Wednesday 18 June 2014 – Afternoon

A2 GCE MATHEMATICS (MEI)

4769/01 Statistics 4

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4769/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer any **three** questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

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Option 1: Estimation

1 $X_1, X_2, \ldots, X_n$ represent n independent observations on the random variable X with probability density function

$$\mathrm{f}(x) = \frac{\theta^3 x^2 e^{-\theta x}}{2}, \quad x > 0,$$

where θ is an unknown parameter ($\theta > 0$). $\overline{X}$ denotes the sample mean of $X_1, X_2, \ldots, X_n$, ie $\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$.

(i) Show that the maximum likelihood estimator of θ is $\hat{\theta} = \frac{3}{\overline{X}}$. **[9]**

(ii) Show that, in the case $n = 1$, $\hat{\theta}$ is a biased estimator of θ. **[8]**

(iii) For large n, the distribution of $\hat{\theta}$ is well approximated by $\mathrm{N}(\theta, \mathrm{H}(\theta))$ where

$$\mathrm{H}(\theta) = \frac{1}{\mathrm{E}\left(-\frac{\mathrm{d}^2 \ln \mathrm{L}}{\mathrm{d}\theta^2}\right)}$$

where L is the likelihood. Show that $\mathrm{H}(\theta) = \frac{\theta^2}{3n}$. For the case where $n = 100$ and the value of $\overline{X}$ is 5.0, evaluate $\hat{\theta}$ and $\mathrm{H}(\hat{\theta})$, and use these values to find an approximate 95% confidence interval for θ. **[7]**

Option 2: Generating Functions

2 **(i)** The probability density function of the random variable X is

$$\mathrm{f}(x) = \frac{x^{k-1}\mathrm{e}^{-x/\phi}}{\phi^k (k-1)!}, \quad x > 0,$$

where k is a known positive integer and ϕ is an unknown parameter ($\phi > 0$). Show that the moment generating function (mgf) of X is

$$\mathrm{M}_X(\theta) = (1-\phi\theta)^{-k}$$

for $\theta < \frac{1}{\phi}$. **[12]**

(ii) Write down the mgf of the random variable $W = \sum_{i=1}^{n} X_i$ where $X_1, X_2, \ldots, X_n$ are independent random variables each with the same distribution as X. **[1]**

(iii) Write down the mgf of the random variable $Y = \frac{2W}{\phi}$. Given that the mgf of the random variable V having the χ^2_m distribution is $\mathrm{M}_V(\theta) = (1-2\theta)^{-m/2}$ (for $\theta < \frac{1}{2}$), deduce the distribution of Y. **[3]**

(iv) Deduce that $\mathrm{P}\left(l < \frac{2W}{\phi} < u\right) = 0.95$ where l and u are the lower and upper $2\frac{1}{2}\%$ points of the χ^2_{2nk} distribution. Hence deduce that a 95% confidence interval for ϕ is given by $\left(\frac{2w}{u}, \frac{2w}{l}\right)$ where w is an observation on the random variable W. **[2]**

(v) For the case $k = 2$ and $n = 10$, use percentage points of the χ^2 distribution to write down, in terms of w, an expression for a 95% confidence interval for ϕ. By considering the mgf of W, find in terms of ϕ the expected length of this interval. **[6]**

Option 3: Inference

3 **(i)** Explain the meaning of the following terms in the context of hypothesis testing: Type I error, Type II error, operating characteristic, power. **[8]**

(ii) A chemical manufacturer is endeavouring to reduce the amount of a certain impurity in one of its bulk products by improving the production process. The amount of impurity is measured in a convenient unit of concentration, and this is modelled by the Normally distributed random variable X. In the old production process, the mean of X, denoted by μ, was 63 and the standard deviation of X was 3.7. Experimental batches of the product are to be made using the new process, and it is desired to examine the hypotheses H_0: $\mu = 63$ and H_1: $\mu < 63$ for the new process. Investigation of the variability in the new process has established that the standard deviation may be assumed unchanged.

The usual Normal test based on $\overline{X}$ is to be used, where $\overline{X}$ is the mean of X over n experimental batches (regarded as a random sample), with a critical value c such that H_0 is rejected if the value of $\overline{X}$ is less than c. The following criteria are set out.

- If in fact $\mu = 63$, the probability of concluding that $\mu < 63$ must be only 1%.
- If in fact $\mu = 60$, the probability of concluding that $\mu < 63$ must be 90%.

Find c and the smallest value of n that is required. With these values, what is the power of the test if in fact $\mu = 58.5$? **[16]**

Option 4: Design and Analysis of Experiments

4 A trial is being made of four experimental methods, A, B, C and D, for carrying out an industrial process. These are being compared with each other and with the standard method M. The trial is conducted according to a completely randomised design. The results, x, are as follows, in a suitable unit.

Method	Results x	Total	Mean
M	25.0 23.0 30.1 27.5 28.8 25.6 29.2 31.6	220.8	27.6
A	37.3 34.9 30.8 40.2	143.2	35.8
B	36.4 36.6 29.2 44.0 34.8	181.0	36.2
C	32.0 40.1 33.0 36.5	141.6	35.4
D	35.0 31.8 39.0 38.2	144.0	36.0
	Grand total	830.6	

You are also given that $\sum x^2 = 28\,260.18$.

(i) The usual statistical model underlying a one-way analysis of variance is given, in the usual notation, by

$$x_{ij} = \mu + \alpha_i + e_{ij}$$

where x_{ij} denotes the jth observation on the ith treatment. State the properties that are assumed for the term that represents experimental error. **[3]**

(ii) Construct the usual analysis of variance table for these data. Stating your hypotheses carefully, test whether there is evidence of differences among the means for the five methods, using a 5% significance level. **[12]**

(iii) In each case using the residual mean square as the estimate of the variance of the experimental error, find a 95% confidence interval for the population mean for method M and a 95% confidence interval for the population mean for method A. What do these confidence intervals suggest about these population means? **[5]**

(iv) The residuals, calculated by subtracting the corresponding method mean from each observation, are given in the table below. For example the first residual for method M is 25.0 – 27.6 = –2.6. Each residual gives a measure of experimental error.

Method	Residuals
M	–2.6 –4.6 2.5 –0.1 1.2 –2.0 1.6 4.0
A	1.5 –0.9 –5.0 4.4
B	0.2 0.4 –7.0 7.8 –1.4
C	–3.4 4.7 –2.4 1.1
D	–1.0 –4.2 3.0 2.2

The diagram in the printed answer book shows a dotplot of the residuals for method M. Complete the diagram by adding the dotplots for the other methods.

Use these dotplots to comment briefly on the assumptions you have stated in part **(i)**. **[4]**

END OF QUESTION PAPER

Wednesday 18 June 2014 – Afternoon

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4769/01 Statistics 4

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Candidate forename		Candidate surname	

Centre number						Candidate number				

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Turn over

1 (i)

(answer space continued on next page)

1 (i) **(continued)**

1 (ii)

(answer space continued on next page)

1 (ii)	**(continued)**

1 (iii)

2 (i)

(answer space continued on next page)

2 (i)	**(continued)**

2 (ii)

2 (iii)

2 (iv)

2 (v)

3 (i)

3 (ii)

(answer space continued on next page)

3 (ii)	**(continued)**

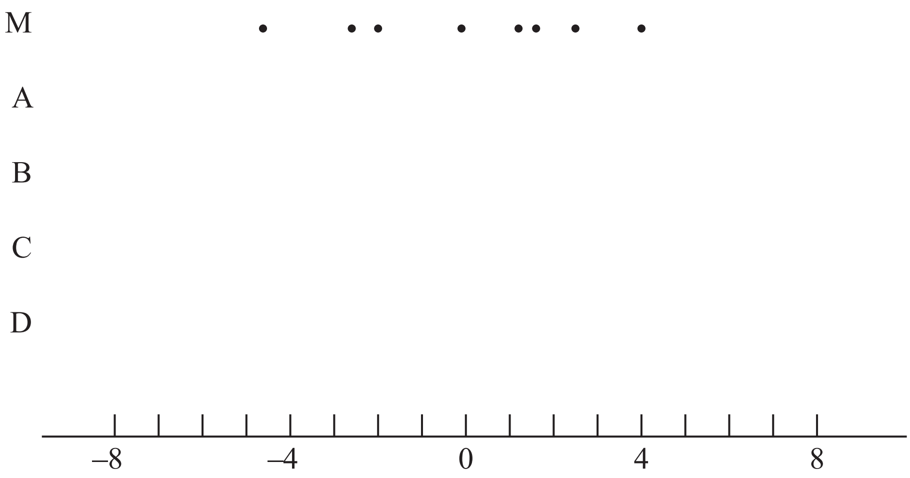
4 (i)	
4 (ii)	

(answer space continued on next page)

4 (ii) **(continued)**

4 (iii)

4 (iv)



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GCE

Mathematics (MEI)

Unit **4769**: Statistics 4

Advanced GCE

Mark Scheme for June 2014

1. Annotations and abbreviations

Annotation in scoris	**Meaning**
BP	**Blank Page – this annotation must be used on all blank pages within an answer booklet (structured or unstructured) and on each page of an additional object where there is no candidate response.**
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

2. Subject-specific Marking Instructions for GCE Mathematics (MEI) Statistics strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep *’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

Candidates are expected to give numerical answers to an appropriate degree of accuracy. 3 significant figures may often be the norm for this, but this always needs to be considered in the context of the problem in hand. For example, in quoting probabilities from Normal tables, we generally expect *some* evidence of interpolation and so quotation to 4 decimal places will often be appropriate. But even this does not always apply – quotations of the standard critical points for significance tests such as 1.96, 1.645, 2.576 (maybe even 2.58 – but not 2.57) will commonly suffice, especially if the calculated value of a test statistic is nowhere near any of these values. Sensible discretion *must* be exercised in such cases.

Discretion must also be exercised in the case of small variations in the degree of accuracy to which an answer is given. For example, if 3 significant figures are expected (either because of an explicit instruction or because the general context of a problem demands it) but only 2 are given, loss of an accuracy ("A") mark is likely to be appropriate; but if 4 significant figures are given, this should not normally be penalised. Likewise, answers which are slightly deviant from what is expected in a very minor manner (for example a Normal probability given, after an attempt at interpolation, as 0.6418 whereas 0.6417 was expected) should not be penalised. However, answers which are *grossly* over- or under-specified should normally result in the loss of a mark. This includes cases such as, for example, insistence that the value of a test statistic is (say) 2.128888446667 merely because that is the value that happened to come off the candidate's calculator. Note that this applies to answers that are given as final stages of calculations; intermediate working should usually be carried out, and quoted, to a greater degree of accuracy to avoid the danger of premature approximation.

The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h Genuine misreading (of numbers or symbols, occasionally even of text) occurs. If this results in the object and/or difficulty of the question being considerably changed, it is likely that all the marks for that question, or section of the question, will be lost. However, misreads are often such that the object and/or difficulty remain substantially unaltered; these cases are considered below.

The simple rule is that *all* method ("M") marks [and of course all independent ("B") marks] remain accessible but at least some accuracy ("A") marks do not. It is difficult to legislate in an overall sense beyond this global statement because misreads, even when the object and/or difficulty remains unchanged, can vary greatly in their effects. For example, a misread of 1.02 as 10.2 (perhaps as a quoted value of a sample mean) may well be catastrophic; whereas a misread of 1.6748 as 1.6746 may have so slight an effect as to be almost unnoticeable in the candidate's work.

A misread should normally attract *some* penalty, though this would often be only 1 mark and should rarely if ever be more than 2. Commonly in sections of questions where there is a numerical answer either at the end of the section or to be obtained and commented on (eg the value of a test statistic), this answer will have an "A" mark that may actually be designated as "cao" [correct answer only]. This should be interpreted *strictly* – if the misread has led to failure to obtain this value, then this "A" mark must be withheld even if all method marks have been earned. It will also often be the case that such a mark is implicitly "cao" even if not explicitly designated as such.

On the other hand, we commonly allow "fresh starts" within a question or part of question. For example, a follow-through of the candidate's value of a test statistic is generally allowed (and often explicitly stated as such within the marking scheme), so that the candidate may exhibit knowledge of how to compare it with a critical value and draw conclusions. Such "fresh starts" are not affected by any earlier misreads.

A misread may be of a symbol rather than a number – for example, an algebraic symbol in a mathematical expression. Such misreads are more likely to bring about a considerable change in the object and/or difficulty of the question; but, if they do not, they should be treated as far as possible in the same way as numerical misreads, *mutatis mutandis*. This also applied to misreads of text, which are fairly rare but can cause major problems in fair marking.

The situation regarding any particular cases that arise while you are marking for which you feel you need detailed guidance should be discussed with your Team Leader.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance
1	**(i)**		$f\ x\ = \dfrac{\theta^3 x^2 e^{-\theta x}}{2}$ $(x > 0)$		
			$L = \dfrac{\theta^{3n}}{2^n} x_1{}^2 x_2{}^2 ... x_n{}^2 e^{-\theta\Sigma x_i}$	M1 A1	M1 for general product form. A1 (a.e.f.) for answer.
			$\ln L = 3n \ln \theta - \theta\, \Sigma x_i$ + constant	M1 A1	M1 for taking logs (base e), correct for their L. cao Ignore whatever is offered instead of "constant" as long as it is a constant (i.e. independent of θ). If it involves θ, M1A0 at most.
			$\dfrac{d \ln L}{d\theta} = \dfrac{3n}{\theta} - \Sigma x_i$	M1	M1 for differentiating correct for their ln L.
			= 0 for ML Est $\hat{\theta}$.	M1	M1 for setting equal to 0.
			$\therefore \hat{\theta} = \dfrac{3n}{\Sigma x_i} = \dfrac{3}{\bar{x}}$.	A1	Allow u.c. or l.c $\bar{x}$. cao i.e. do not f.t. from incorrect derivative. Beware printed answer.
			Confirmation that this is a maximum:	M1	
			$\dfrac{d^2 \ln L}{d\theta^2} = -\dfrac{3n}{\theta^2} < 0$.	A1	Allow other ways to confirm maximum, but $d^2 \ln L/d\theta^2$ will be needed later and may be rewarded there if M0 has been awarded here.
				[9]	

Question			**Answer**	**Marks**	**Guidance**
1	**(ii)**		We want $\mathrm{E}\left(\frac{3}{X}\right)=\int_0^\infty \frac{3}{2}\theta^3 x\mathrm{e}^{-\theta x}\mathrm{d}x$	M1 M1	M1 for realising that this expectation is needed. M1 for correct integral, including limits, with an *attempt* to evaluate it.
			$=\frac{3}{2}\theta^3\int_0^\infty x\mathrm{e}^{-\theta x}\mathrm{d}x=\frac{3}{2}\theta^3\times\frac{1}{\theta}\times$ mean of exponential dist with ptr θ	A1 M1 M1	A1 for expressing integral in form shown here. M1 for taking out the $1/\theta$ factor. M1 for use of mean of exponential distribution. *Or for explicit integration by parts:* *1st term A1, 2nd term A1, 1st term zero A1,*
			$=\frac{3}{2}\theta^3\times\frac{1}{\theta}\times\frac{1}{\theta}$	A1	$\left[-\frac{1}{\theta^2}\mathrm{e}^{-\theta x}\right]_0^\infty$ *A1, Result (including 3/2) A1.*
			$=\frac{3}{2}\theta$.	A1	
			This is not equal to θ, therefore the estimator is biased.	E1	The word "biased" must appear. Statement that this is not equal to θ does *not* earn the mark on its own.
				[8]	
1	**(iii)**		$\mathrm{E}\left(-\frac{\mathrm{d}^2\ln\mathrm{L}}{\mathrm{d}\theta^2}\right)=\frac{3n}{\theta^2}$, $\therefore \mathrm{H}\ \hat{\theta}\ =\frac{\theta^2}{3n}$.	B1	Beware printed answer for H(θ). M1 for $\frac{\mathrm{d}^2\ln\mathrm{L}}{\mathrm{d}\theta^2}$ can be earned here if not awarded in (i).
			$\hat{\theta}=3/5=0.6$	B1	Allow 3/5 or 0.6.
			$\mathrm{H}\ \hat{\theta}\ =\frac{\hat{\theta}^2}{3n}=\frac{0.36}{300}=0.0012$	B1	
			Approximate confidence interval is	M1 B1	M1 for "centre" of 0.6 (f.t. candidate's value). B1 for 1.96 used. [Do *not* allow any *t* point.]
			0.6 ± 1.96√0.0012 0.6 ± 0.067(8963)	M1	M1 for √0.0012 (f.t. candidate's value).
			(0.532(1) , 0.667(9))	A1	cao (i.e. no f.t of candidate's values). If more than 4 d.p. are offered, A0 (over-specification).
				[7]	

Question			Answer	Marks	Guidance
2	**(i)**		$\mathrm{f}\ \ x\ = \frac{x^{k-1}\mathrm{e}^{-x/\phi}}{\phi^k\ \ k-1\ !}$, $x > 0$		
			$\mathrm{M}_X\ \ \theta\ = \mathrm{E}\ \ \mathrm{e}^{\theta x}\ = \int_0^\infty \frac{\mathrm{e}^{\theta x-\frac{x}{\phi}}x^{k-1}}{\phi^k\ \ k-1\ !}\mathrm{d}x$	M1	Correct integral with *attempt* to integrate by parts.
			$= \frac{1}{\phi^k\ \ k-1\ !}\left\{\left[x^{k-1}\frac{\mathrm{e}^{\theta x-\frac{x}{\phi}}}{\theta-\frac{1}{\phi}}\right]_0^\infty - \int_0^\infty \frac{\mathrm{e}^{\theta x-\frac{x}{\phi}}}{\theta-\frac{1}{\phi}}\ \ k-1\ \ x^{k-2}\mathrm{d}x\right\}$	A1 B1 A1	A1 for first term in integral by parts B1 for sight of first term with limits. (follow-through if it simply "disappears" by magic!). A1 for second term in integral by parts.
					[These 3 marks can be earned if x^k occurs]
			$= \frac{1}{\phi^k\ \ k-1\ !}\frac{\phi\ \ k-1}{1-\phi\theta}\int_0^\infty \mathrm{e}^{\theta x-\frac{x}{\phi}}x^{k-2}\ \mathrm{d}x$	M1 M1	Re-arrangement into a form that re-creates the same form as the original integral with *k* – 2 instead of *k* – 1. Attempt at "reduction formula" method.
			$= \frac{1}{\phi^k\ \ k-1\ !}\frac{\phi^2\ \ k-1\ \ \ k-2}{1-\phi\theta\ \ ^2}\int_0^\infty \mathrm{e}^{\theta x-\frac{x}{\phi}}x^{k-3}\ \mathrm{d}x$		
					Note that some marks may be implicit in a candidate's work, and/or other methods might have been tried.
			= …..		
			$= \frac{1}{\phi^k\ \ k-1\ !}\frac{\phi^{k-1}\ \ k-1\ \ \ k-2\ \ ...1}{1-\phi\theta\ \ ^{k-1}}\int_0^\infty \mathrm{e}^{\theta x-\frac{x}{\phi}}x^0\ \mathrm{d}x$	A1 A1 A1	A1 for power *k* – 1 of ϕ in numerator. A1 for (*k* – 1)(*k* – 2)…1 in numerator. A1 for power *k* – 1 of 1 – $\phi\theta$ in denominator. Final integral and integration explicit for following marks (AG)

Question			Answer	Marks	Guidance
			The integral on the previous line is $\left[\dfrac{\mathrm{e}^{\theta x-\frac{x}{\phi}}}{\theta-\frac{1}{\phi}}\right]_0^\infty$.	M1 A1	M1 for *attempt* to deal with the *k* = 0 term. A1 for $\dfrac{\phi}{1-\phi\theta}$ (may be implicit in candidate's work).
			$=\dfrac{1}{\phi^k\ \ k-1\ !}\dfrac{\phi^k\ \ k-1\ !}{1-\phi\theta\ ^k}\ =\ 1-\phi\theta\ ^{-k}$.	A1	Beware printed answer.
				[12]	Alternative methods leading more directly to the correct solution will be given full credit.
2	**(ii)**		$\mathrm{M}_W\ \ \theta\ =\ 1-\phi\theta\ ^{-nk}$	B1 **[1]**	
2	**(iii)**		$\mathrm{M}_Y\ \ \theta\ =\left(1-\dfrac{2\phi\theta}{\phi}\right)^{-nk}\ =\ 1-2\theta\ ^{-nk}$	B1	
			This is mgf of χ^2_{2nk}, so (by inversion theorem/uniqueness property)	M1	For some allusion to inversion theorem/uniqueness property
			distribution of *Y* is χ^2_{2nk}.	B1 **[3]**	May be awarded for correct conclusion even if the preceding M1 was not awarded.

Question			Answer	Marks	Guidance
2	(iv)		We have $\frac{2W}{\phi} \sim \chi^2_{2nk}$, and $0.95 = \mathrm{P}\ \ l < \chi^2_{2nk} < u\ \ .$		Must link 0. 95, χ^2_{2nk} to l and u, thence to $Y = \frac{2W}{\phi}$
			$\therefore 0.95 = \mathrm{P}\left(l < \frac{2W}{\phi} < u \right).$		(AG)
			Inserting an observed value *w* of *W* gives 2*w*/*u* < ϕ and 2*w*/*l* > ϕ so that a 95% confidence interval for ϕ is (2*w*/*u*, 2*w*/*l*).	B2,1,0	Final CI must not be embedded in a probability statement.
				[2]	
2	(v)		We have χ^2_{40}.	B1	Follow-through from here if not 40 d.f.
			Confidence interval is $\left(\frac{2w}{59.34}, \frac{2w}{24.43} \right)$.	A1	Both critical points correct, using candidate's d.f.
			We need E(*W*).	M1	For realising E(W) is needed.
			Mgf of *W* is $\mathrm{M}_W\ \ \theta\ \ =\ \ 1-\phi\theta\ \ ^{-20}$ (see part (ii)).		*nk* = 20 may be seen below.
			E(*W*) =M'(0).	M1	For attempt to find first derivative at zero, or by expanding the series.
			$\mathrm{M}'(\theta) = -20(1 - \phi\theta)^{-21}(-\phi)$ $\therefore \mathrm{M}'(0) = 20\phi$.	A1	
			$\therefore$ E(length of CI) = $\frac{2 \times 20\phi}{24.43} - \frac{2 \times 20\phi}{59.34}$	A1	cao (no follow-through from candidate's incorrect d.f)
			$= 40\phi\,(0.040933 - 0.016852) = 40\phi\,(0.0240812) = 0.963(25)\phi$.		Usual penalty for over-specification of final answer. (3 or 4 sf, awrt 0.963)
				[6]	

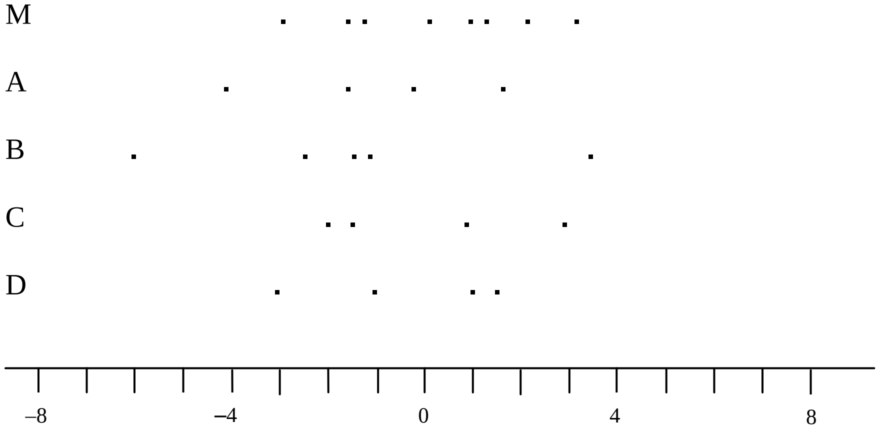
Question			Answer	Marks	Guidance
3	**(i)**		Type I error: rejecting null hypothesis when it is true	B1 B1	Allow B1 out of 2 for P(...).
			Type II error: accepting null hypothesis when it is false	B1 B1	Allow B1 out of 2 for P(...).
			OC: P(accepting null hypothesis as a function of the parameter under investigation)	B1 B1	P(Type II error | the true value of the parameter) scores B1+B1.
			Power: P(rejecting null hypothesis as a function of the parameter under investigation)	B1 B1	P(Type I error | the true value of the parameter) scores B1+B1. Definition of power as "1 – OC" scores zero.
				[8]	

Question			Answer	Marks	Guidance
3	(ii)		$X \sim \mathrm{N}(\mu, \sigma = 3.7)$ H$_0$: $\mu = 63$ H$_1$: $\mu < 63$		**Note**. Candidates might not exhibit their work in the style of this mark scheme. Provided a candidate's work is clear, award marks commensurately.
			We require		
			$0.01 = \mathrm{P}\ \ \text{reject H}_0 \mid \mu = 63\ \ = \mathrm{P}\ \ \bar{X} < c \mid \mu = 63$	M1	May be implied
			$= \ \mathrm{P}\ \ \mathrm{N}\ \ 63, 3.7^2/n\ \ < c \quad = \ \mathrm{P}\left(\mathrm{N}\ \ 0,1\ \ < \dfrac{c-63}{3.7/\sqrt{n}}\right)$	M1 M1	M1 for distribution of $\bar{X}$ (may be implied) M1 for standardising (ignore "c")
			$\therefore \dfrac{c-63}{3.7/\sqrt{n}} = -2.326$	B1	B1 for –2 326 used.
			We also require $0.90 = \mathrm{P}\ \ \text{reject H}_0 \mid \mu = 60$		
			$= \ \mathrm{P}\ \ \mathrm{N}\ \ 60, 3.7^2/n\ \ < c \quad = \ \mathrm{P}\left(\mathrm{N}\ \ 0,1\ \ < \dfrac{c-60}{3.7/\sqrt{n}}\right)$	M1* M1dep*	M1 for distribution of $\bar{X}$ M1 for standardising (ignore "c")
			$\therefore \dfrac{c-60}{3.7/\sqrt{n}} = 1.282$	B1	B1 for 1.282 used.
			$\therefore$ we have $c = 63 - \dfrac{8.6062}{\sqrt{n}}$ and $c = 60 + \dfrac{4.7434}{\sqrt{n}}$	M1 A1	M1 for two equations for *c* and *n*. A1 if both correct (f.t. from any previous errors).
			Attempt to solve.	M1	
			$c = 61.0658$ [allow 61.1 or awrt]	B1	cao (can allow if st error is incorrect)
			$\sqrt{n} = 4.45(053)$, $n = 19.8(07)$	A1	cao (awrt 19.8)
			Take *n* as "next integer up" from candidate's value.	B1	
			$\mathrm{P}\ \ \text{reject H}_0 \mid \mu = 58.5$	M1	Correct "translation" of power of test for $\mu = 58.5$.
			$= \ \mathrm{P}\ \ \mathrm{N}\ \ 58.5, 3.7^2/20\ \ < 61.0658$	M1	Use of *candidate's* values of *c* and *n* (NB *n* must be an *integer* here).

Question			Answer	Marks	Guidance
			$= \quad \text{P}\left(\text{N} \;\; 0,1 \;\; < \dfrac{2.5658}{3.7/\sqrt{20}} = 3.10 \right) \; = \; 0.9990$.	A1 **[16]**	cao (awrt 0.999)
4	**(i)**		*e*$_{ij}$ are independently Normally distributed with mean zero and variance σ^2 (i.e. a constant)	B1 B1 B1 **[3]**	Must have *both* independent and Normal. Must clearly indicate *constant* variance. A correct notational statement such as e_{ij} ~ ind N(0, σ^2) earns all three marks.

Question			Answer	Marks	Guidance
4	(ii)		"Correction factor" CF = $830.6^2/25$ = 27595.854		
			Total SS = 28260.18 – CF = 664.326 Between methods SS = $\frac{220.8^2}{8}+\frac{143.2^2}{4}+\frac{181.0^2}{5}+\frac{141.6^2}{4}+\frac{144.0^2}{4}-\text{CF}$	M1	M1 for correct method for all three sums of squares (may be implied by *correct* numerical answers from calculator).
			= 27969.48 – CF = 373.626	A1	A1 if all three calculated sums of squares are correct.
			Residual SS (by subtraction) = 664.326 –373.626 = 290.700		
			Source of variation SS df MS MS ratio Between treatments 373.626 4 93.4065 6.42(6) Residual 290.700 20 14.535 Total 664.326 24	B1 M1 M1 A1	B1 if d.f. 4 and 20 are both correct. M1 for method of finding mean squares. M1 for method of finding mean square ratio. A1 cao for MS ratio correct.
			Null hypothesis: $\alpha_1=\alpha_2=...=\alpha_5\ (=0)$.	B1	Accept $\alpha_1=\alpha_2=\ldots$ with no explicit upper end to sequence.
			Alternative hypothesis: not all α_i are equal (zero).	B1	B0 for the alternative hypothesis if this is wrongly stated or implied to be that the α_i are all different.
					Accept hypotheses stated verbally provided it is clear that *population* parameters (means) are being referred to.
			Refer MS ratio to $F_{4,20}$.	M1	No f.t. from here if d.f. are wrong.
			Upper 5% point is 2.87.	A1	No f.t. if wrong (or if not quoted).
			Significant, reject H_o	A1	either
			therefore the evidence suggests that there are differences among the method means	E1	Verbal conclusion in context, and not "too assertive".
				[12]	

Question			Answer	Marks	Guidance
4	(iii)				**Use of Normal distribution gets 0/5**
			Confidence intervals are of form $\bar{x} \pm \frac{t\sqrt{14.535}}{\sqrt{n}}$.	M1	For general form of CI with candidate's Res. Mean Square.
			t refers to t_{20}.	M1	For use of t_{20}. (Allow candidate's res. d.f.)
			Double-tailed 5% point is 2.086.	A1	cao.
			For M: $27.6 \pm \frac{2.086\sqrt{14.535}}{\sqrt{8}} = 27.6 \pm 2.81 = (24.79, 30.41)$.		
			For A: $35.8 \pm \frac{2.086\sqrt{14.535}}{\sqrt{4}} = 35.8 \pm 3.98 = (31.82, 39.78)$.	A1	A1 cao for *both* intervals correct. Allow 3 s.f.
			Confidence intervals do not overlap – suggests population means differ.	E1	Not "too assertive". F.t. from candidate's confidence intervals even if they are wrong.
				[5]	

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td>4</td><td>(iv)</td><td></td><td>
</td><td></td><td>This dotplot (i.e. for M) is provided in the PAB.
.</td></tr>
<tr><td></td><td></td><td></td><td></td><td>B1</td><td>if any dotplot is reasonably correct.</td></tr>
<tr><td></td><td></td><td></td><td></td><td>B1</td><td>another B1 if they all are</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td>Do not insist on too much accuracy. "Rough and ready" dotplots are acceptable provided they are reasonably correct. The exact positions of the dots are shown in the table of residuals in the question, which is repeated here for convenience:
<table>
<tr><th>Method</th><th>Residuals</th></tr>
<tr><td>M</td><td>–2.6 –4.6 2.5 –0.1 1.2 –2.0 1.6 4.0</td></tr>
<tr><td>A</td><td>1.5 –0.9 –5.0 4.4</td></tr>
<tr><td>B</td><td>0.2 0.4 –7.0 7.8 –1.4</td></tr>
<tr><td>C</td><td>–3.4 4.7 –2.4 1.1</td></tr>
<tr><td>D</td><td>–1.0 –4.2 3.0 2.2</td></tr>
</table></td></tr>
<tr><td></td><td></td><td></td><td>Common variance seems unlikely (e.g. residuals for B are much more spread out than those for C).</td><td>E1</td><td></td></tr>
<tr><td></td><td></td><td></td><td>Normality also seems unlikely – there is only a partial suggestion, for some of the methods, of clustering near 0 such as we would expect under Normality.</td><td>E1
[4]</td><td>Allow other sensible comments.</td></tr>
</table>

Unit level raw mark and UMS grade boundaries June 2014 series
AS GCE / Advanced GCE / AS GCE Double Award / Advanced GCE Double Award

GCE Mathematics (MEI)

			Max Mark	a	b	c	d	e	u
4751/01	(C1) MEI Introduction to Advanced Mathematics	Raw	72	61	56	51	46	42	0
		UMS	100	80	70	60	50	40	0
4752/01	(C2) MEI Concepts for Advanced Mathematics	Raw	72	57	51	45	39	33	0
		UMS	100	80	70	60	50	40	0
4753/01	(C3) MEI Methods for Advanced Mathematics with Coursework: Written Paper	Raw	72	58	52	47	42	36	0
4753/02	(C3) MEI Methods for Advanced Mathematics with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4753/82	(C3) MEI Methods for Advanced Mathematics with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
4753	(C3) MEI Methods for Advanced Mathematics with Coursework	UMS	100	80	70	60	50	40	0
4754/01	(C4) MEI Applications of Advanced Mathematics	Raw	90	68	61	54	47	41	0
		UMS	100	80	70	60	50	40	0
4755/01	(FP1) MEI Further Concepts for Advanced Mathematics	Raw	72	63	57	51	45	40	0
		UMS	100	80	70	60	50	40	0
4756/01	(FP2) MEI Further Methods for Advanced Mathematics	Raw	72	60	54	48	42	36	0
		UMS	100	80	70	60	50	40	0
4757/01	(FP3) MEI Further Applications of Advanced Mathematics	Raw	72	57	51	45	39	34	0
		UMS	100	80	70	60	50	40	0
4758/01	(DE) MEI Differential Equations with Coursework: Written Paper	Raw	72	63	56	50	44	37	0
4758/02	(DE) MEI Differential Equations with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4758/82	(DE) MEI Differential Equations with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
4758	(DE) MEI Differential Equations with Coursework	UMS	100	80	70	60	50	40	0
4761/01	(M1) MEI Mechanics 1	Raw	72	57	49	41	34	27	0
		UMS	100	80	70	60	50	40	0
4762/01	(M2) MEI Mechanics 2	Raw	72	57	49	41	34	27	0
		UMS	100	80	70	60	50	40	0
4763/01	(M3) MEI Mechanics 3	Raw	72	55	48	42	36	30	0
		UMS	100	80	70	60	50	40	0
4764/01	(M4) MEI Mechanics 4	Raw	72	48	41	34	28	22	0
		UMS	100	80	70	60	50	40	0
4766/01	(S1) MEI Statistics 1	Raw	72	61	53	46	39	32	0
		UMS	100	80	70	60	50	40	0
4767/01	(S2) MEI Statistics 2	Raw	72	60	53	46	40	34	0
		UMS	100	80	70	60	50	40	0
4768/01	(S3) MEI Statistics 3	Raw	72	61	54	47	41	35	0
		UMS	100	80	70	60	50	40	0
4769/01	(S4) MEI Statistics 4	Raw	72	56	49	42	35	28	0
		UMS	100	80	70	60	50	40	0
4771/01	(D1) MEI Decision Mathematics 1	Raw	72	51	46	41	36	31	0
		UMS	100	80	70	60	50	40	0
4772/01	(D2) MEI Decision Mathematics 2	Raw	72	46	41	36	31	26	0
		UMS	100	80	70	60	50	40	0
4773/01	(DC) MEI Decision Mathematics Computation	Raw	72	46	40	34	29	24	0
		UMS	100	80	70	60	50	40	0
4776/01	(NM) MEI Numerical Methods with Coursework: Written Paper	Raw	72	54	48	43	38	32	0
4776/02	(NM) MEI Numerical Methods with Coursework: Coursework	Raw	18	14	12	10	8	7	0
4776/82	(NM) MEI Numerical Methods with Coursework: Carried Forward Coursework Mark	Raw	18	14	12	10	8	7	0
4776	(NM) MEI Numerical Methods with Coursework	UMS	100	80	70	60	50	40	0
4777/01	(NC) MEI Numerical Computation	Raw	72	55	47	39	32	25	0
		UMS	100	80	70	60	50	40	0
4798/01	(FPT) Further Pure Mathematics with Technology	Raw	72	57	49	41	33	26	0
		UMS	100	80	70	60	50	40	0

GCE Statistics (MEI)

			Max Mark	a	b	c	d	e	u
G241/01	(Z1) Statistics 1	Raw	72	61	53	46	39	32	0
		UMS	100	80	70	60	50	40	0
G242/01	(Z2) Statistics 2	Raw	72	55	48	41	34	27	0
		UMS	100	80	70	60	50	40	0
G243/01	(Z3) Statistics 3	Raw	72	56	48	41	34	27	0
		UMS	100	80	70	60	50	40	0