

OCR

Oxford Cambridge and RSA

Friday 24 June 2016 – Morning

A2 GCE MATHEMATICS (MEI)

4769/01 Statistics 4

QUESTION PAPER

Candidates answer on the Printed Answer Book.

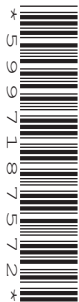
OCR supplied materials:

- Printed Answer Book 4769/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer any **three** questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

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- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Option 1: Estimation

- 1 The random variable X has a Cauchy distribution centred on m . Its probability density function (pdf) is $f(x)$ where

$$f(x) = \frac{1}{\pi} \frac{1}{1 + (x - m)^2}, \quad \text{for } -\infty < x < \infty .$$

- (i) Sketch the pdf. Show that the mode and median are at $x = m$. [6]
- (ii) A sample of size 1, consisting of the observation x_1 , is taken from this distribution. Show that the maximum likelihood estimate (MLE) of m is x_1 . [3]
- (iii) Now suppose that a sample of size 2, consisting of observations x_1 and x_2 , is taken from the distribution. By considering the logarithm of the likelihood function or otherwise, show that the MLE, \hat{m} , satisfies the cubic equation

$$\left(2\hat{m} - (x_1 + x_2)\right)\left(\hat{m}^2 - (x_1 + x_2)\hat{m} + 1 + x_1x_2\right) = 0. \quad [5]$$

- (iv) Obtain expressions for the three roots of this equation. Show that if $|x_1 - x_2| < 2$ then only one root is real. How do you know, without doing further calculations, that in this case the real root will be the MLE of m ? [5]
- (v) Obtain the three possible values of \hat{m} in the case $x_1 = -2$ and $x_2 = 2$. Evaluate the likelihood function for each value of \hat{m} and comment on your answer. [5]

Option 2: Generating Functions

2 The random variable X has probability density function $f(x)$ where

$$f(x) = \lambda e^{-\lambda x}, \quad x > 0.$$

(i) Obtain the moment generating function (mgf) of X . [3]

(ii) Use the mgf to find $E(X)$ and $\text{Var}(X)$. [6]

The random variable Y is defined as follows:

$$Y = X_1 + \dots + X_n,$$

where the X_i are independently and identically distributed as X .

(iii) Write down expressions for $E(Y)$ and $\text{Var}(Y)$.

Obtain the mgf of Y . [4]

(iv) Find the mgf of Z where $Z = \frac{Y - \frac{n}{\lambda}}{\frac{\sqrt{n}}{\lambda}}$. [4]

(v) By considering the logarithm of the mgf of Z , show that the distribution of Z tends to the standard Normal distribution as n tends to infinity. [7]

Option 3: Inference

- 3 A large department in a university wished to compare the standards of literacy and numeracy of its students. A random sample of 24 students was taken and sub-divided, randomly, into two groups of 12. The students in one group took a literacy assessment (scores denoted by x); the students in the other group took a numeracy assessment (scores denoted by y). The two assessments were designed to give the same distributions of scores when taken by random samples from the general population.

The scores obtained by the students on the two assessments are shown in the table.

x	23	42	43	46	48	48	50	54	58	59	62	65
y	44	36	63	55	53	58	63	80	61	57	83	54

$$\sum x = 598 \quad \sum x^2 = 31196 \quad \sum y = 707 \quad \sum y^2 = 43543$$

- (i) Carry out an appropriate t test, at the 5% level of significance, to compare the standards of literacy and numeracy. [12]
- (ii) State the distributional assumptions required for the t test to be valid.

Name the test that you would use if the assumptions required for the t test are thought not to hold. State the hypotheses for this new test.

Explain, in general terms, which of the two tests is more powerful, and why. [6]

A statistician at the university looked at the data and commented that a paired sample design would have been better.

- (iii) Explain how a paired sample design would be applied in this context, and how the data would be analysed. Explain also why it would be better than the design used. [6]

Option 4: Design and Analysis of Experiments

- 4 The cardiovascular unit of a hospital is studying the effect on patients' heart rates of three different light exercises, A, B and C. Patients are given an exercise to do and the increases in their pulse rates are measured after 5 minutes. There are 16 patients in the study: 5 are chosen randomly and allocated to exercise A, 6 to exercise B, and 5 to exercise C.

The data obtained are as follows.

A	B	C
63	69	56
41	72	44
42	52	65
51	64	48
47	54	53
	57	

	A	B	C
Sum of data	244	368	266
Sum of squares	12 224	22 910	14 410

- (i) State the usual one-way analysis of variance model.

Explain what the terms in the model mean in this context.

State the distributional assumptions required for the standard test.

Carry out the test at the 5% level of significance and report your conclusions. [18]

- (ii) Someone unfamiliar with analysis of variance analysed these data. They used three t tests to compare A with B, B with C, and C with A. The test comparing A with B was significant at the 5% level; the other two tests were not significant at the 5% level.

Comment on this analysis, explaining whether it is better than, worse than or equivalent to the analysis carried out in part (i). Your comments should include consideration of the independence of the t tests and the overall level of significance of the procedure. [6]

END OF QUESTION PAPER

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4769/01 Statistics 4

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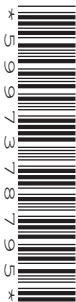
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Candidate forename		Candidate surname	
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Centre number						Candidate number				
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1 (iii)	(continued)

1 (iv)	

1 (v)	
2 (i)	

2 (ii)	

2 (iii)	

2 (v)	(continued)

Question 3(i) begins on page 10

3 (iii)	

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4 (ii)	(continued)

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GCE

Mathematics (MEI)

Unit **4769**: Statistics 4

Advanced GCE

Mark Scheme for June 2016

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of candidates of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, Cambridge Nationals, Cambridge Technicals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support, which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

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1. Annotations and abbreviations

Annotation in scoris	Meaning
✓ and ✕	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

2. Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

- a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

- b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

- c The following types of marks are available.

M

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B

Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- d When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep *’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

- f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

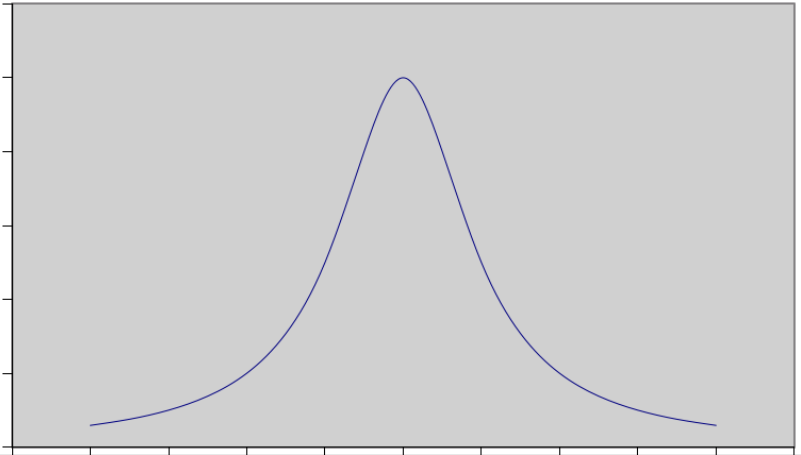
If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question	Answer	Marks	Guidance	
1	<p>(i)</p>  <p style="text-align: center;">m</p> <p>Centred on $x = m$</p> <p>Mode: observe that $f(x)$ takes its maximum when $x - m = 0$ (or differentiate and set to zero)</p> <p>Median: integral is $(1/\pi) \arctan(x - m)$. Evaluated between $-\infty$ and m or m and ∞ this evaluates to 0.5.</p> <p>(ii)</p> <p>Likelihood function is $f(x_1)$ Differentiate wrt m and set to zero: $\frac{1}{\pi} \frac{2(x_1 - m)}{(1 + (x_1 - m)^2)^2} = 0$ Giving $x_1 = m$</p>	<p>G2</p> <p>M1E1</p> <p>M1A1 [6]</p> <p>B1 M1</p> <p>A1 [3]</p>	<p>Derivative is</p> $-\frac{1}{\pi} \frac{2(x - m)}{(1 + (x - m)^2)^2}$ <p>“symmetry” SC B1</p> <p>Answer given</p>	

Question	Answer	Marks	Guidance	
(iii)	Likelihood function is $f(x_1) f(x_2)$ The derivative of the log likelihood wrt m is $\frac{x_1 - m}{1 + (x_1 - m)^2} + \frac{x_2 - m}{1 + (x_2 - m)^2}$ Setting to zero and convincing algebra to given result	B1 M1A1 M1A1 [5]		
(iv)	$\hat{m} = \frac{1}{2}(x_1 + x_2)$ $\hat{m} = \frac{1}{2}(x_1 + x_2) \pm \frac{1}{2}\sqrt{(x_1 - x_2)^2 - 4}$ Convincing argument that if $ x_1 - x_2 < 2$ then only 1 root is real Convincing argument that the ML function is positive, continuous and tends to zero at $\pm \infty$. Hence the only turning point is a maximum	B1 M1A1 E1 E1 [5]	Or equivalent	
(v)	The three values of \hat{m} are: $0, \pm\sqrt{3}$ Likelihood at zero is $\frac{1}{25\pi^2}$ or $\frac{0.04}{\pi^2}$ (approx. 0.0040) Likelihood at $\pm\sqrt{3}$ is $\frac{1}{16\pi^2}$ or $\frac{0.0625}{\pi^2}$ (approx. 0.0063) So $\pm\sqrt{3}$ are joint MLE, 0 is not MLE.	B1B1 B1 B1 E1 [5]	B1 for 0, B1 both Accept approx. values	
		[24]		

Question	Answer	Marks	Guidance
2	<p>(i) $M_X(\theta) = E(e^{X\theta}) = \int_0^\infty e^{x\theta} \lambda e^{-\lambda x} dx$</p> $= \int_0^\infty \lambda e^{x(\theta-\lambda)} dx$ $= \left[\frac{\lambda}{\theta-\lambda} e^{x(\theta-\lambda)} \right]_0^\infty$ $= \frac{\lambda}{(\lambda-\theta)} \text{ or } \left(1 - \frac{\theta}{\lambda}\right)^{-1}$ <p>(ii) Differentiate once and substitute $\theta = 0$ $M'_X(\theta) = \lambda(\lambda-\theta)^{-2}$</p> <p>Hence $E(X) = 1/\lambda$</p> $M''_X(\theta) = 2\lambda(\lambda-\theta)^{-3}$ $E(X^2) = 2/\lambda^2$ $\text{Var}(X) = 2/\lambda^2 - 1/\lambda^2 = 1/\lambda^2$ <p>(iii) $E(Y) = n/\lambda$</p> $\text{Var}(Y) = n/\lambda^2$ <p>Use the convolution theorem:</p> $M_Y(\theta) = (M_X(\theta))^n = \left(\frac{\lambda}{\lambda-\theta}\right)^n$	<p>M1</p> <p>A1</p> <p>A1</p> <p>[3]</p> <p>M1A1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>[6]</p> <p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>[4]</p>	<p>With limits shown</p> <p>Indefinite integral</p> <p>Correct www</p> <p>2nd derivative</p> <p>FT (ii) , both</p> <p>For a product</p> <p>cao</p> <p>Condone omission of the fact that upper limit requires $\theta < \lambda$</p> <p>OR Expanding the mgf and identifying coefficients of θ and θ^2, M1 A1 A1</p>

Question	Answer	Marks	Guidance	
(iv)	Use $M_{aX+b}(\theta) = e^{b\theta} M_X(a\theta)$ To obtain $M_Z(\theta)$ as $e^{-\sqrt{n}\theta} \left(\frac{\lambda}{\lambda - \theta(\frac{\lambda}{\sqrt{n}})} \right)^n$ Simplifying to $e^{-\sqrt{n}\theta} \left(\frac{1}{1 - \frac{\theta}{\sqrt{n}}} \right)^n$	M1 A1 A1 A1 [4]	For factor $e^{-\sqrt{n}\theta}$ oe Second factor This version may be seen in (v)	
(v)	Logarithm is $-\sqrt{n}\theta - n \ln \left(1 - \frac{\theta}{\sqrt{n}} \right)$ $= -\sqrt{n}\theta + n \left(\frac{\theta}{\sqrt{n}} + \frac{\theta^2}{2n} + \dots \right)$ First two terms cancel and $\frac{1}{2}\theta^2$ seen Next term seen or “remaining terms have powers of n in denominator” Hence mgf tends to $\exp\left(\frac{1}{2}\theta^2\right)$ Which is mgf of standard Normal	M1 A1 M1 A1 A1 E1 E1 [7]	Expression with two terms For series expansion Answer given so working must be convincing	
		[24]		

Question	Answer	Marks	Guidance
3	<p>(i)</p> <p>$H_0: \mu_1 = \mu_2$ $H_1: \mu_1 \neq \mu_2$ where μ_1 and μ_2 are the means in the underlying population</p> <p>$\bar{x} = \frac{598}{12} = 49.8333$ $\bar{y} = \frac{707}{12} = 58.9167$</p> <p>$\sum (x - \bar{x})^2 = 31196 - \frac{598^2}{12} = 1395.66667$; [$s_x^2 = 126.87\dots, s_x = 11.264\dots$]</p> <p>$\sum (y - \bar{y})^2 = 43543 - \frac{707^2}{12} = 1888.91667$; [$s_y^2 = 171.719\dots, s_y = 13.104\dots$]</p> <p>Pooled variance estimate = $\frac{(1395.666\dots + 1888.916\dots)}{(11+11)} = 149.299$</p> <p>Test statistic: $\frac{58.9167 - 49.8333}{\sqrt{149.299} \sqrt{\frac{1}{12} + \frac{1}{12}}} = 1.8209$</p> <p>5% two-tailed critical value for t_{22} is 2.0739.</p> <p>Hence no reason to reject H_0, no reason to suppose that standards of literacy and numeracy are different in the underlying population, on average.</p> <p>(ii)</p> <p>Scores in the underlying population distributed Normally With common variance Wilcoxon rank sum test (or Mann-Whitney 2 sample test) H_0: literacy scores and numeracy scores have the same distribution H_1: literacy scores and numeracy scores have the same distribution but for a shift in location The t test will be more powerful because it uses the magnitudes of the data rather than just their ranks.</p>	<p>B1 B1 B1 M1 A1 M1A1 M1 A1 B1 M1 A1 [12] B1 B1 B1 B1 B1 [6]</p>	<p>Zero if sample means used. B1 if not clearly population means</p> <p>Accept alternative forms if correctly used later</p> <p>$\frac{11s_x^2 + 11s_y^2}{22}$</p> <p>Correct construction, their s, \bar{x}, \bar{y}</p> <p>2.0772 by interpolation from tables no reason to reject H_0 context</p> <p>Accept same median and different medians</p>

Question	Answer	Marks	Guidance
(iii)	<p>In a paired sample design, all the students in the sample would do both assessments.</p> <p>The order in which the students do the assessments should be randomised and/or blocked for balance.</p> <p>The data used in the test would be the differences in their scores.</p> <p>A single sample t test (or Wilcoxon if Normality cannot be assumed) would be used</p> <p>This would be better than the two sample design used because the variation between students would be factored out.</p> <p>The design would therefore be more sensitive to differences between literacy and numeracy.</p>	<p>B1</p> <p>B1</p> <p>B1</p> <p>B1</p> <p>B1</p> <p>B1</p> <p>[6]</p>	<p>This part of the question is entirely descriptive and candidates may present answers which are correct but expressed differently. Marks should be awarded accordingly.</p>

Question	Answer	Marks	Guidance																								
<p>4 (i)</p>	<p>$Y_{ij} = \mu + \alpha_i + \varepsilon_{ij}$ where: Y_{ij} is the jth value in the ith group μ is the global mean in the underlying population α_i is the ‘treatment effect’ in the ith group ε_{ij} is a random error term In this context, μ measures the average effect of the exercise regimes, and the α_i represent the differences from the mean for the three regimes ε_{ij} iid $N(0, \sigma^2)$</p> <p>H_0: the three exercise regimes give the same (population) increase in mean pulse rate H_1: the three exercise regimes do not give the same (population) increase in mean pulse rate</p> $\sum \frac{T_i^2}{n_i} - \frac{T^2}{n} = \frac{244^2}{5} + \frac{368^2}{6} + \frac{266^2}{5} - \frac{878^2}{16} = 448.8167$ $\sum \sum y_{ij}^2 - \frac{T^2}{n} = 49544 - \frac{878^2}{16} = 1363.75$ <p>ANOVA table:</p> <table border="1" data-bbox="360 1038 1256 1157"> <thead> <tr> <th>Source of Variation</th> <th>Sum Sq</th> <th>df</th> <th>MS</th> <th>F ratio</th> <th>F critical</th> </tr> </thead> <tbody> <tr> <td>Between Groups</td> <td>448.8167</td> <td>2</td> <td>224.41</td> <td>3.1885</td> <td>3.8056</td> </tr> <tr> <td>Within Groups</td> <td>914.9333</td> <td>13</td> <td>70.379</td> <td></td> <td></td> </tr> <tr> <td>Total</td> <td>1363.75</td> <td>15</td> <td></td> <td></td> <td></td> </tr> </tbody> </table> <p>Result not significant. Insufficient evidence to suppose that the exercise regimes have different effects on pulse rate on average</p>	Source of Variation	Sum Sq	df	MS	F ratio	F critical	Between Groups	448.8167	2	224.41	3.1885	3.8056	Within Groups	914.9333	13	70.379			Total	1363.75	15				<p>B1 B1 B1 B1 B1 E1 B1 B1 M1A1 M1A1 A1Ft B1 A1Ft B1 M1 A1 [18]</p>	<p>Or $\mu_i - \mu$ Accept “residual” Context explained at least once Distributional assumption Or: $\alpha_1 = \alpha_2 = \alpha_3 (= 0)$ Not all α_i the same Ft their Total SS-BGSS Df all 3 F ratio Ft their Sum Sqs F critical; 3.81 from tables</p>
Source of Variation	Sum Sq	df	MS	F ratio	F critical																						
Between Groups	448.8167	2	224.41	3.1885	3.8056																						
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Question		Answer	Marks	Guidance	
	(ii)	<p>The analysis using three tests is not equivalent to ANOVA, and the multiple comparisons procedure is worse than ANOVA</p> <p>The three tests are not independent</p> <p>The significance level of the whole procedure is therefore impossible to assess</p> <p>A comparison with the different result obtained in (i), and why this may be so.</p>	<p>B1</p> <p>B1</p> <p>B1</p> <p>B1</p> <p>B1</p> <p>B1</p> <p>[6]</p>	Other points could be made	<p>eg</p> <p>Multiple comparisons are likely to generate more type I errors than the nominal significance level would suggest.</p> <p>However, multiple comparisons are useful post hoc to identify where the largest differences have occurred</p>
			[24]		

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