40	0
	UMS	100	80	70	60	50	40	0
4756/01 (FP2) MEI Further Methods for Advanced Mathematics	Raw	72	60	54	48	42	36	0
	UMS	100	80	70	60	50	40	0
4757/01 (FP3) MEI Further Applications of Advanced Mathematics	Raw	72	57	51	45	39	34	0
	UMS	100	80	70	60	50	40	0
4758/01 (DE) MEI Differential Equations with Coursework: Written Paper	Raw	72	63	56	50	44	37	0
4758/02 (DE) MEI Differential Equations with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4758/82 (DE) MEI Differential Equations with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
4758 (DE) MEI Differential Equations with Coursework	UMS	100	80	70	60	50	40	0
4761/01 (M1) MEI Mechanics 1	Raw	72	57	49	41	34	27	0
	UMS	100	80	70	60	50	40	0
4762/01 (M2) MEI Mechanics 2	Raw	72	57	49	41	34	27	0
	UMS	100	80	70	60	50	40	0
4763/01 (M3) MEI Mechanics 3	Raw	72	55	48	42	36	30	0
	UMS	100	80	70	60	50	40	0
4764/01 (M4) MEI Mechanics 4	Raw	72	48	41	34	28	22	0
	UMS	100	80	70	60	50	40	0
4766/01 (S1) MEI Statistics 1	Raw	72	61	53	46	39	32	0
	UMS	100	80	70	60	50	40	0
4767/01 (S2) MEI Statistics 2	Raw	72	60	53	46	40	34	0
	UMS	100	80	70	60	50	40	0
4768/01 (S3) MEI Statistics 3	Raw	72	61	54	47	41	35	0
	UMS	100	80	70	60	50	40	0
4769/01 (S4) MEI Statistics 4	Raw	72	56	49	42	35	28	0
	UMS	100	80	70	60	50	40	0
4771/01 (D1) MEI Decision Mathematics 1	Raw	72	51	46	41	36	31	0
	UMS	100	80	70	60	50	40	0
4772/01 (D2) MEI Decision Mathematics 2	Raw	72	46	41	36	31	26	0
	UMS	100	80	70	60	50	40	0
4773/01 (DC) MEI Decision Mathematics Computation	Raw	72	46	40	34	29	24	0
	UMS	100	80	70	60	50	40	0
4776/01 (NM) MEI Numerical Methods with Coursework: Written Paper	Raw	72	54	48	43	38	32	0
4776/02 (NM) MEI Numerical Methods with Coursework: Coursework	Raw	18	14	12	10	8	7	0
4776/82 (NM) MEI Numerical Methods with Coursework: Carried Forward Coursework Mark	Raw	18	14	12	10	8	7	0
4776 (NM) MEI Numerical Methods with Coursework	UMS	100	80	70	60	50	40	0
4777/01 (NC) MEI Numerical Computation	Raw	72	55	47	39	32	25	0
	UMS	100	80	70	60	50	40	0
4798/01 (FPT) Further Pure Mathematics with Technology	Raw	72	57	49	41	33	26	0
	UMS	100	80	70	60	50	40	0

GCE Statistics (MEI)

		Max Mark	a	b	c	d	e	u
G241/01 (Z1) Statistics 1	Raw	72	61	53	46	39	32	0
	UMS	100	80	70	60	50	40	0
G242/01 (Z2) Statistics 2	Raw	72	55	48	41	34	27	0
	UMS	100	80	70	60	50	40	0
G243/01 (Z3) Statistics 3	Raw	72	56	48	41	34	27	0
	UMS	100	80	70	60	50	40	0