

**ADVANCED GCE
MATHEMATICS (MEI)**

Decision Mathematics Computation

THURSDAY 12 JUNE 2008

4773/01

Morning
Time: 2 hours 30 minutes

Additional materials (enclosed): None

Additional materials (required):
Answer Booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)



INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- Additional sheets, including computer print-outs, should be fastened securely to the Answer Booklet.

COMPUTING RESOURCES

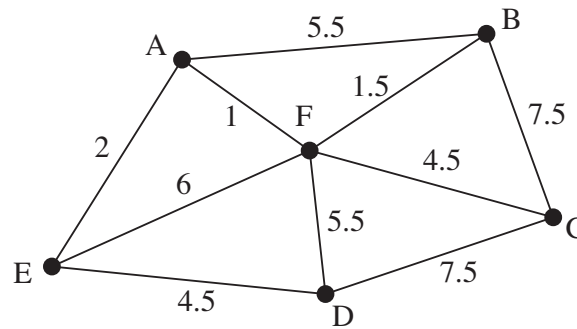
- Candidates will require access to a computer with a spreadsheet program, a linear programming package and suitable printing facilities throughout the examination.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- The total number of marks for this paper is **72**.
- In each of the questions you are required to write spreadsheet or other routines to carry out various processes.
- For each question you attempt, you should submit print-outs showing the routine you have written and the output it generates.
- You are not expected to print out and submit everything your routine produces, but you are required to submit sufficient evidence to convince the examiner that a correct procedure has been used.

This document consists of **6** printed pages and **2** blank pages.

- 1 The vertices of the network represent six villages in a rural area in a developing country. The arc weights represent journey times (hours) along connecting tracks.



Health centres are to be built in some villages, so that all villages are within 5 hours journey time of a health centre.

The matrix shows the complete set of least journey times between the villages.

	A	B	C	D	E	F
A	0	2.5	5.5	6.5	2	1
B	2.5	0	6	7	4.5	1.5
C	5.5	6	0	7.5	7.5	4.5
D	6.5	7	7.5	0	4.5	5.5
E	2	4.5	7.5	4.5	0	3
F	1	1.5	4.5	5.5	3	0

- (i) Let X_A, X_B, \dots be indicator variables representing whether or not a health centre is built in village A, B, ...
- Write down an inequality using these variables which, when satisfied, will ensure that village A is within 5 hours journey of a health centre. Explain your inequality by reference to the matrix above. [5]
- (ii) Formulate an LP, the solution to which will give the minimum number of health centres required and their locations. [5]
- (iii) Run your LP and interpret the solution. [3]
- (iv) Modify your LP so as to find a second optimal solution to the problem. [2]
- (v) Find a third optimal solution. [1]
- (vi) Explain why it is not necessary to force your variables to be indicator variables, and why it is preferable not to do so. [2]

- 2 For a small population of organisms it is approximately the case that in any one-second interval either a birth occurs, or a death, or both, or neither. There is never more than one birth or one death in a one-second interval.

Let n be the number of organisms in existence at the beginning of a one-second interval.

The probability of a birth occurring in that interval is $\alpha \times n$, where α is a small positive number.

Similarly, the probability of a death occurring in that interval is $\beta \times n$, where β is a small positive number.

- (i) Build a spreadsheet to simulate the development of a population of size 10 over a one-minute period.

Print out your spreadsheet and show the formulae which you use. [6]

- (ii) Using $\alpha = 0.01$ and $\beta = 0.04$, run your simulation 10 times.

Tabulate your results and estimate the probability of extinction over the one-minute period. [3]

- (iii) Investigate how the probability of extinction varies for different β s, given that α remains at 0.01.

Tabulate your results. [3]

You are now to investigate what happens if the population is increased by immigration. In each one-second period there may be either 0 or 1 immigrants arrive into the population. The probability of an arrival is γ .

- (iv) Modify your spreadsheet to incorporate immigration.

Print out your spreadsheet and show the formulae which you use. [4]

- (v) Investigate how immigration affects your results in part (iii), given that $\gamma = 0.05$. [2]

3 (a)

	A	B	C	D	E	F	G
A	–	8	–	–	3	–	–
B	8	–	5	7	–	–	–
C	–	5	–	1	–	–	8
D	–	7	1	–	2	–	–
E	3	–	–	2	–	4	–
F	–	–	–	–	4	–	6
G	–	–	8	–	–	6	–

Table 3.1

Table 3.1 represents the capacities of arcs in a flow network.

Formulate and solve an LP to find the maximum flow from A to G.

[9]

(b)

	S1	S2	S3	S4	S5	S6	S7
W1	–	8	–	–	3	–	–
W2	8	–	5	7	–	–	–
W3	–	5	–	1	–	–	8
W4	–	7	1	–	2	–	–
W5	3	–	–	2	–	4	–
W6	–	–	–	–	4	–	6
W7	–	–	8	–	–	6	–

Table 3.2

Table 3.2 has the same numerical entries as Table 3.1, but its rows are labelled W1 to W7, representing warehouses, and its columns are labelled S1 to S7, representing shops supplied from those warehouses. Each numerical entry represents the cost of moving a single unit from the corresponding warehouse to the corresponding shop.

Each warehouse has 10 units in stock. Each shop requires 10 units.

Formulate and solve an LP to find a minimum cost distribution pattern.

[9]

- 4 (a) Solve the recurrence relation $u_{n+2} = \frac{3}{2}u_{n+1} - \frac{1}{2}u_n$, given that $u_0 = 5$ and $u_1 = 3$.

Give the values of u_2 , u_3 , u_{10} and $u_{1000000}$. [13]

- (b) Successive waves arriving on a beach are found to have heights which follow approximately the recurrence relation $w_{n+2} = \frac{3}{2}w_{n+1} - w_n + 5$, with $w_0 = 5$ and $w_1 = 3$.

(i) Construct a spreadsheet to model wave height. [2]

(ii) Give the values of w_2 , w_3 , w_{10} and w_{20} . [2]

(iii) State the limitation of using a spreadsheet to model a recurrence relation. [1]

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1.

<p>(i) $XA + XB + XE + XF \geq 1$</p> <p>Indicator variables correspond to matrix column A (or row A) entries which are less than or equal to 5. Ensures that at least one such indicator is 1.</p> <p>(ii) Min $XA+XB+XC+XD+XE+XF$ st $XA+XB+XE+XF \geq 1$ $XA+XB+XE+XF \geq 1$ $XC+XF \geq 1$ $XD+XE \geq 1$ $XA+XB+XD+XE+XF \geq 1$ $XA+XB+XC+XE+XF \geq 1$</p> <p>(iii) 2 centres, at F&D or E&C or E&F</p> <p>(iv) e.g. add $XF=0$ to force solution E and C</p> <p>(v) Three solutions are F & D, E & C, E & F.</p> <p>(vi) Problem is unimodular (or convincing argument). In the interests of efficiency (and parsimony).</p>	<p>M1 A1 ">" OK</p> <p>B1 indicator vars B1 ≤ 5 B1</p> <p>B1</p> <p>M1 A3 (-1 each error/ omission) allow (correct) reduced set of inequalities</p> <p>M1 A1 A1</p> <p>M1 A1 B1</p> <p>B1 B1</p>
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2.

(i) e.g. (candidates should show formulae)	<p>B1 handling parameters B1 births B1 deaths B1 use of "rand" B1 use of "if" B1 updating population</p>																																				
<table border="0"> <tr> <td>$\alpha =$</td> <td>0.01</td> <td>10</td> <td>P(birth)</td> <td>P(death)</td> <td>rand</td> <td>rand</td> <td>birth</td> <td>death</td> </tr> <tr> <td>$\beta =$</td> <td>0.04</td> <td>9</td> <td>0.1</td> <td>0.4</td> <td>0.4261</td> <td>0.3537</td> <td>0</td> <td>1</td> </tr> <tr> <td></td> <td></td> <td>8</td> <td>0.09</td> <td>0.36</td> <td>0.257</td> <td>0.1405</td> <td>0</td> <td>1</td> </tr> <tr> <td></td> <td></td> <td>8</td> <td>0.08</td> <td>0.32</td> <td>0.8854</td> <td>0.8632</td> <td>0</td> <td>0</td> </tr> </table>	$\alpha =$	0.01	10	P(birth)	P(death)	rand	rand	birth	death	$\beta =$	0.04	9	0.1	0.4	0.4261	0.3537	0	1			8	0.09	0.36	0.257	0.1405	0	1			8	0.08	0.32	0.8854	0.8632	0	0	
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(ii) e.g. 1 1 3 2 0 2 2 3 3 0 – 0.2	M1 A1 B1																																				
(iii) e.g.	<p>M1 A1 decent range A1 reasonable outcomes</p>																																				
<table border="0"> <tr> <td>β</td> <td></td> <td>0.01</td> <td>0.02</td> <td>0.03</td> <td>0.04</td> <td>0.05</td> <td>0.06</td> <td>0.07</td> </tr> <tr> <td>prob extinction</td> <td>0</td> <td>0</td> <td>0.1</td> <td>0.2</td> <td>0.5</td> <td>0.7</td> <td>0.8</td> <td></td> </tr> </table>	β		0.01	0.02	0.03	0.04	0.05	0.06	0.07	prob extinction	0	0	0.1	0.2	0.5	0.7	0.8																				
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(iv) Addition of another rand + another if + extra add-on	<p>B1 M1 A1 B1</p>																																				
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3.

<p>(a) max AB+AE st AB-BC-BD+CB+DB=0 BC+DC-CB-CD-CG=0 BD+ED+CD-DB-DC-DE=0 AE-ED-EF+DE+FE=0 EF-FE-FG=0 AB<8 AE<3 BC<5 CB<5 BD<7 DB<7 CD<1 DC<1 DE<2 ED<2 EF<4 FE<4 CG<8 FG<6 END</p>	<p>B1 objective M1 flow balance A2 constraints M1 capacity A1 constraints</p>																																													
<p>OBJECTIVE FUNCTION VALUE</p> <p>1) 10.00000</p>	<p>M1 run</p>																																													
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<p>Max flow of 10 with flows of 7 from A to B, ... etc.</p>	<p>B1 interpretation</p>																																													

<p>(b)</p> <p>min $8X_{12}+3X_{15}+8X_{21}+5X_{23}+7X_{24}+5X_{32}+X_{34}+8X_{37}+7X_{42}+X_{43}+2X_{45}+3X_{51}+2X_{54}+4X_{56}+4X_{65}+6X_{67}+8X_{73}+6X_{76}$</p> <p>st $X_{12}+X_{15}=10$ $X_{21}+X_{23}+X_{24}=10$ $X_{32}+X_{34}+X_{37}=10$ $X_{42}+X_{43}+X_{45}=10$ $X_{51}+X_{54}+X_{56}=10$ $X_{65}+X_{67}=10$ $X_{73}+X_{76}=10$ $X_{21}+X_{51}=10$ $X_{12}+X_{32}+X_{42}=10$ $X_{23}+X_{43}+X_{73}=10$ $X_{24}+X_{34}+X_{54}=10$ $X_{15}+X_{45}+X_{65}=10$ $X_{56}+X_{76}=10$ $X_{37}+X_{67}=10$</p> <p>END</p>	<p>M1 objective A1</p> <p>M1 supply A1 constraints</p> <p>M1 demand A1 constraints</p>																																																									
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<p>Cost = 310 by sending 10 from W1 to S5, ... etc.</p>	<p>B1 interpretation</p>																																																									

4.

<p>(a) Auxiliary equation: $2\lambda^2 - 3\lambda + 1 = 0$ $(2\lambda - 1)(\lambda - 1) = 0$ $\lambda = 1$ or $\frac{1}{2}$</p> <p>$u_n = A + B(\frac{1}{2})^n$</p> <p>$5 = A + B$ $3 = A + \frac{1}{2}B$</p> <p>$u_n = 1 + 4(\frac{1}{2})^n$</p> <p>$u_2 = 2, u_3 = 1.5, u_{10} = 1.003906$ $u_{1000000} \approx 1$</p>	<p>M1 A1 M1 A1</p> <p>B1 B1</p> <p>B1 B1</p> <p>M1 A1</p> <p>B1 B1 B1</p>																																										
<p>(b)(i) & (ii)</p> <table border="0"> <tr><td>0</td><td>5</td></tr> <tr><td>1</td><td>3</td></tr> <tr><td>2</td><td>4.5</td></tr> <tr><td>3</td><td>8.75</td></tr> <tr><td>4</td><td>13.625</td></tr> <tr><td>5</td><td>16.6875</td></tr> <tr><td>6</td><td>16.40625</td></tr> <tr><td>7</td><td>12.92188</td></tr> <tr><td>8</td><td>7.976563</td></tr> <tr><td>9</td><td>4.042969</td></tr> <tr><td>10</td><td>3.087891</td></tr> <tr><td>11</td><td>5.588867</td></tr> <tr><td>12</td><td>10.29541</td></tr> <tr><td>13</td><td>14.85425</td></tr> <tr><td>14</td><td>16.98596</td></tr> <tr><td>15</td><td>15.62469</td></tr> <tr><td>16</td><td>11.45108</td></tr> <tr><td>17</td><td>6.551926</td></tr> <tr><td>18</td><td>3.376808</td></tr> <tr><td>19</td><td>3.513287</td></tr> <tr><td>20</td><td>6.893122</td></tr> </table>	0	5	1	3	2	4.5	3	8.75	4	13.625	5	16.6875	6	16.40625	7	12.92188	8	7.976563	9	4.042969	10	3.087891	11	5.588867	12	10.29541	13	14.85425	14	16.98596	15	15.62469	16	11.45108	17	6.551926	18	3.376808	19	3.513287	20	6.893122	<p>M1 A1</p> <p>A1 3.087891</p> <p>A1 6.893122</p>
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<p>(iii) Limited wrt to (very) long-term</p>	<p>B1</p>																																										

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General Comments

Most candidates were quite well organised this year. It helps enormously if output is organised into a logical order, and if it is annotated. However, see comments below regarding excesses of output in some questions and lack of evidence in others.

Comments on Individual Questions

1) LP modelling

Modelling is always difficult, and this question tried to give as much help as possible to candidates in formulating this set covering problem. However, there were still many who failed to note that the variables were indicator variables.

In part (i) some did not include the "zero distance" village in their constraint. Those that succeeded with the modelling generally succeeded with the rest of the question, though not all dealt with part (iv) by introducing an extra constraint, or at least they did not show any evidence of having done so.

2) Simulation

There was a huge problem with responses to this question – the majority of candidates failed to capture the fact that birth and death rates are dependent on population size. By far the most common response was to model with a constant birth rate of 10×0.01 and a constant death rate of 10×0.04 . This misses the point!

A subsidiary problem occurred with candidates who attempted to model both births and deaths by using just one random variable, instead of two independent random variables. This is OK provided that the probabilities are correct – they should be $(1-n\alpha)(1-n\beta)$, $n\alpha(1-n\beta)$, $(1-n\alpha)n\beta$ and $n\alpha n\beta$ – but it was not intended for candidates to show this level of skill in probability modelling.

There was a small problem with ensuring that a full minute was simulated. Many candidates started at $t=1$, and so had simulated only 59 seconds by time $t=60$.

In parts (ii), (iii) and (v) too many candidates failed to produce evidence of their work, whilst a number produced copious pages of repetitive output. There were still a few candidates who do not ensure that cells are wide enough to display all of the formulae.

3) Networks

This was a fairly straightforward two-part question. The question labelling and wording both indicated that, although specified by the same matrix, the problems were different.

In part (a) some candidates encountered difficulties by allowing back flows to the source and/or from the sink. This had the potential to derail the optimisation with, for instance, flows of $AB=8$, $BA=8$, $AE=3$ and $EA=3$ being preferred to the correct answer.

Some solutions included the redundant constraint $AB+AE = CG+FG$, whilst a number included all the variables in the objective function.

Part (b) presented no such difficulties. Some printouts lost the end of the objective function (this needed splitting on to two lines). There were inevitably some typing errors in variable names – these could have been identified if candidates had checked the list of variables in their solution output.

Candidates sometimes viewed "interpretation" as rewriting what was plain to see, rather than as interpreting back to the given scenario.

4) **Recurrence relations**

Candidates were largely well drilled in the algebraic solution. The spreadsheet was also well handled.

There were many alternative points made about the limitations of the spreadsheet solution, some valid but of marginal significance, and some false. Not many candidates gave the required answer that, using a spreadsheet, there is a (relatively small) limit to how far one can go into the sequence (accepting that Excel's programming capability could be used to circumvent this constraint).