OCR
Oxford Cambridge and RSA

Tuesday 20 June 2017 – Afternoon

AS GCE MATHEMATICS (MEI)

4776/01 Numerical Methods

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4776/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

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- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (36 marks)

1 **(i)** Calculate the relative error in using the approximation $\pi \approx \frac{355}{113}$. **[2]**

The diameter of a circle is 226.3 cm. It is decided to use the approximation $\pi \approx \frac{355}{113}$ and to round the diameter to the nearest whole number. Find the magnitude of the relative error in using these approximations to calculate

(ii) the circumference of the circle, **[2]**

(iii) the area of the circle. **[2]**

2 The following data were collected during an experiment.

x	1	2	4
y	0	4.5	29.7

(i) State, with a reason, whether or not it is possible to use Newton's forward difference interpolation formula to construct a polynomial of degree 2 for these data. **[1]**

(ii) Use a suitable method to construct an interpolating polynomial of degree 2 for this data. Give your answer in the form $ax^2 + bx + c$. **[5]**

(iii) Use your answer to part **(ii)** to estimate the value of y when $x = 2.5$. **[2]**

3 **(i)** Show that the equation $x^5 - 3x - 1 = 0$ has a root α, such that $-1 < \alpha < 0$. **[2]**

(ii) Use the iteration

$$x_{r+1} = \frac{x_r^5 - 1}{3},$$

to find α correct to five decimal places. Start at $x = 0$. **[3]**

You are given that there is another root to the equation, β, such that $\beta \approx 1.39$.

(iii) Determine whether the iteration in **(ii)** may be used to find β to greater accuracy. **[3]**

4 The formula $I = \frac{(Z_2 + Z_1)^2}{(Z_2 - Z_1)^2}$ occurs in physics.

You are given that $Z_1 = 21.1$ and $Z_2 = 20.9$, correct to one decimal place.

(i) Calculate the range of possible values of I. **[4]**

(ii) Explain why this range is so large. **[2]**

5 The function f(x) has the values shown in the table.

x	0.3	0.35	0.375	0.425	0.45	0.5
f(x)	0.897 31	0.879 32	0.871 16	0.856 80	0.850 70	0.840 90

(i) Explain why it is not possible to use the forward difference method to estimate f′(0.4) using these data. **[1]**

(ii) Calculate three estimates of f′(0.4) using the central difference method. Give your answers correct to 4 decimal places. Hence state the value of f′(0.4) to the accuracy which seems justified. **[5]**

(iii) Given that f(0.4) = 0.863 63, find an approximation to f(0.41). **[2]**

Section B (36 marks)

6 **(i)** By calculating $0.1^{(0.1^3)}$, $0.01^{(0.01^3)}$ and $0.001^{(0.001^3)}$ show that $x^{(x^3)}$ tends to 1 as x tends to 0. **[2]**

(ii) Identify the difficulty with using the trapezium rule to evaluate $\int_0^1 x^{(x^3)}\,dx$.

Use your answers to part **(i)** to suggest a way of addressing this difficulty. **[2]**

(iii) Use the mid-point rule with $h = 1$, $h = 0.5$ and $h = 0.25$ to estimate the value of $\int_0^1 x^{(x^3)}\,dx$.

Hence give the value of the integral to the accuracy that appears justified. **[5]**

Further estimates of the integral using the trapezium rule for different values of h are given in the following table.

h	T
1	1
0.5	0.958 502
0.25	0.945 321
0.125	0.941 606
0.062 5	0.940 643

(iv) Obtain four Simpson's Rule estimates of the integral. Give your answers correct to six decimal places. Hence give the value of the integral to the accuracy that appears justified. **[5]**

(v) State the theoretical value for the ratio of differences of a sequence of estimates to a definite integral using Simpson's Rule. Use this value to obtain an improved approximation to $\int_0^1 x^{(x^3)}\,dx$. State the value of the integral as accurately as you can. **[4]**

7 **(i)** Show that the equation $3x^5 - 5x^3 - 1 = 0$ has a root α, such that $1 < \alpha < 2$, and a root β such that $-1 < \beta < 0$. **[3]**

(ii) Obtain the Newton-Raphson iteration

$$x_{r+1} = x_r - \frac{3x_r^5 - 5x_r^3 - 1}{15x_r^4 - 15x_r^2}. \qquad (*)$$

[2]

(iii) Use (*) with $x_0 = 2$ to obtain the value of α correct to 5 decimal places. **[3]**

(iv) Explain why it is not possible to use (*) with $x_0 = 0$ or $x_0 = -1$ to obtain a value for β. **[2]**

(v) Show numerically that using (*) with $x_0 = -0.3$ leads to a third root, γ. Obtain the value of γ correct to 5 significant figures. **[3]**

(vi) Given that $-0.9 < \beta < -0.3$, use interval bisection to obtain a value for β with a maximum possible error of 0.018 75. **[4]**

(vii) How many further applications of bisection are needed to obtain a value for β with a maximum possible error of less than 5×10^{-5}? **[1]**

END OF QUESTION PAPER

OCR
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Tuesday 20 June 2017 – Afternoon

AS GCE MATHEMATICS (MEI)

4776/01 Numerical Methods

PRINTED ANSWER BOOK

Candidates answer on this Printed Answer Book.

OCR supplied materials:
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- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

Candidate forename		Candidate surname	

Centre number						Candidate number				

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Section A (36 marks)

1 (i)	
1 (ii)	
1 (iii)	

2 (i)	
2 (ii)	
2 (iii)	

3 (i)	
3 (ii)	
3 (iii)	

4 (i)	
4 (ii)	

5 (i)	
5 (ii)	

5 (iii)	

Section B (36 marks)

6 (i)	
6 (ii)	
6 (iii)	

6 (iv)

6 (v)	
7 (i)	

7 (ii)

7 (iii)

7 (iv)	

7 (v)	

7 (vi)	
7 (vii)	

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Oxford Cambridge and RSA

GCE

Mathematics (MEI)

Unit **4776**: Numerical Methods

Advanced Subsidiary GCE

Mark Scheme for June 2017

Annotations and abbreviations

Annotation in scoris	**Meaning**
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance	
1	(i)		$\dfrac{\frac{355}{113}-\pi}{\pi}$	M1		ignore extra terms *Condone consistent use of opposite sign convention.*
			8.4×10^{-8}	A1 [2]		
1	(ii)		$8.4\times10^{-8}+\dfrac{0.3}{226.3}$ or $\dfrac{\frac{355}{113}\times226-\pi\times226.3}{\pi\times226.3}$	M1	Attempt at a relative error	
			0.00133	A1 [2]		to 3 or more sig figs
1	(iii)		$8.4\times10^{-8}+2\times\dfrac{0.3}{226.3}$ or $\dfrac{\frac{355}{113}\times113^2-\pi\times113.15^2}{\pi\times113.15^2}$	M1	Attempt at a relative error. Accept double the error.	to 3 or more sig figs
			0.00265 or 0.00266	A1 [2]		
2	(i)		Not possible since data not evenly spaced.	B1 [1]		

Question			Answer	Marks	Guidance	
2	(ii)			M1	Correct use of the *x* or *y* values in the interpolation formula.	
			$\frac{(x-1)(x-4)}{(2-1)(2-4)}\times 4.5$ oe	M1	Fully correct use of the interpolation formula for one non zero term.	ignore zero term
			$\frac{(x-1)(x-2)}{(4-1)(4-2)}\times 29.7$ oe	A1	For both.	
			$2.7x^2-3.6x+0.9$	A2(-1)	for each error.	
				[5]		
2	(iii)		$2.7\times 2.5^2-3.6\times 2.5+0.9$	M1	Evidence of substitution of x = 2.5 in their trinomial.	
			8.775 cao	A1		
				[2]		
3	(i)		evaluation of f(−1) and f(0)	M1		
			− 1 and 1 seen	A1		
				[2]		
3	(ii)		$\frac{0^5-1}{3}$ soi	M1		
			− 0.33333, − 0.33471, − 0.33473, − 0.33473	A1	First two nonzero correct.	
			− 0.33473	A1	Indication that α = -0.33473	
				[3]		

Question			Answer	Marks	Guidance	
3	(iii)		$g'(x)=\frac{5x^4}{3}$	B1	alternatively 1.39629…seen	following start value of 1.39
			evaluation of $g'(1.39)$ NB 6.22	M1	1.4358, 1.700…, 4.410…,	
			$g'(1.39) > 1$ so iteration diverges	A1 [3]	555.78 or 1.78×10^{13}, clearly diverging	
4	(i)		$\frac{(21.15+20.85)^2}{(21.15-20.85)^2}$	M1	Upper and lower bonds on Z_1 and Z_2 of ±0.05 in all four cases.	
			$\frac{(21.05+20.95)^2}{(21.05-20.95)^2}$	M1	Evaluation of two rational expressions.	
			19 600 < I < 176 400	A2 [4]	one for each	
4	(ii)		numbers in denominator very close together, so a small change leads to a large relative change in their difference	E1		
			squaring this difference before division magnifies the effect	E1 [2]		
5	(i)		the value of f(0.4) is unavailable	E1 [1]		

Question			Answer	Marks	Guidance	
5	(ii)		eg $\dfrac{0.84090-0.89731}{0.5-0.3}$	M1	correct application of central difference formula for any of the three calculations	
			− 0.28205	A1		
			− 0.2862	A1		
			− 0.2872	A1		
			− 0.29 seems secure	A1	cao	
				[5]		
5	(iii)		$0.86363 - 0.29 \times 0.01$	M1	**FT** their − 0.29 Any acceptable interpolation method.	**NB** 0.86073
			≈ 0.861	A1	Allow four decimal places that round to 0.861.	
				[2]		
6	(i)		0.997700, 0.999995, 0.999999993	B2	B1 for any two of these	
				[2]		
6	(ii)		0^0 is undefined	B1		
			but 1 is a reasonable approximation	B1		
				[2]		

Question			Answer	Marks	Guidance	
6	(iii)		correct use of midpoint rule $M_1 = 0.91700$ $M_2 = 0.93214$ $M_4 = 0.93789$ 0.9 is certain, 0.94 is probable	**M1** **A1** **A1** **A1** **A1** **[5]**	accept either	
6	(iv)		correct use of $\dfrac{4T_{2n} - T_n}{3}$ 0.944669 0.940927 0.940368 0.940322 0.940 seems secure, 0.9403 is possible	**M1** **A3** **A1** **[5]**	Or $\dfrac{2M_n + T_n}{3}$ Accept answers that round to five correct decimal places **A2** for three correct, **A1** for two correct accept either	
6	(v)		$r = \dfrac{1}{16}$ $0.940322 + \dfrac{1}{16}(0.940322 - 0.940368)$ or $\dfrac{16 \times 0.940322 - 0.940368}{15}$ 0.940319 so $I \approx 0.94032$	**B1** **M1** **A1** **A1** **[4]**	Or any acceptable method. Or any value that rounds to 0.94032	

Question			Answer	Marks	Guidance	
7	(i)		evaluation of f(x) at $x = 1, 2, -1$ and 0.	M1		
			$f(1) = -3$ and $f(2) = 55$	A1		
			$f(-1) = 1$ and $f(0) = -1$	A1		
				[3]		
7	(ii)		$5\times 3\times x^4 - 3\times 5\times x^2$ oe	B1		
			substitution of f(x) and f′(x) in $x_{r+1} = x_r - \frac{f(x_r)}{f'(x_r)}$			
			completion to $x_{r+1} = x_r - \frac{3x_r^5 - 5x_r^3 - 1}{15x_r^4 - 15x_r^2}$.	B1	**NB** answer given	
				[2]		
7	(iii)		$2 - \frac{3\times 2^5 - 5\times 2^3 - 1}{15\times 2^4 - 15\times 2^2}$ soi	M1		
			2 1.69444444 1.48870712 1.37890021 1.34599146 1.34322828 1.34320975 1.34320975	A1	two or more correct iterates seen to 3 d.p. or more	
			1.34321	A1	cao	
				[3]		

Question			Answer	Marks	Guidance	
7	(iv)		gradient is zero at each point consequently in each case the tangent can never meet the x-axis	**E1** **E1** **[2]**		do not allow "math error" for second **E1**
7	(v)		– 1.0100448 – 4.2416907, – 3.4263544, – 2.7830921 – 1.2173	**B1** **B1** **B1** **[3]**	1st iterate next three	accept to 3 d.p. or more for first two marks NB from – 1.2173274
7	(vi)		estimate mpe – 0.6 0.3 − 0.75 0.15 − 0.675 0.075 − 0.6375 0.03075 − 0.65625 0.01875 − 0.65625	**M1** **A1** **A1** **A1** **[4]**	correct application of bisection method first two estimates cao	must see all 5 iterates for this mark
7	(vii)		$n = 9$	**B1** **[1]**		

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PART OF THE CAMBRIDGE ASSESSMENT GROUP

4776 Numerical Methods (Written Examination)

General Comments:

Most of the candidates seemed well prepared for this paper. The routine numerical work was generally done well and the candidates performed well on most of the questions on this paper. Several of the questions had sections that required the candidates to comment on the results of their calculations. The interpretation of their results was often inadequate. The candidates can carry out the algorithms correctly, but are less sure of the theoretical conditions underlying these algorithms.

In two questions there were sequences of values that were converging on an approximation to either a derivative or an integral. The candidates were asked to comment on the justification of the accuracy of their answer. There was a tendency to choose the final term of the sequence as the most accurate. The candidates should make a judgement based upon examining all the terms of the sequence. This approach usually results in an estimate that results in a value whose accuracy has fewer significant figures that the final term of the sequence.

Comments on Individual Questions:

Question No. 1.

This question involved relative errors and was generally well answered. A significant number were penalised for giving negative numbers for parts (ii) and (iii). In part (i) nearly all the candidates correctly evaluated the relative error in the approximation for π. A few candidates miscopied the $\frac{355}{113}$ as $\frac{355}{133}$ and so could only gain credit for the method. The answers to part (ii) were more varied. The elegant way was to add the relative error in part (i) to the relative error obtained by approximating the diameter of the circle from 226.3 cm to 226 cm as indicated in the question. Most candidates used an alternative method of finding the relative error by using both the approximations for π and the diameter in a single rational expression. A common error was to ignore the instruction in the question to round the value of the diameter and use the approximation to π. Naturally, this approach lead to the same answer as part (i). These comments for part (ii) can be applied to part (iii) where the area instead of the circumference was required.

Question No. 2

The finding of a Lagrange Interpolating polynomial was well understood and very well answered. Part (i) asked why a Newton forward difference interpolation formula could not be used to construct a second degree polynomial from the given data. The fact that it was not possible, since the data were not evenly spaced, was almost universally well-known and given as the reason. In part (ii) the initial formula for the Lagrange method was normally correctly stated. The subsequent algebra to produce the required quadratic polynomial was carried out successfully by most candidates. In part (iii) the answer to part (ii) was used to estimate the value of y when $x = 2.5$. This was usually straightforward, but some candidates with an incorrect polynomial just quoted a wrong value. They could have received some credit if they had shown some evidence of substituting of $x = 2.5$ into their trinomial.

Question No. 3

Most candidates found this question straightforward. Showing the location of the root in part (i) and performing the iteration to find the root to five decimal places in part (ii) presented no difficulties. Part (iii) was more of a challenge for the candidates. The question stated that there was another root $\beta \approx 1.39$ and the task was to determine whether the iteration in part **(**ii**)** could be used to find β to greater accuracy. Many candidates started with $x = 1.39$ and stopped after two or three iterations and stated that the iteration was diverging. This statement about divergence, although true, was based on insufficient evidence. More iterations were needed to conclusively demonstrate the divergence. The iteration formula was $x_{r+1} = \frac{x_r^5 - 1}{3}$ and the better candidates used a theoretical approach and showed that substituting the value of $x = 1.39$ into the derivative $\frac{5x^4}{3}$ of the right hand side gave a value greater than 1, guaranteeing divergence. A few candidates misunderstood the theory and differentiated the left hand side of the original equation $x^5 - 3x - 1 = 0$ to try and demonstrate divergence.

Question No. 4

In part (i) the candidates were given the formula $I = \frac{(Z_2 + Z_1)^2}{(Z_2 - Z_1)^2}$ and told that the values of Z_1 and Z_2 were correct to one decimal place. The question then asked for the range of possible values of *I*. The majority of candidates used the upper and lower bounds of Z_1 and Z_2 correctly and used them to calculate values for *I*. A significant number of attempts misunderstood this question and failed to realise that the values of the variables needed to be consistent in the calculation; many used different values for Z_1 and Z_2 in the numerator and denominator, leading to inaccurate limits for *I*. Even with incorrect values for the range it was still possible to attempt part (ii) and explain why the range was so large. The answers seen to part (ii) were often poor and usually not creditworthy.

Standard responses, such as "the problem was ill-conditioned" or "subtraction of unequal values" did not receive credit unless specific features of the calculation were used to illustrate the point. The fact that the difference between the numbers in the denominator is a small positive number was missed or not clearly stated by the majority of the candidates.

With the numbers in the denominator being very close together a small change leads to a large relative change in their difference and squaring this difference before division magnifies the effect.

Question No. 5

Almost all the candidates knew that you could not use the forward difference formula to estimate the derivative *f '(0.4)* since the value for *f(0.4)* was not given in the table.

The central difference method was used in finding the three estimates of *f '(0.4)* in part (ii) and was generally done well. Sometimes the differences in the denominator were increased by a factor of ten due to misplaced decimal points thus causing an error in the estimate. Stating the value of *f '(0.4)* to the accuracy which seemed justified was not done as well. Many candidates treated their estimate of -0.2872 for the derivative *f '(0.4)* as the best and quoted this to three or four decimal places. When a sequence of values is created, it is necessary to consider all three values before justifying the accuracy. The previous estimate for *f '(0.4)* was -0.2862 and since both these estimates round to -0.29 this seems to be the secure answer.

Having been given the value of *f(0.4)* in part (iii) then using the value of the gradient times 0.01, the difference between *f(0.4)* and *f(0.41),* was the easiest way to find the approximation for *f(0.41)*. A significant number of candidates chose to use various methods of linear interpolation to find the approximation. These methods were not always of sufficient accuracy and a common error was to use 0.1 instead of 0.01 as the difference between 0.4 and 0.41. Since the value of *f(0.4)* was given to five decimal places most answers for the value of *f(0.41)* were also recorded to five decimal places. Candidates often overlooked the fact that the value of the gradient was an approximation to two decimal places and the difference between 0.4 and 0.41 is also two decimal places. Hence, at most only four decimal places can be considered in justifying the accuracy and the answer should be quoted to three decimal places to be secure.

Question No. 6

Finding $x^{(x^2)}$ for $x = 0.1$, 0.01 and 0.001 was straightforward, except that some candidates did not use the full accuracy of their calculator for the 0.001 value.

The hint in part (i) was not fully understood by the candidates in part (ii). There were not very many convincing arguments for both questions in part (ii). A reasonable number of candidates said in one form or another that 0^0 was undefined, but fewer used part (i) to suggest that a value of 1 at the origin would enable the trapezium rule to be used.

The application of the midpoint rule in part (iii) was understood and done well. All three values should be considered before deciding on the accuracy of the integral. In this case both the certain answer of 0.9 and the probable answer of 0.94 were acceptable.

The candidates were given a table of values containing five estimates of the integral using the trapezium rule for use in part (iv). The task was to obtain four Simpson's Rule estimates of the integral and hence give the value of the integral to the accuracy that appeared justified. Those candidates who used $\frac{4T_{2n} - T_n}{3}$ normally produced the four required answers. Those who chose to use the weighted mean of $\frac{2M_n + T_n}{3}$ could only produce three answers before having to do further work. This either involved reverting to the trapezium formula or calculating another midpoint rule. The latter approach sometimes caused inaccuracies in the final Simpson's Rule estimate. Considering the sequence of four values, the answers of 0.940 being secure and 0.9403 as possible were acceptable.

In part (v) nearly all the candidates knew the 0.0625 theoretical value for the ratio of differences of a sequence of estimates to a definite integral using Simpson's Rule. Most used this value to try and improve their estimate for the integral, but not always successfully. The correct answer of $I \approx 0.94032$ was seldom achieved. It is not clear that the candidates understood that each section of question 6 was guiding them through increasing improvements in the estimates of the $\int_0^1 x^{(x^2)}\,dx$.

Question No. 7

Part (i) was a routine exercise in showing that the equation $3x^5 - 5x^3 - 1 = 0$ had a root α, such that $1 < \alpha < 2$, and a root β such that $-1 < \beta < 0$. The vast majority correctly answered this exercise, but some candidates did not provide the values at the end points of the two intervals to justify their statements about a change of sign.

The requirement in part (ii) was to obtain the Newton-Raphson iteration formula for the above equation. The answer was displayed so most candidates had no problems in obtaining the correct answer. Since the answer was displayed, it is incumbent on the candidates to give their answer in the same form. Several candidates presented their result as an equation in x rather than x_r.

Part (iii) was a straightforward application of the Newton-Raphson method to find α and it was generally done well.

The following part (iv) asked for an explanation of why it was not possible to use the Newton-Raphson iteration formula with $x_0 = 0$ or $x_0 = -1$ to obtain a value for β. This part was not answered very well by most of the candidates and illustrated the difference between carrying out the algorithms and understanding the theory that underlines them. The usual answer was that at $x_0 = 0$ and $x_0 = -1$ the value of $f'(x_r)$ was 0 and this caused a math error and the iteration could not be performed. Very few candidates mentioned the fact that $f'(x_r) = 0$ showed that the gradient was zero and so the tangent was parallel to the x-axis and cut not intercept it to provide a new iterate.

Part (v) required another application of the Newton-Raphson iteration formula to show that starting with x_0 = -0.3, a value close to the root β, the iteration produced a new root γ = -1.2173. This application was quite slow to converge and took several iterations to reach the requested answer of five significant figures. There was confusion amongst some candidates about the distinction between significant figures and decimal places, which usually cost them the final answer mark.

The bisection method was used in part (vi) to estimate the value of β. The method was well understood and most candidates chose the correct intervals as the maximum possible error was reduced. The tables were not always set out in a readable format and occasionally the final answer was left as an interval containing β rather than the value of β itself.

The final part (vii) asked how many further applications of bisection were needed to obtain a value for *β* with a maximum possible error of less than 5×10^{-5}. This was only a 1 mark section and the answers varied from those who appeared to have guessed, those who just wrote down the correct answer of 9 and those who worked out the answer was 9. The majority of candidates did get the mark.

Unit level raw mark and UMS grade boundaries June 2017 series

For more information about results and grade calculations, see www.ocr.org.uk/ocr-for/learners-and-parents/getting-your-results

AS GCE / Advanced GCE / AS GCE Double Award / Advanced GCE Double Award

GCE Mathematics (MEI)										
				Max Mark	a	b	c	d	e	u
4751	01	C1 – MEI Introduction to advanced mathematics (AS)	Raw	72	63	58	53	49	45	0
			UMS	100	80	70	60	50	40	0
4752	01	C2 – MEI Concepts for advanced mathematics (AS)	Raw	72	55	49	44	39	34	0
			UMS	100	80	70	60	50	40	0
4753	01	(C3) MEI Methods for Advanced Mathematics with Coursework: Written Paper	Raw	72	54	49	45	41	36	0
4753	02	(C3) MEI Methods for Advanced Mathematics with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4753	82	(C3) MEI Methods for Advanced Mathematics with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
			UMS	100	80	70	60	50	40	0
4754	01	C4 – MEI Applications of advanced mathematics (A2)	Raw	90	67	61	55	49	43	0
			UMS	100	80	70	60	50	40	0
4755	01	FP1 – MEI Further concepts for advanced mathematics (AS)	Raw	72	57	52	47	42	38	0
			UMS	100	80	70	60	50	40	0
4756	01	FP2 – MEI Further methods for advanced mathematics (A2)	Raw	72	65	58	52	46	40	0
			UMS	100	80	70	60	50	40	0
4757	01	FP3 – MEI Further applications of advanced mathematics (A2)	Raw	72	64	56	48	41	34	0
			UMS	100	80	70	60	50	40	0
4758	01	(DE) MEI Differential Equations with Coursework: Written Paper	Raw	72	63	56	50	44	37	0
4758	02	(DE) MEI Differential Equations with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4758	82	(DE) MEI Differential Equations with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
			UMS	100	80	70	60	50	40	0
4761	01	M1 – MEI Mechanics 1 (AS)	Raw	72	57	49	41	34	27	0
			UMS	100	80	70	60	50	40	0
4762	01	M2 – MEI Mechanics 2 (A2)	Raw	72	56	48	41	34	27	0
			UMS	100	80	70	60	50	40	0
4763	01	M3 – MEI Mechanics 3 (A2)	Raw	72	58	50	43	36	29	0
			UMS	100	80	70	60	50	40	0
4764	01	M4 – MEI Mechanics 4 (A2)	Raw	72	53	45	38	31	24	0
			UMS	100	80	70	60	50	40	0
4766	01	S1 – MEI Statistics 1 (AS)	Raw	72	61	55	49	43	37	0
			UMS	100	80	70	60	50	40	0
4767	01	S2 – MEI Statistics 2 (A2)	Raw	72	56	50	45	40	35	0
			UMS	100	80	70	60	50	40	0
4768	01	S3 – MEI Statistics 3 (A2)	Raw	72	63	57	51	46	41	0
			UMS	100	80	70	60	50	40	0
4769	01	S4 – MEI Statistics 4 (A2)	Raw	72	56	49	42	35	28	0
			UMS	100	80	70	60	50	40	0
4771	01	D1 – MEI Decision mathematics 1 (AS)	Raw	72	52	46	41	36	31	0
			UMS	100	80	70	60	50	40	0
4772	01	D2 – MEI Decision mathematics 2 (A2)	Raw	72	53	48	43	39	35	0
			UMS	100	80	70	60	50	40	0
4773	01	DC – MEI Decision mathematics computation (A2)	Raw	72	46	40	34	29	24	0
			UMS	100	80	70	60	50	40	0
4776	01	(NM) MEI Numerical Methods with Coursework: Written Paper	Raw	72	58	53	48	43	37	0
4776	02	(NM) MEI Numerical Methods with Coursework: Coursework	Raw	18	14	12	10	8	7	0
4776	82	(NM) MEI Numerical Methods with Coursework: Carried Forward Coursework Mark	Raw	18	14	12	10	8	7	0
			UMS	100	80	70	60	50	40	0
4777	01	NC – MEI Numerical computation (A2)	Raw	72	55	48	41	34	27	0

Oxford Cambridge and RSA

		UMS	100	80	70	60	50	40	0
4798	01 FPT - Further pure mathematics with technology (A2)	Raw	72	57	49	41	33	26	0
		UMS	100	80	70	60	50	40	0

GCE Statistics (MEI)

			Max Mark	a	b	c	d	e	u
G241	01 Statistics 1 MEI (Z1)	Raw	72	61	55	49	43	37	0
		UMS	100	80	70	60	50	40	0
G242	01 Statistics 2 MEI (Z2)	Raw	72	55	48	41	34	27	0
		UMS	100	80	70	60	50	40	0
G243	01 Statistics 3 MEI (Z3)	Raw	72	56	48	41	34	27	0
		UMS	100	80	70	60	50	40	0

GCE Quantitative Methods (MEI)

			Max Mark	a	b	c	d	e	u
G244	01 Introduction to Quantitative Methods MEI	Raw	72	58	50	43	36	28	0
G244	02 Introduction to Quantitative Methods MEI	Raw	18	14	12	10	8	7	0
		UMS	100	80	70	60	50	40	0
G245	01 Statistics 1 MEI	Raw	72	61	55	49	43	37	0
		UMS	100	80	70	60	50	40	0
G246	01 Decision 1 MEI	Raw	72	52	46	41	36	31	0
		UMS	100	80	70	60	50	40	0

Level 3 Certificate and FSMQ raw mark grade boundaries June 2017 series

For more information about results and grade calculations, see www.ocr.org.uk/ocr-for/learners-and-parents/getting-your-results

Level 3 Certificate Mathematics for Engineering												
				Max Mark	a*	a	b	c	d	e	u
H860	01	Mathematics for Engineering		This unit has no entries in June 2017							
H860	02	Mathematics for Engineering									

Level 3 Certificate Mathematical Techniques and Applications for Engineers											
				Max Mark	a*	a	b	c	d	e	u
H865	01	Component 1	Raw	60	48	42	36	30	24	18	0

Level 3 Certificate Mathematics - Quantitative Reasoning (MEI) (GQ Reform)										
				Max Mark	a	b	c	d	e	u
H866	01	Introduction to quantitative reasoning	Raw	72	54	47	40	34	28	0
H866	02	Critical maths	Raw	60*	48	42	36	30	24	0
		*Component 02 is weighted to give marks out of 72	Overall	144	112	97	83	70	57	0

Level 3 Certificate Mathematics - Quantitive Problem Solving (MEI) (GQ Reform)										
				Max Mark	a	b	c	d	e	u
H867	01	Introduction to quantitative reasoning	Raw	72	54	47	40	34	28	0
H867	02	Statistical problem solving	Raw	60*	41	36	31	27	23	0
		*Component 02 is weighted to give marks out of 72	Overall	144	103	90	77	66	56	0

Advanced Free Standing Mathematics Qualification (FSMQ)										
				Max Mark	a	b	c	d	e	u
6993	01	Additional Mathematics	Raw	100	72	63	55	47	39	0

Intermediate Free Standing Mathematics Qualification (FSMQ)										
				Max Mark	a	b	c	d	e	u
6989	01	Foundations of Advanced Mathematics (MEI)	Raw	40	35	30	25	20	16	0