

Wednesday 30 May 2012 – Afternoon

FSMQ ADVANCED LEVEL

6993 Additional Mathematics

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:

- Printed Answer Book 6993

Other materials required:

- Scientific or graphical calculator

Duration: 2 hours



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given correct to three significant figures where appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **100**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

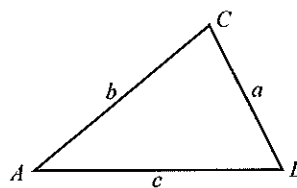
INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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Formulae Sheet: 6993 Additional Mathematics

In any triangle ABC

Cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$



Binomial expansion

When n is a positive integer

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \dots + \binom{n}{r} a^{n-r}b^r + \dots + b^n$$

where

$$\binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

Section A

1 (i) Find the range of values of x satisfying $x^2 - 4x + 3 \leq 0$. [3]

(ii) Show this range on the number line provided. [1]

2 A die has 6 faces numbered one to six. The die is biased so that when it is thrown the probability of obtaining a six is $\frac{1}{5}$.

The die is thrown 5 times.

Find the probability of obtaining

(i) at least 1 six, [2]

(ii) exactly 3 sixes. [4]

3 The function $f(x) = x^3 + ax + 6$ is such that when $f(x)$ is divided by $(x - 3)$ the remainder is 12.

(i) Show that the value of a is -7 . [2]

(ii) Factorise $f(x)$. [3]

4 A car moves from rest with constant acceleration on a straight road. When the car passes a point A it is travelling at 10 m s^{-1} and when it passes a point B further along the road it is travelling at 16 m s^{-1} .

The car takes 10 seconds to travel from A to B.

Find

- the distance AB,
- the constant acceleration. [4]

5 (i) Show that the equation $3\cos^2\theta = \sin\theta + 1$ can be written as $3\sin^2\theta + \sin\theta - 2 = 0$. [2]

(ii) Solve this equation to find values of θ in the range $0^\circ < \theta < 360^\circ$ that satisfy

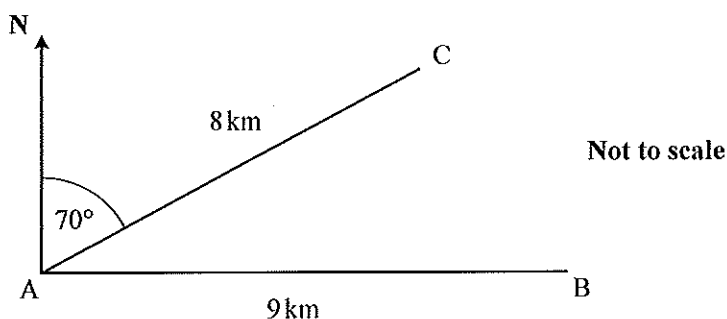
$$3\cos^2\theta = \sin\theta + 1. \quad [4]$$

6 The equation of a curve is $y = 2x^3 - 9x^2 + 12x$.

(i) Show that the curve has a stationary point where $x = 2$. [4]

(ii) Determine whether the stationary value where $x = 2$ is a maximum or minimum. [2]

7 A yachtsman wishes to sail from a port, A, to another port, B, which is 9 km due East of A. Because of the wind he is unable to sail directly East and sails 8 km on a bearing of 070° to point C.



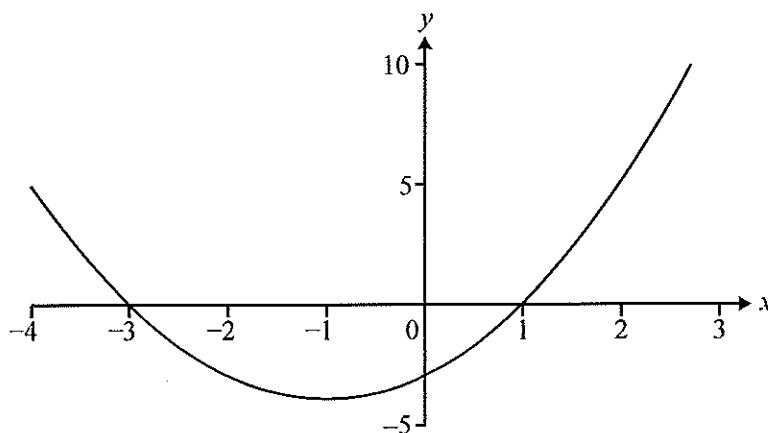
Calculate

(i) the distance he is now from port B, [3]

(ii) the angle ABC and hence the bearing on which he must sail to reach port B from point C, correct to the nearest degree. [4]

- 8 (i) Show that $\int_0^2 (x^2 + 2x - 3) dx = \frac{2}{3}$. [3]

The diagram shows part of the curve $y = x^2 + 2x - 3$.



- (ii) Marc claims that the total area between the curve, the x -axis and the lines $x = 0$ and $x = 2$ is $\frac{2}{3}$.

Explain why he is wrong.

[1]

- (iii) Calculate the total area between the curve, the x -axis and the lines $x = 0$ and $x = 2$.

[3]

- 9 The height above the ground of a seat on a fairground big wheel is h metres. At time t minutes after the wheel starts, h is given by

$$h = 7 - 5\cos(480t)^\circ.$$

- (i) Write down the initial height above the ground of the seat (when $t = 0$). [1]
- (ii) Find the greatest height reached by the seat. [2]
- (iii) Calculate the time of the first occasion when the seat is 9 metres above the ground. Give your answer correct to the nearest second. [4]

Section B

10 A (1, 10), B (8, 9) and C (7, 2) are three points.

(i) Find the coordinates of the midpoint, M, of AC. [1]

(ii) Find the equation of the circle with AC as diameter. [4]

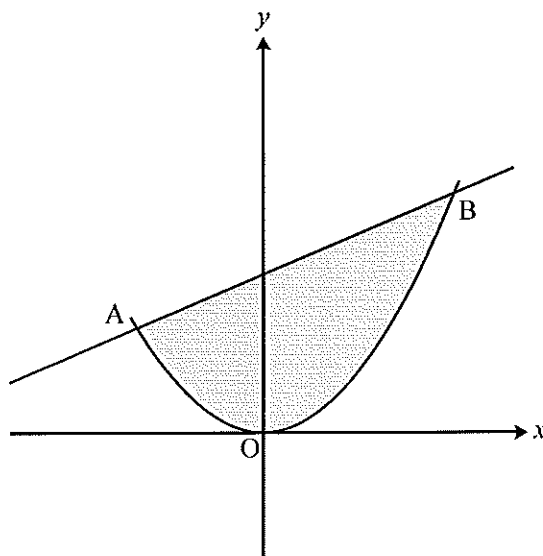
(iii) Show that B lies on this circle. [1]

(iv) Prove that AM and BM are perpendicular. [3]

(v) BD is a diameter of this circle. Find the coordinates of D. [3]

11 The shaded region in the diagram shows a wooden shape.

The curve has equation $y = \frac{1}{2}x^2$ and the coordinates of A are (-2, 2).



The line AB is the normal to the curve at the point A.

(i) Find the equation of the line AB. [5]

(ii) Find the coordinates of the point B where the line AB meets the curve again. [3]

(iii) Find the shaded area. [4]

- 12 The Highway Code gives a table of shortest stopping distances (d feet) for a vehicle travelling at v miles per hour.

The formula used for this table is given by

$$d = av^2 + bv.$$

Two entries in the table are given below.

v mph	d feet
30	75
60	240

- (i) By forming and solving a pair of simultaneous equations in a and b , show that the formula is

$$d = \frac{v^2}{20} + v. \quad [5]$$

- (ii) Find the difference between the stopping distances for a car travelling at 65 mph and a car travelling at 70 mph. [3]
- (iii) Many drivers maintain a distance of 50 feet or less when driving on a motorway.

Use the formula in part (i) to find the speed at which the shortest stopping distance is 50 feet. [4]

Question 13 is printed overleaf

- 13 (i) Find the coefficients a , b and c in the expansion

$$(2 + h)^3 = 8 + ah + bh^2 + ch^3. \quad [3]$$

- (ii) The graph of the equation $y = x^3$ passes through the points P and Q which have x -coordinates 2 and $2 + h$ respectively.

Show that the gradient of the chord PQ is $\frac{(2 + h)^3 - 8}{h}$. [3]

- (iii) Express $\frac{(2 + h)^3 - 8}{h}$ as a quadratic function of h . [2]

- (iv) As the value of h decreases, the point Q gets closer and closer to the point P on the curve. As h gets closer to 0 the chord PQ gets closer to being the tangent to the curve at P.

Deduce the value of the gradient of the tangent at P. [1]

- (v) Kareen uses the same method to deduce the value of the gradient of the tangent at the point (2, 16) on the curve $y = x^4$.

The first three lines of her working are given below and in the answer booklet.

Take P to be the point (2, 16)

Take Q to be the point (2 + h, (2 + h)⁴)

The gradient of the chord PQ is given by $\frac{(2+h)^4 - 16}{h} =$

Complete Kareen's working. [3]

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Additional FSMQ

Free Standing Mathematics Qualification

6993: Additional Mathematics

Mark Scheme for June 2012

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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Annotations and abbreviations

Annotation in scoris	Meaning
✓ and ✕	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	Meaning
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions

- a Annotations should be used whenever appropriate during your marking.
- b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

- c The following types of marks are available.

M

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B

Mark for a correct result or statement independent of Method marks.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

- f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.
- g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

- h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

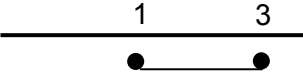
Viewing tips for this paper

In general, set your screen to 'fit width.'

You may find it helpful to set to 'fit height' for the some questions:

[if you set a view, it stays for subsequent scripts]. If the writing is too small, you may wish to zoom in.

Section A

Question		Answer	Marks	Guidance
1	(i)	$(x \pm 1)(x \pm 3) (\leq 0)$ $\Rightarrow 1 \leq x \leq 3$ www	M1 A1 A1 [3]	SC Test integers and select 1 and 3 B1 Accept $x \leq 3$ and $1 \leq x$ Or : from 1 to 3 inclusive (must imply inclusion of end points).
		Alternative: Draw curve for parabola the right way up Correct points on x-axis answer	M1 A1 A1	
	(ii)		B1 [1]	Filled in circles must be evident. SC B1 if correct but M0 in (i). Accept alternative conventions. Answer must be a range (ie just a set of points is 0).

Question		Answer	Marks	Guidance
2	(i)	$= 1 - \left(\frac{4}{5}\right)^5$ $= 0.672(32) = \frac{2101}{3125}$	M1 A1 [2]	$1 - p^5$ <p>p does not have to be 0.8 for this mark but the power must be 5. (ie p could be 0.2)</p>
		Alternative: P(1) + ... + P(5) 5 terms added, each term with powers correct M1 Answer A1	Condone missing coeffs for M1	Terms are: 0.4096, 0.2048, 0.0512, 0.0064, 0.00032.
	(ii)	$10p^3q^2 = 10 \times 0.2^3 \times 0.8^2$ $= 0.0512 = \frac{32}{625} \text{ www}$	M1 B1 A1 A1 [4]	Must include powers of p and q and $\binom{5}{3}$ or 5C_3 (which need not be evaluated Powers Coefficient soi Accept 0.051 but not 0.05 Can be obtained by listing.

Question		Answer	Marks	Guidance
3	(i)	$f(3) = 12$ $\Rightarrow 27 + 3a + 6 = 12$ $\Rightarrow 3a = -21$ $\Rightarrow a = -7$	M1 A1 [2]	
		Alternative: Substitute $a = -7$ M1 and show that $R = 12$ A1		If this method is used then if long division is used then $x^3 - 3x^2$ must be seen. NB Answer given so long division must be totally correct for A1
	(ii)	$f(1) = 0$ or $(x - 1)$ seen $\Rightarrow f(x) = (x - 1)(x - 2)(x + 3)$	M1 A1 A1 [3]	Divide, try factor theorem for at least one value, or obtain a 3-term quadratic factor by inspection. Using or getting a correct factor or root Answer Divide means you need to see the x^2 in the quotient and x^3 and x^2 terms correct in the initial dividing line.

Question	Answer	Marks	Guidance
4	$s = \left(\frac{u+v}{2} \right) t \Rightarrow s = 13 \times 10 = 130$ <p style="text-align: right;">www</p> $v = u + at \Rightarrow a = \frac{16-10}{10} = 0.6$ <p style="text-align: right;">www</p>	<p>M1 A1</p> <p>M1 A1</p> <p style="text-align: center;">[4]</p>	<p>In any order using any valid formulae. Ignore units</p> <p>eg $s = \frac{(10+16)}{2} \times 10$</p> <p>eg $16 = 10 + 10a$ Alternative order: $a = \frac{16-10}{10} = 0.6$ $\Rightarrow s = 10 \times 10 + \frac{1}{2} \times 0.6 \times 10^2 = 130$</p> <p>MR $u = 0$ and $v = 10$ gives $s = 50$, $a = 1$ Or $u = 0$ and $v = 16$ gives $s = 80$ and $a = 1.6$ M1 A0 M1 A0</p>

Question		Answer	Marks	Guidance
5	(i)	$3 - 3\sin^2\theta = \sin\theta + 1$ $\Rightarrow 3\sin^2\theta + \sin\theta - 2 = 0$ www	M1 A1 [2]	Sight of and use of $\cos^2\theta = 1 - \sin^2\theta$ Must see = 0 NB answer given
	(ii)	$(3\sin\theta - 2)(\sin\theta + 1) = 0$ $\Rightarrow \sin\theta = -1$ or $\sin\theta = \frac{2}{3}$ $\Rightarrow \theta = 270^\circ, 41.8^\circ, 138.2^\circ$	M1 A1 A2 [4]	Solve to obtain $\sin\theta = \pm 1$ or $\sin\theta = \pm \frac{2}{3}$ Sight of both values All 3 with no extras in range Ignore -90° A1 for one or two values Or: all 3 values correct but extra values in range. Anything that rounds to 41.8° and 138° Allow 138° but not 42° SC2 $\sin\theta = 1, -\frac{2}{3}$ $\Rightarrow \theta = 90^\circ, 318.2^\circ, 221.8^\circ$ (only) (Allow 318° and 222°)

Question		Answer	Marks	Guidance
6	(i)	$\frac{dy}{dx} = 6x^2 - 18x + 12$ <p>When $x = 2$, $\frac{dy}{dx} = 24 - 36 + 12 = 0$</p>	M1 A1 M1 dep A1 [4]	Differentiation All three terms Sub $x = 2$ into or factorise their derived function. Get 0 or set = 0 and get 2. At least 2 terms with powers reduced by 1 (NB: beware division by x). Do not condone division by 6 before substituting for x. NB answer given. Numerical values must be seen Second M1 dep on first M1
	(ii)	$\frac{d^2y}{dx^2} = 12x - 18$ <p>When $x = 2$, $\frac{d^2y}{dx^2} > 0$ giving a minimum</p>	M1 A1 [2]	Diffn their derived function correctly. BOD no arithmetic computations seen. Using the function $2x - 3$ can earn M1 A0.
		Alternative: Sign of gradient either side of $x = 2$ M1 Or: Values of y either side of $x = 2$ and the value of y at stationary point. M1 Correct answer (provided l.h. $x > 1$) A1		Allow sketch of function indicating left stationary value is maximum and right one is minimum. For A1 LH x greater than 1

Question		Answer	Mark	Guidance
7	(i)	$(CB^2 =) 8^2 + 9^2 - 2 \times 8 \times 9 \times \cos 20$ $= 9.684$ $\Rightarrow CB = 3.11$	M1 A1 A1 [3]	8, 9 must be used, any angle soi Anything that rounds to 3.11 Ignore units
	(ii)	$\frac{\sin ABC}{8} = \frac{\sin their\ 20}{their\ 3.11}$ $\Rightarrow \sin ABC = 0.879$ $\Rightarrow ABC = 61.55^\circ$ $\Rightarrow \text{Bearing} = 152^\circ$	M1 A1ft A1 A1ft [4]	Correct application of sine rule Must be same angle as used in (i) and their CB Anything that rounds to 62° www Anything that rounds to 152° 90 + their ABC
		Alternative methods: Cosine Rule: $\cos ABC = \frac{9^2 + their\ CB^2 - 8^2}{2 \times 9 \times their\ CB}$ $= 0.4767$ M1 A1ft Then angle and bearing A1 A1ft OR: Perpendicular from C and use of sin twice M1 $h = 8 \sin their\ 20 = 2.736$ A1ft $\sin ABC = \frac{2.736}{their\ CB}$ Then angle and bearing M1 A1ft Or: Find other angle by sine rule M1 A1 Angle ACB = 98.45 giving ABC = 61.55 A1 Bearing = $180 - (98.45 - 70) = 152$ A1ft		Correct application of cos rule Must be same angle as used in (i) and their CB NB Question asks for ABC so if not found 3/4 Angle = 81.55 can earn M1 A0 A0 (for ABC) A1ft only

Question		Answer	Marks	Guidance
8	(i)	$\int_0^2 (x^2 + 2x - 3) dx = \left[\frac{x^3}{3} + x^2 - 3x \right]_0^2$ $= \left(\frac{8}{3} + 4 - 6 \right) - (0) \quad \text{oe}$ $= \frac{2}{3} \quad \text{www}$	M1 A1 A1 [3]	Integrate All three terms Completion to $\frac{2}{3}$. Test for integration is “are there at least two terms with the power increased by 1?” Care that the process is not just multiplying each term by x . Working must be seen as the answer is given. Ignore absence of “- 0”.
	(ii)	Because the curve crosses the x-axis in the range	B1 [1]	Because one bit is +ve and the other is -ve. Any reference to $x = -3$ will be 0. If there is an additional statement give 0.
	(iii)	$\left[\frac{x^3}{3} + x^2 - 3x \right]_0^1 \quad \text{or} \quad \left[\frac{x^3}{3} + x^2 - 3x \right]_1^2$ $= \pm 1 \frac{2}{3} \quad \text{or} \quad 2 \frac{1}{3}$ $\Rightarrow \text{Total area} = 1 \frac{2}{3} + 2 \frac{1}{3} = 4$	M1 A1 A1 [3]	Calculation of their integral between 0 & 1 or 1 & 2 One of the areas

Question		Answer	Marks	Guidance
9	(i)	$h = 7 - 5 \times \cos 0 = 2$	B1 [1]	
	(ii)	$h = 7 - (-5) = 12$	M1 A1 [2]	Set $\cos \theta = -1$
	(iii)	$9 = 7 - 5 \cos(480t)$ $\Rightarrow \cos(480t) = -0.4$ oe $\Rightarrow 480t = 113.578$ $\Rightarrow t = 0.2366$ $\Rightarrow \text{time} = 0.2366 \text{ mins} = 14 \text{ sec}$	M1 A1 A1 A1 [4]	Substitute $h = 9$ soi Allow 114 leading to $t = 0.2375$

Section B

Question		Answer	Marks	Guidance
10	(i)	(4,6)	B1 [1]	
	(ii)	Distance MC: $\sqrt{(4-7)^2 + (6-2)^2}$ = 5 Equation of circle: $(x-4)^2 + (y-6)^2 = 5^2 (= 25)$	M1 A1 M1 A1 [4]	Attempt to find radius or diameter by pythagoras. soi Must include their M and their r^2 Can be expanded form.
		Alternative: Equation of circle on AC as diameter: $(x-1)(x-7) + (y-10)(y-2) = 0$ $\Rightarrow x^2 - 8x + 7 + y^2 - 12y + 20 = 0$ $\Rightarrow (x-4)^2 + (y-6)^2 = 25$ isw		
	(iii)	B lies on circle as $(8-4)^2 + (9-6)^2 = 16 + 9 = 25$	B1 [1]	Working must be convincing
	(iv)	gradient of AM = $\left(\frac{10-6}{1-4}\right) = \frac{4}{-3}$ gradient of BM = $\left(\frac{9-6}{8-4}\right) = \frac{3}{4}$ Since $\frac{4}{-3} \times \frac{3}{4} = -1$ the lines are perpendicular	B1 B1 B1 [3]	One gradient (need not be simplified) Second gradient (need not be simplified) Demonstration that $m_1 \times m_2 = -1$ is satisfied and all working to derive gradients shown .

Question	Answer	Marks	Guidance
	Alternative: Use of Pythagoras M1 5, 5, $\sqrt{50}$ seen and used A1 Arithmetic correct and final statement A1		Attempt to find all three lengths
(v)	$B \text{ to } M = \begin{pmatrix} -4 \\ -3 \end{pmatrix} \Rightarrow M \text{ to } D = \begin{pmatrix} -4 \\ -3 \end{pmatrix}$ $\Rightarrow D \text{ is } (0,3)$	M1 A1, A1 [3]	Idea of BM = MD soi Each value
	Alternative: Centre as midpoint: Idea M1 $\left(\frac{8+x}{2}\right) = 4 \Rightarrow x = 0$ $\left(\frac{9+y}{2}\right) = 6 \Rightarrow y = 3$ Each value A1 A1		
	Alternative: Equation BM is $y = \frac{3}{4}x + 3$ Sub in eqn for circle $\Rightarrow x^2 - 8x = 0$ $\Rightarrow x = 0$ Sub to give $y = 3$ Idea M1 Each value A1 A1		

Question		Answer	Marks	Guidance
11	(i)	$\frac{dy}{dx} = x$ At A gradient of tangent = -2 so gradient of normal = $\frac{1}{2}$. \Rightarrow Eqn of AB is $y - 2 = \frac{1}{2}(x + 2)$ $\Rightarrow 2y = x + 6$ oe	M1 A1 A1ft M1dep A1 [5]	Differentiation If no differentiation then 0/5 Follow through their gradient of tangent. Using $(-2, 2)$ and their normal gradient 3 terms only
	(ii)	line meets curve when $x^2 = x + 6$ $\Rightarrow x^2 - x - 6 = 0$ $\Rightarrow (x - 3)(x + 2) = 0$ \Rightarrow At B $x = 3, y = \frac{9}{2}$	M1 A1 A1 [3]	Equate <i>their</i> straight line to given curve. Quadratic
	(iii)	Area between = Area under line – area under curve = $16.25 - 5.833 = 10.4$ = $10\frac{5}{12}$	M1 M1 M1dep A1 [4]	Attempt to evaluate area under curve by integration soi Attempt to evaluate area under their straight line by trapezium or integration soi Subtracting areas, dep on both M marks Answer Seen by power increased by 1. Care not to multiply by x <i>Ignore absence of limits for first 3 marks</i>

Question		Answer	Marks	Guidance
12	(i)	Substitute: $75 = 900a + 30b$ $240 = 3600a + 60b$ Solve: $\Rightarrow a = \frac{1}{20}, b = 1 \Rightarrow d = \frac{1}{20}v^2 + v$	B1 B1 M1 A1 A1 [5]	Allow unsimplified coefficients Solve <i>a</i> <i>b</i> ie equal coefficients and subtract or correct substitution. NB Answers given so algebra for first value found must be convincing.
	(ii)	$D = \left(\frac{4900}{20} + 70 \right) - \left(\frac{4225}{20} + 65 \right)$ $= 38.75$	M1 A1 A1 [3]	Calculation at each value and subtraction attempted For either 315 or 276.25 soi Allow 38.8 Or 33.75 or 5
	(iii)	Substitute: $50 = \frac{1}{20}v^2 + v$ or $v^2 + 20v - 1000 = 0$ $\Rightarrow v = \frac{-20 \pm \sqrt{400 + 4000}}{2}$ $\approx 23.2 \text{ mph}$	M1 A1 M1 A1 [4]	Substitute Quadratic (in any form) isw Solving their quadratic using correct formula or completion of square M1 A1 or B2 answer with no working Correct application of completion of square is $(v + 10)^2 = k$ seen SCM1 for trial and improvement with values between 20 and 25. A1 ans correct to 3 sf SCB2 If answer given with no quadratic. Final answer is anything that rounds to 23.2 <i>Ignore negative values</i>

Question		Answer	Marks	Guidance
13	(i)	$(2+h)^3 = 8 + 3.4h + 3.2h^2 + h^3$ $= (8+)12h + 6h^2 + h^3$	B1 B1 B1 [3]	For each coefficient or term that is correct Ignore incorrect identification of coefficients after expansion Mark final line ie allow answer left in simplified expansion form.
	(ii)	$\frac{(2+h)^3 - 2^3}{(2+h) - 2}$ $\text{Gradient} = \frac{(2+h)^3 - 8}{2+h-2} = \frac{(2+h)^3 - 8}{h}$	B1 B1 B1 [3]	Change in y Change in x Only award if you are satisfied that the algebra is correct Accept description in words
	(iii)	$\frac{(2+h)^3 - 8}{h} = \frac{8 + 12h + 6h^2 + h^3 - 8}{h}$ $= \frac{12h + 6h^2 + h^3}{h} = 12 + 6h + h^2$	M1 A1 [2]	Or using their part (i)
	(iv)	<i>Their 12 in (iii)</i>	B1 [1]	Dependent on (iii) being a polynomial. This answer must be consistent with (iii)
	(v)	$(2+h)^4 = 16 + 32h + 24h^2 + 8h^3 + h^4$ Gradient of chord = $32 + 24h + 8h^2 + h^3$ Giving 32www	B1 B1 B1 [3]	Allow $16 + 32h +$ (higher orders of h) Allow $32 +$ (higher orders of h) Dependent on previous work

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Additional Mathematics

Free Standing Mathematics Qualification **6993**

OCR Report to Centres

June 2012

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of candidates of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, OCR Nationals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

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This report on the examination provides information on the performance of candidates which it is hoped will be useful to teachers in their preparation of candidates for future examinations. It is intended to be constructive and informative and to promote better understanding of the specification content, of the operation of the scheme of assessment and of the application of assessment criteria.

Reports should be read in conjunction with the published question papers and mark schemes for the examination.

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CONTENTS

Additional Mathematics FSMQ (6993)

OCR REPORT TO CENTRES

Content	Page
Additional Mathematics – 6993	1

Additional Mathematics – 6993

General Comments

Performance was similar to last year. A creditable number of candidates scored a mark above 90. However, it continues to cause distress to see that 10% of candidates scored 10 or less. There remains a significant number of candidates who appear to have been entered inappropriately; this could not have been a good experience for them.

Algebraic and calculus notation was often poor; comments on individual questions will indicate where this is not simply a matter of sloppy writing, but results in the loss of (sometimes many) marks.

There were a number of equations to be solved and it was surprising to find a number of candidates resorting to trial and improvement methods to find a solution. In some questions the method of solution was part of the test and so these marks could not be awarded. In a significant number of cases, while the process seemed to be successful, the final result was not given to the right number of significant figures and so more marks were lost. At this level, candidates should be comfortable with using the formula or using factorisation methods to find the roots of a quadratic equation, for instance.

Comments on Individual Questions

Section A

- 1 (i) A significant number of candidates started the paper with a correct solution. The most common error was to check just integers. A few were unable to factorise the quadratic expression.
(ii) A follow through mark was allowed here; a number of candidates did not indicate the ends of the range correctly.
- 2 (i) Generally this was correct, though a number misread the question and found $P(\text{exactly } 1)$. A small number added five terms rather than take one term from 1.
(ii) Generally well done. Only a few failed to include the coefficient or get the powers wrong. A few approximated their answer to only 1 significant figure.
- 3 (i) Those that applied the factor theorem usually obtained the correct result. Many of those who depended on long division failed to get the algebra correct. This is one of a number of "show that..." in the paper. Candidates need to be aware that in these situations it is crucial that every step is shown in order to convince the examiner that the correct process is being used to obtain the result. Candidates who miss out essential steps will not gain full marks.
(ii) A significant number of candidates did not read the question properly and gave the answer to the question "solve $f(x) = 0$ " rather than simply factorise $f(x)$. The most common error was to start with the expression $(x - 3)$, presumably because it was given in part (i), but ignoring the fact that in part (i) this expression was shown not to be a factor.
- 4 Many candidates were able to obtain at least 2 out of the 4 marks for either the distance or the acceleration. A common mistake was to use different initial speeds for the two parts of the question or take the initial speed to be 0 for both parts. Candidates need to ensure they are familiar with the formulae required as there were several instances of wrong formulae quoted.
- 5 (i) Candidates found this question difficult. Many did not know the formula required to prove this part. The two most common mistakes for those who had an idea of the correct formula were to use $\cos^2 \theta = \sin^2 \theta - 1$ or to use the correct formula but not deal with the factor of 3 correctly. Signs were not considered carefully enough resulting in an incorrect

- equation which was then corrected by changing the signs.
- (ii) Many candidates did not recognise this as a quadratic they could solve by factorising. Of those who did and went on to solve it correctly, many had a problem with the negative root and decided that -90° was not a root and so discarded it rather than adding 360° to it. Other mistakes included not realising that the positive value has two answers.
- 6 (i) Most candidates were able to differentiate accurately. Candidates then either substituted $x = 2$ to obtain a value of 0 for the gradient or they set the derived function equal to 0 and solved the resulting quadratic equation.
- (ii) Approaches to the second part of the question were more varied. Values of the gradient or y either side of the stationary point need to be “close to” in order to avoid difficulties with the other turning value so using the value $x = 1$ was not given credit. It is worth noting that the use of the second derivative to determine the nature of a stationary point is not stated in the specification and is not a requirement. However, it is a perfectly acceptable method and the vast majority of candidates took this route. An essential part of the process is to be able to assert that the second derivative is positive (ie the gradient function is an increasing function at the point where it is zero). For full marks, therefore, it was necessary for candidates to confirm the point that $6 > 0$.
- 7 (i) The most common error in this part was to take the angle of the triangle at A to be 30° and not 20° .
- (ii) Many candidates found the angle ACB rather than ABC. While this was used successfully to find the bearing, the question asked for the angle ABC and so a mark was lost by some candidates. A number could not determine the bearing from their angle.
- 8 (i) For a convincing demonstration of the answer that was given, the full working of the integral when $x = 2$ was necessary. While the omission of the working for $x = 0$ was condoned, the jump to the answer given from the integrand was not enough for full marks.
- (ii) Some reference to the fact that the curve crossed the axis in the interval and that the values of the area cancelled themselves was necessary. A number of candidates misunderstood the question and referred instead to the fact that the curve crossed the axis between $x = -3$ and $x = 1$ and so the limits were wrong.
- (iii) A good number of candidates obtained the correct answer to this part.
- 9 (i) Most were able to set $t = 0$ to find the answer.
- (ii) Very few, however, were able to set $\cos(480t)$ to -1 for this part. Many decided that $t = 1$ would result in the maximum height without further check.
- (iii) A number of arithmetic and trigonometrical errors in this part prevented many from obtaining full marks. For some the negative sign was lost.

Section B

- 10 A question that, in part, tested some core skills relating to coordinate geometry but also included some testing parts that challenged many.
- (i) Most candidates could find the midpoint.
- (ii) Likewise, the value of the radius was usually found and the equation of the circle obtained.
- (iii) The arithmetic seen was usually accepted, though few candidates explained what they were doing or came to a conclusion.
- (iv) The product of the two gradients was to be -1 for verification that the lines were perpendicular. Unfortunately, without adequate working it was not possible to discern whether candidates had used a circular argument to obtain the result. This was a clear example of the need to show all working.
- (v) A few took the formula for the midpoint between two points to find the other end of the line. A few more worked the vector idea successfully. Rather too many found the equation of the line BM and then did not know how to proceed. By substituting and

- solving the resulting quadratic to obtain the coordinates of B and D the required result could have been obtained, and a few did so, but it involved rather more work than the other methods for the 3 marks allocated.
- 11 The question was clearly worded and most candidates knew what was required of them. A good 3 part question which provided a full range of marks with only the very best candidates getting close to 12 marks. Of those who failed to score more than nine marks most went wrong on part (i) and failed to find the correct equation of line AB.
- (i) Quite a few candidates failed to differentiate here. Of those who did differentiate successfully a significant number failed to substitute $x = -2$ to obtain the gradient. Many candidates did not know the relationship between the gradient of a tangent and that of a normal.
 - (ii) Most candidates realised that they should equate their line with the quadratic and most who had the equation of AB correct could see this through to a correct answer. For some, the fact that the coordinates of A were given was a useful check in part (ii) when finding the coordinates of B and some recognised this in their answer.
 - (iii) Most candidates at all levels understood that the method was to subtract the area under the curve from that under the line. However, there were very few fully correct answers even from candidates who had the correct equation for AB, with a failure to use limits in integration accurately.
- 12 Strong candidates had few problems and scored full marks. Conversely, weak candidates had little idea and were only able to tackle part (ii).
- (i) Although many candidates set up the equations correctly and realised how to solve, there were as many candidates who immediately created problems for themselves. It was not uncommon to see 30^2 and 60^2 unevaluated until one of the variables was eliminated. Some continued to have these unevaluated when eliminating and misconceptions continued: doubling 30^2 say was often given as 60^2 . Some candidates seemed to think av^2 meant a^2v^2 .
 - (ii) This was found to be a straightforward source of 3 marks by almost all. Some forgot to subtract, others misread, others insisted their (incorrect) formula generated in (i) was the correct formula to use rather than the printed one!
 - (iii) Most gained the first two marks for substitution. The next stage was not done well: it was common to see candidates multiply through by 20 and write $v^2 + v - 1000 = 0$ (or equivalent).
The solving process was generally done well by those who used the formula or completed the square on *their* quadratic equation. The trial and improvement method was very much in evidence in this question.
- 13 As was expected from the last question on the paper a majority of candidates struggled with this non-standard question on the procedure of differentiation from first principles..
- (i) The most common mistake was to use Pascal's triangle to acquire the coefficients in the expansion but incorrectly thinking that the expansion being asked for was $(1 + h)^3$ rather than $(2 + h)^3$ so $a = 3$, $b = 3$ and $c = 1$ was quite common.
 - (ii) This part was done very well by those who realised that they had to first find the y coordinates of P and Q and then consider that the gradient between the two points on the chord is simply the change in y values divided by the change in x values. As this was a 'show that' it was expected that candidates would do a little bit more than simply stating the numerator (which was given).
 - (iii) Many good candidates did not realise that all that was required was to substitute their expansion from part (i) into the gradient of the chord given in (ii) to obtain a quadratic expression in h . Instead they started the expansion of $(2 + h)^3$ from scratch and in a number of cases obtained a different answer than the one they gave in (i). Other errors from those that attempted this part was not to cancel the 8s and so their answer was no longer polynomial; a number multiplied by h rather than dividing by h and, although not penalised, many put their quadratic expression equal to 0.

- (iv) All this part required was for the candidate to substitute $h = 0$ into their quadratic function found in (iii) but many found the more formal mathematical language used in this part to be beyond their comprehension
- (v) Those that understood what was required in the first four parts of this question had no difficulty in deriving the required answer. Many managed to score the first two marks for arriving at the correct expression for the gradient of the chord but once again failed to obtain the correct answer of 32 by again setting $h = 0$.

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