OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE UNIT **4751/01**

MATHEMATICS (MEI)

Introduction to Advanced Mathematics (C1)

TUESDAY 16 JANUARY 2007

Morning
Time: 1 hour 30 minutes

Additional materials:
Answer booklet (8 pages)
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are **not** permitted to use a calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- There is an **insert** for use in Question **11**.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

WARNING

You are not allowed to use a calculator in this paper

This document consists of **4** printed pages and an insert.

[Turn over

Section A (36 marks)

1 Find, in the form $y = ax + b$, the equation of the line through $(3, 10)$ which is parallel to $y = 2x + 7$. [3]

2 Sketch the graph of $y = 9 - x^2$. [3]

3 Make a the subject of the equation

$$2a + 5c = af + 7c.$$ [3]

4 When $x^3 + kx + 5$ is divided by $x - 2$, the remainder is 3. Use the remainder theorem to find the value of k. [3]

5 Calculate the coefficient of x^4 in the expansion of $(x + 5)^6$. [3]

6 Find the value of each of the following, giving each answer as an integer or fraction as appropriate.

(i) $25^{\frac{3}{2}}$ [2]

(ii) $\left(\frac{7}{3}\right)^{-2}$ [2]

7 You are given that $a = \frac{3}{2}$, $b = \frac{9 - \sqrt{17}}{4}$ and $c = \frac{9 + \sqrt{17}}{4}$. Show that $a + b + c = abc$. [4]

8 Find the set of values of k for which the equation $2x^2 + kx + 2 = 0$ has no real roots. [4]

9 **(i)** Simplify $3a^3b \times 4(ab)^2$. [2]

(ii) Factorise $x^2 - 4$ and $x^2 - 5x + 6$.

Hence express $\frac{x^2 - 4}{x^2 - 5x + 6}$ as a fraction in its simplest form. [3]

10 Simplify $(m^2 + 1)^2 - (m^2 - 1)^2$, showing your method.

Hence, given the right-angled triangle in Fig. 10, express p in terms of m, simplifying your answer. [4]

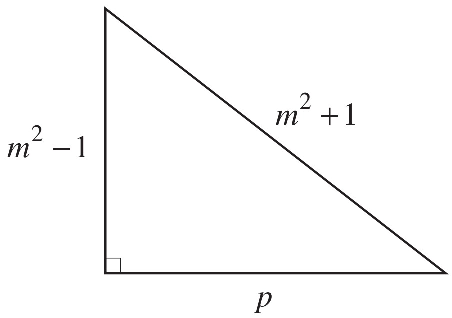


Fig. 10

Section B (36 marks)

11 **There is an insert for use in this question.**

The graph of $y = x + \frac{1}{x}$ is shown on the insert. The lowest point on one branch is $(1, 2)$. The highest point on the other branch is $(-1, -2)$.

(i) Use the graph to solve the following equations, showing your method clearly.

(A) $x + \frac{1}{x} = 4$ [2]

(B) $2x + \frac{1}{x} = 4$ [4]

(ii) The equation $(x - 1)^2 + y^2 = 4$ represents a circle. Find in exact form the coordinates of the points of intersection of this circle with the y-axis. [2]

(iii) State the radius and the coordinates of the centre of this circle.

Explain how these can be used to deduce from the graph that this circle touches one branch of the curve $y = x + \frac{1}{x}$ but does not intersect with the other. [4]

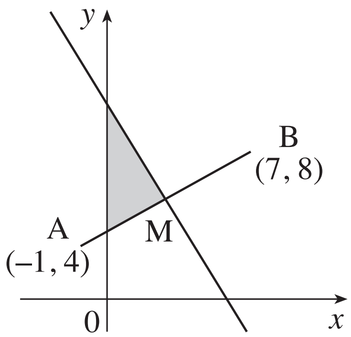
12 Use coordinate geometry to answer this question. Answers obtained from accurate drawing will receive no marks.

A and B are points with coordinates $(-1, 4)$ and $(7, 8)$ respectively.

(i) Find the coordinates of the midpoint, M, of AB.

Show also that the equation of the perpendicular bisector of AB is $y + 2x = 12$. [6]

(ii) Find the area of the triangle bounded by the perpendicular bisector, the y-axis and the line AM, as sketched in Fig. 12. [6]



Not to scale

Fig. 12

13

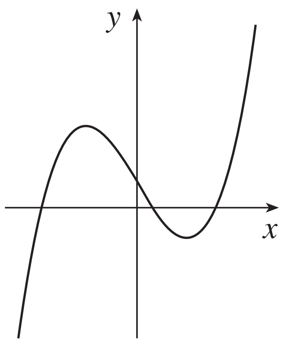


Fig. 13

Fig. 13 shows a sketch of the curve $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = x^3 - 5x + 2$.

(i) Use the fact that $x = 2$ is a root of $\mathrm{f}(x) = 0$ to find the exact values of the other two roots of $\mathrm{f}(x) = 0$, expressing your answers as simply as possible. [6]

(ii) Show that $\mathrm{f}(x - 3) = x^3 - 9x^2 + 22x - 10$. [4]

(iii) Write down the roots of $\mathrm{f}(x - 3) = 0$. [2]

RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE UNIT
MATHEMATICS (MEI)

Introduction to Advanced Mathematics (C1)

4751/01

INSERT

TUESDAY 16 JANUARY 2007

Morning
Time: 1 hour 30 minutes

Candidate Name						
Centre Number						
Candidate Number						

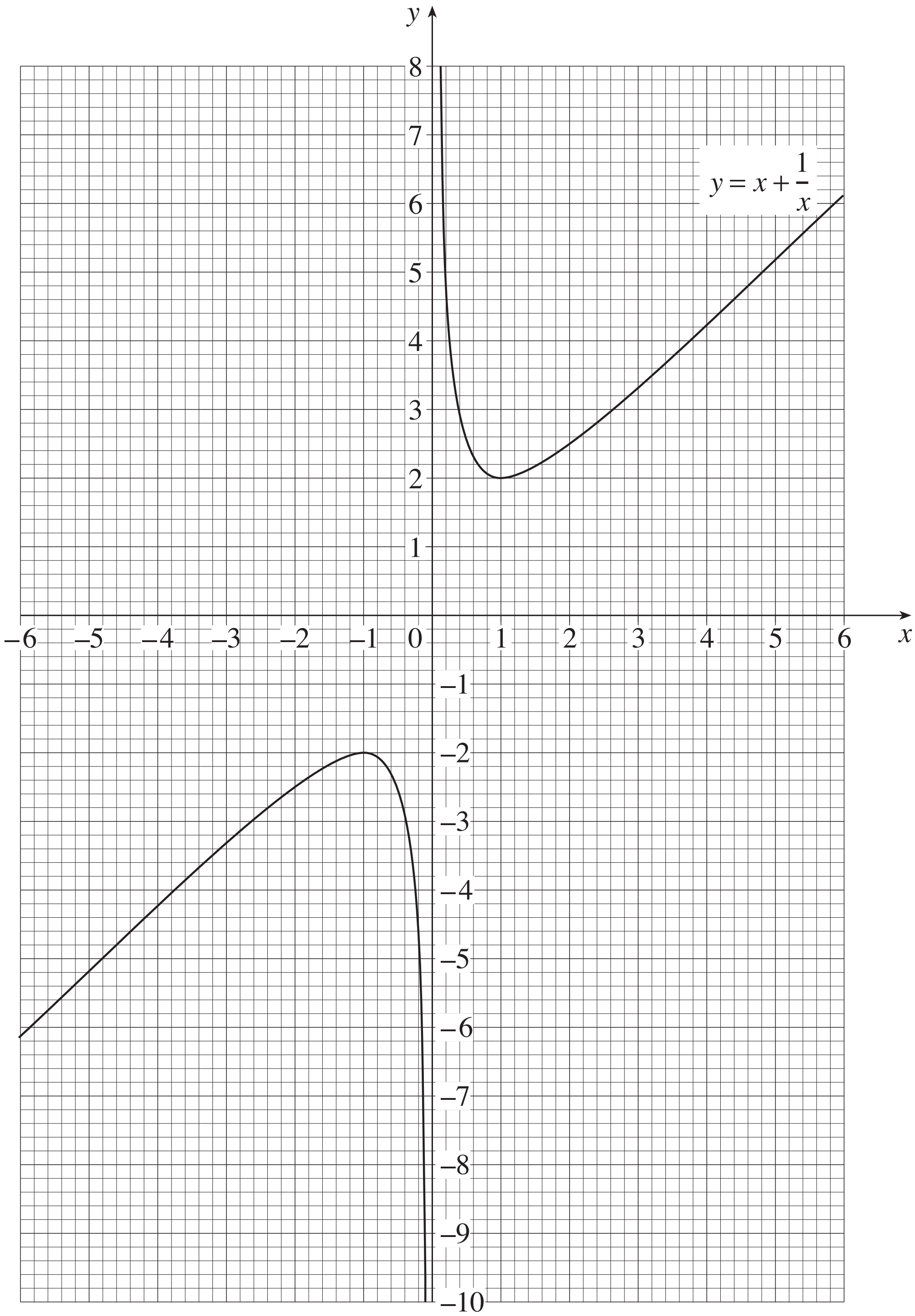
INSTRUCTIONS TO CANDIDATES

- This insert should be used in Question **11**.
- Write your name, centre number and candidate number in the spaces provided above and **attach the page to your answer booklet**.

This insert consists of 2 printed pages.

[Turn over

11 (i)



Mark Scheme 4751
January 2007

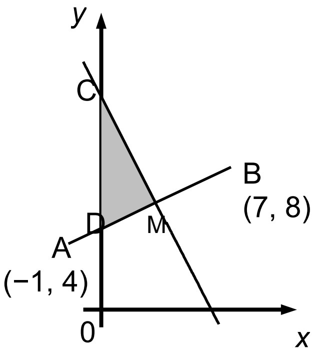
Section A

1	$y = 2x + 4$	3	M1 for $m = 2$ stated [M0 if go on to use $m = -½$] or M1 for $y = 2x + k$, $k \neq 7$ and M1indep for $y - 10 = m(x - 3)$ or (3, 10) subst in $y = mx + c$; allow 3 for $y = 2x + k$ and $k = 4$	3
2	neg quadratic curve intercept (0, 9) <u>through</u> (3, 0) and (−3, 0)	1 1 1	condone (0, 9) seen eg in table	3
3	$[a =]\frac{2c}{2-f}$ or $\frac{-2c}{f-2}$ as final answer	3	M1 for attempt to collect *a*s and *c*s on different sides and M1 ft for $a(2 - f)$ or dividing by $2 - f$; allow M2 for $\frac{7c-5c}{2-f}$ etc	3
4	f(2) = 3 seen or used $2^3 + 2k + 5 = 3$ o.e. $k = -5$	M1 M1 B1	allow M1 for divn by $(x - 2)$ with $x^2 + 2x + (k + 4)$ or $x^2 + 2x - 1$ obtained <u>alt:</u> M1 for $(x - 2)(x^2 + 2x - 1) + 3$ (may be seen in division) then M1dep (and B1) for $x^3 - 5x + 5$ <u>alt</u> divn of $x^3 + kx + 2$ by $x - 2$ with no rem.	3
5	375	3	allow $375x^4$; M1 for 5^2 or 25 used or seen with x^4 and M1 for 15 or $\frac{6\times5}{2}$ oe eg $\frac{6!}{4!2!}$ or 1 6 15 … seen [$^6C_4$ not sufft]	3
6	(i) 125 (ii) $\frac{9}{49}$ as final answer	2 2	M1 for $25^{\frac{1}{2}} = \sqrt{25}$ soi or for $\sqrt{25^3}$ M1 for $a^{-1} = \frac{1}{a}$ soi eg by 3/7 or 3/49	4
7	showing $a + b + c = 6$ o.e $bc = \frac{9^2-17}{16}$ =64/16 o.e. correctly obtained completion showing $abc = 6$ o.e.	1 M1 A1 A1	simple equiv fraction eg 192/32 or 24/4 correct expansion of numerator; may be unsimplified 4 term expansion; M0 if get no further than $\left(\sqrt{17}\right)^2$; M0 if no evidence before 64/16 o.e. may be implicit in use of factors in completion	4

8	$b^2 - 4ac$ soi use of $b^2 - 4ac$ **<** 0 $k^2 < 16$ [may be implied by $k < 4$] $-4 < k < 4$ or $k > -4$ and $k < 4$ isw	M1 M1 A1 A1	may be implied by $k^2 < 16$ deduct one mark in qn for ≤ instead of <; allow equalities earlier if final inequalities correct; condone *b* instead of *k*; if M2 not earned, give SC2 for qn [or M1 SC1] for *k* [=] 4 and − 4 as answer]	4
9	(i) $12a^5b^3$ as final answer (ii) $\frac{(x+2)(x-2)}{(x-2)(x-3)}$ $\frac{x+2}{x-3}$ as final answer	2 M2 A1	1 for 2 'terms' correct in final answer M1 for each of numerator or denom. correct or M1, M1 for correct factors seen separately	5
10	correct expansion of both brackets seen (may be unsimplified), or difference of squares used $4m^2$ correctly obtained [p =] [±]2m cao	M2 A1 A1	M1 for one bracket expanded correctly; for M2, condone done together and lack of brackets round second expression if correct when we insert the pair of brackets	4

Section B

11	**iA**	0.2 to 0.3 and 3.7 to 3.8	1+1	[tol. 1mm or 0.05 throughout qn]; if 0, allow M1 for drawing down lines at both values	2
	iB	$x + \frac{1}{x} = 4 - x$ their $y = 4 - x$ drawn 0.2 to 0.35 and 1.65 to 1.8	M1 M1 B2	condone one error allow M2 for plotting positive branch of $y = 2x + 1/x$ [plots at (1,3) and (2,4.5) and above other graph] or for plot of $y = 2x^2 - 4x + 1$ 1 each	4
	ii	(0, ±√3)	2	condone $y = \pm\sqrt{3}$ isw; 1 each or M1 for $1 + y^2 = 4$ or $y^2 = 3$ o.e.	2
	iii	centre (1, 0) radius 2 touches at (1, 2) [which is distance 2 from centre] all points on other branch > 2 from centre	1+1 1 1	allow seen in (ii) allow ft for both these marks for centre at (−1, 0), rad 2; allow 2 for good sketch or compass-drawn circle of rad 2 centre (±1, 0)	4

12	**i**	(3, 6) grad AB = (8 − 4)/(7 − −1) or 4/8 grad normal = −2 or ft perp bisector is *y* − 6 = −2(*x* − 3) or ft their grad. of normal (not AB) and/or midpoint correct step towards completion	2 M1 M1 M1 A1	1 each coord indep obtained for use of $m_1m_2 = -1$; condone stated/used as −2 with no working only if 4/8 seen or M1 for showing grad given line = −2 and M1 for showing (3, 6) fits given line	 6
	ii	Bisector crosses *y* axis at C (0, 12) seen or used AB crosses *y* axis at D (0, 4.5) seen or used ½ × (12 − their 4.5) × 3 (may be two triangles M1 each) 45/4 o.e. without surds, isw  alt allow integration used: $\int_0^3 (-2x+12)\mathrm{d}x\ [=27]$ obtaining AB is *y* − 8 = their ½ (*x* − 7) oe [*y* = ½ *x* + 4.5] $\int_0^3 (\frac{1}{2}x+4.5)\mathrm{d}x$ = 63/4 o.e. cao their area under CB − their area under AB = 45/4 o.e. cao	M1 B2 M2 A1 M1 M1 M1 A1 M1 A1	may be implicit in their area calcn M1 for 4 + their grad AB or for eqn AB is *y* − 8 = their ½ (*x* − 7) oe with coords of A or their M used or M1 for $[MC]^2 = 3^2 + 6^2$ or 45 or $[MD]^2 = 3^2 + 1.5^2$ or 11.25 oe and M1 for ½ × their MC × MD; all ft their M <u>MR</u>: AMC used not DMC: lose B2 for D but then allow ft M1 for MC^2 or MA^2 $[=4^2 + 2^2]$ and M1 for ½ × MA × MC and A1 for 15 <u>MR</u>: intn used as D(0, 4) can score a max of M1, B0, M2 (eg M1 for their DM = √13), A0 condone poor notation allow if seen, with correct line and limits seen/used as above ft from their AB allow only if at least some valid integration/area calculations for these trapezia seen if combined integration, so 63/4 not found separately, mark equivalently for Ms and allow A2 for final answer	6
13	**i**	*x* − 2 is factor soi attempt at divn by *x* − 2 as far as $x^3 - 2x^2$ seen in working $x^2 + 2x - 1$ obtained attempt at quad formula or comp square $-1\pm\sqrt{2}$ as final answer	M1 M1 A1 M1 A2	eg may be implied by divn or other factor (x^2 ….−1) or (x^2 + 2*x*...) or B3 www ft their quadratic A1 for $\frac{-2\pm\sqrt{8}}{2}$ seen; or B3 www	6

	ii	$f(x-3) = (x-3)^3 - 5(x-3) + 2$ $(x-3)(x^2-6x+9)$ or other constructive attempt at expanding $(x-3)^3$ eg 1 3 3 1 soi $x^3 - 9x^2 + 27x - 27$ $-5x + 15$ [+2]	B1 M1 A1 B1	or $(x-5)(x-2+\sqrt{2})(x-2-\sqrt{2})$ soi or ft from their (i) for attempt at multiplying out 2 brackets or valid attempt at multiplying all 3 alt: A2 for correct full unsimplified expansion or A1 for correct 2 bracket expansion eg $(x-5)(x^2-4x+2)$	4
	iii	5 $2 \pm \sqrt{2}$ or ft	B1 B1	condone factors here, not roots if B0 in this part, allow SC1 for their roots in (i) − 3	2

4751 - Introduction to Advanced Mathematics (C1)

General Comments

As usual, candidates for this paper gained the full range of marks. This time, much of the straightforward work was in section A, with some of the work in section B proving stretching for the most able.

The trend of some centres entering all their C1 candidates in January has continued, so that the mark distribution includes very low marks from some very weak candidates who will need to improve dramatically if they are to succeed at AS Mathematics – such candidates demonstrated little idea of how to proceed on any of the straightforward questions in this paper where E grade candidates would be expected to score. There were also many excellent scripts from strong candidates, although fewer than usual gained full marks, mainly due to performance on question 11(i)(*B*).

The removal of graph paper from the list of additional materials on the front of the paper resulted in a reduction in the number of candidates drawing graphs when requested to sketch them, although some centres still issued graph paper to all the candidates.

In general, time was not an issue. Some candidates petered out towards the end of question 13, but there were few for whom this appeared to be because they had run out of time. As ever, a long method used unnecessarily in any question takes valuable time from other questions.

Comments on Individual Questions

Section A

1 This was generally well-answered, although a few candidates confused parallel and perpendicular lines.

2 Most candidates knew the shape of an inverted quadratic graph and labelled the intersection of the graph with the y-axis. Omitting to label the intersections with the x-axis was a common error.

3 The correct rearrangement of this formula was common, but weaker candidates often failed to realise the need to collect the terms in a and then to factorise. A few candidates did not simplify $7c - 5c$.

4 Those who used the remainder theorem by starting with f(2) = 3 were usually successful. However, some attempted long division, which was difficult in this case and was rarely done successfully.

5 Many successfully calculated the coefficient, although some could not cope with the arithmetic, particularly if starting from ${}^6C_4$, whilst others found the 15 from Pascal's triangle but omitted the 5^2 factor.

6 The first part was well done, with the successful using the strategy $\left(\sqrt{25}\right)^3$. As expected, those who used $\sqrt{25^3}$ rarely got further, whilst poorer candidates interpreted $25^{\frac{3}{2}}$ as $3\sqrt{25}$ or $\sqrt[3]{25^2}$. In part (ii). most candidates knew that the reciprocal was involved, but many could not proceed beyond $\frac{1}{\left(\frac{49}{9}\right)}$ or made errors in squaring one or both terms, such as giving 3/49 as their answer.

7 There were more problems with the fractions than with the surds, with some candidates taking half a page to add the fractions, and the denominator frequently being wrong when the fractions were multiplied. Some used the given answer to discover their errors, but some happily wrongly cancelled their wrong answers to 'obtain' the required result. However, there were also many correct answers, efficiently obtained and showing clearly what the candidates had done.

8 The less able often used trial and error to try to solve the equation. Better candidates used $b^2 - 4ac$ but fewer used clearly the fact than this should be negative. Some of the most able candidates gave $k < 4$ as the solution to $k^2 < 16$, omitting the requirement $k > -4$. Some candidates only worked with $b^2 - 4ac = 0$ and were unable to convert correctly to the desired inequality.

9 The first part was found straightforward by many, but some invented their own algebras instead of using the laws of indices. In the second part, there were many correct solutions, but some 'cancelled' $\frac{x+2}{x-3}$ and gave their final answer as $-\frac{2}{3}$. Some very weak candidates had no idea how to factorise the quadratic expressions.

10 Missing brackets caused the main error – those who expanded separately and then subtracted tended to do better. Those who successfully found $4m^2$ sometimes ignored the 'Hence' and started again, or could not cope with finding the square root of $4m^2$. Final answers such as $p = 4\ m^2$ or $4m$ were common. Some weak candidates who worked carefully picked up several marks here, whilst some stronger but careless candidates lost the accuracy marks.

Section B

11 (i) This part was answered particularly poorly with a high proportion of candidates ignoring the instructions to use the insert and to use the graph to solve the equations. Analytical methods were not accepted here. In part (*A*), many candidates did partly solve the equation $x + \frac{1}{x} = 4$ by reading off at $y = 4$ as expected, but often gave only one of the two roots, not realising that the line intersects the curve twice.

The examiners accept that wording such as 'By drawing a straight line on the graph of $y = x + \frac{1}{x}$, solve the equation…' might have assisted more candidates to know what was required in part (*B*) – it was pleasing to see some good solutions using the line $y = 4 - x$, but these were rare. A few candidates correctly used the graph to plot appropriate values of the curve $y = 2x + \frac{1}{x}$ and read off at $y = 4$, and this was accepted. Many candidates omitted part (*B*).

(ii) This part was done well by many candidates, although giving only the positive solution to $y^2 = 3$ was common. Some found the intersections with the *x*-axis instead of the *y*-axis.

(iii) Most candidates obtained a mark for correctly giving the radius as 2 in this part and somewhat fewer gave (1, 0) as the centre of the circle, with (−1, 0) the minority view. The explaining proved a challenge to many, to the extent that a significant proportion did not attempt it. There were some excellent explanations also, and drawing the circle on the insert was a good way of supporting the case (and deemed sufficient to earn the 2 marks).

12 Most candidates scored more marks on this question than in the other two questions in section B, and a mark in the range 6 to 9 was common.

(i) The vast majority were successful in determining the midpoint of AB. Where errors were made it was either in the arithmetic involving negative numbers, giving an answer of (4, 6), or in using an incorrect 'formula' to give (4,2). Candidates usually made a successful attempt at the rest of part (i). Most found the gradient of AB, then of the line perpendicular to it and then used the coordinates of M to obtain the given equation of the line. A significant number of candidates determined the gradients of AB and the given line, and then showed them to be perpendicular; however, they did not always show that the given line did pass through the midpoint. A few candidates failed to make it clear that they understood that the gradient of the perpendicular bisector was −2, and a few also did not make the use of the product of the gradients explicit. When the answer is given in the question, candidates need to make their methods very clear. However, many candidates gained full marks on this part.

(ii) Many candidates successfully found the intersections of AB and the perpendicular bisector with the *y*-axis. However, only a minority of the candidates used the straightforward method of determining the length of the 'base' of the triangle (along the *y*-axis) and multiplying by the 'height' i.e. the *x*-coordinate of the midpoint – this method easily gave the correct answer of 45/4 square units. Most chose to determine the lengths of the other two sides of the triangle. This method gave answers involving surds and many left them like this, hence losing the final accuracy mark for failing to give the answer in a simple form. Some candidates were confused by the reference to the line AM in the question, or by the fact that the point A appeared to be close to the *y*-axis: instead of using the point of intersection of AB with the *y*-axis, they used either the point A (−1,4) or the point (0,4) to calculate one side of the triangle. However, the mark scheme made allowance for these potential misreads, allowing candidates to pick up 4 or 3 of the 6 marks in this part.

13 (i) Weak candidates often floundered around at the start, attempting trials using the factor theorem, not taking the hint from the wording of the question that the other roots would not be integers. Most candidates appreciated that $x - 2$ was a factor, with the majority attempting long division rather than inspection or coefficient methods. Errors in division leading to an answer such as $x^2 + 2x + 1$ meant that the subsequent method mark for attempting the quadratic formula or completing the square was lost, since in factorising the difficulty was not the same. Again, in spite of the wording, many candidates assumed that the quadratic expression found would factorise. Those who used the quadratic formula often gained one accuracy mark for correctly finding the roots in the form $\frac{-2 \pm \sqrt{8}}{2}$, but only about half of them were able to simplify to $-1 \pm \sqrt{2}$,

whereas those who used completing the square arrived at this result more easily.

(ii) Those candidates who gained full marks in this part usually did so by substituting $x - 3$ for x in the expression for f(x). Some strong candidates successfully completed the task using the product of the factors as their first step, having done part (iii) first, but with the difficulty of the surds, errors tended to creep in during this process. Some candidates did not show sufficient evidence of how they had obtained the given expression for f($x - 3$) from their starting point. Since there was some working backwards (sensibly done when candidates found their errors and were able to correct them), in such situations the examiners require steps to be shown. Some candidates did not realise what was going on and attempted to find f(3) or to divide the given expression by ($x - 3$).

(iii) Some candidates had given up on this question by now, and some attempted trials. Some realised that the graph was a translation of the graph of y = f(x) and were able simply to state the roots as intended. A few realised that 5 was a root and started again using the methods of part (i) to find the other roots, and had time to do so.