RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS **4756**

Further Methods for Advanced Mathematics (FP2)

Tuesday **6 JUNE 2006** Afternoon 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions in Section A and **one** question from section B.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is 72.

This question paper consists of 4 printed pages.

Section A (54 marks)

Answer all the questions

1 **(a)** A curve has polar equation $r = a\left(\sqrt{2} + 2\cos\theta\right)$ for $-\frac{3}{4}\pi \leqslant \theta \leqslant \frac{3}{4}\pi$, where a is a positive constant.

(i) Sketch the curve. [2]

(ii) Find, in an exact form, the area of the region enclosed by the curve. [7]

(b) **(i)** Find the Maclaurin series for the function $\mathrm{f}(x) = \tan\left(\frac{1}{4}\pi + x\right)$, up to the term in x^2. [6]

(ii) Use the Maclaurin series to show that, when h is small,

$$\int_{-h}^{h} x^2 \tan\left(\tfrac{1}{4}\pi + x\right)\mathrm{d}x \approx \tfrac{2}{3}h^3 + \tfrac{4}{5}h^5.$$ [3]

2 **(a)** **(i)** Given that $z = \cos\theta + \mathrm{j}\sin\theta$, express $z^n + \dfrac{1}{z^n}$ and $z^n - \dfrac{1}{z^n}$ in simplified trigonometric form. [2]

(ii) By considering $\left(z - \dfrac{1}{z}\right)^4 \left(z + \dfrac{1}{z}\right)^2$, find A, B, C and D such that

$$\sin^4\theta\cos^2\theta = A\cos 6\theta + B\cos 4\theta + C\cos 2\theta + D.$$ [6]

(b) **(i)** Find the modulus and argument of $4 + 4\mathrm{j}$. [2]

(ii) Find the fifth roots of $4 + 4\mathrm{j}$ in the form $r\mathrm{e}^{\mathrm{j}\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$.

Illustrate these fifth roots on an Argand diagram. [6]

(iii) Find integers p and q such that $(p + q\mathrm{j})^5 = 4 + 4\mathrm{j}$. [2]

3 **(i)** Find the inverse of the matrix $\begin{pmatrix} 4 & 1 & k \\ 3 & 2 & 5 \\ 8 & 5 & 13 \end{pmatrix}$, where $k \neq 5$. [6]

(ii) Solve the simultaneous equations

$$\begin{aligned} 4x + y + 7z &= 12 \\ 3x + 2y + 5z &= m \\ 8x + 5y + 13z &= 0 \end{aligned}$$

giving x, y and z in terms of m. [5]

(iii) Find the value of p for which the simultaneous equations

$$\begin{aligned} 4x + y + 5z &= 12 \\ 3x + 2y + 5z &= p \\ 8x + 5y + 13z &= 0 \end{aligned}$$

have solutions, and find the general solution in this case. [7]

Section B (18 marks)

Answer one question

Option 1: Hyperbolic functions

4 **(i)** Starting from the definitions of $\sinh x$ and $\cosh x$ in terms of exponentials, prove that

$$1 + 2\sinh^2 x = \cosh 2x.$$ [3]

(ii) Solve the equation

$$2\cosh 2x + \sinh x = 5,$$

giving the answers in an exact logarithmic form. [6]

(iii) Show that $\int_0^{\ln 3} \sinh^2 x \, dx = \frac{10}{9} - \frac{1}{2}\ln 3$. [5]

(iv) Find the exact value of $\int_3^5 \sqrt{x^2 - 9} \, dx$. [4]

[Question 5 is printed overleaf.]

Option 2: Investigation of curves

This question requires the use of a graphical calculator.

5 A curve has parametric equations

$$x = \theta - k\sin\theta, \quad y = 1 - \cos\theta,$$

where k is a positive constant.

(i) For the case $k = 1$, use your graphical calculator to sketch the curve. Describe its main features. [4]

(ii) Sketch the curve for a value of k between 0 and 1. Describe briefly how the main features differ from those for the case $k = 1$. [3]

(iii) For the case $k = 2$:

(A) sketch the curve; [2]

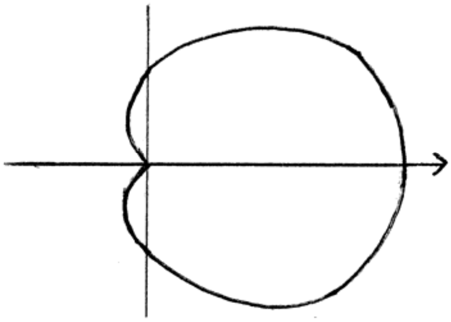
(B) find $\frac{dy}{dx}$ in terms of θ; [2]

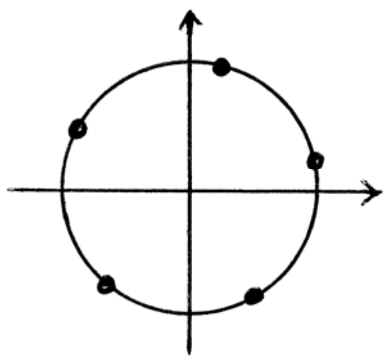
(C) show that the width of each loop, measured parallel to the x-axis, is

$$2\sqrt{3} - \frac{2\pi}{3}.$$ [5]

(iv) Use your calculator to find, correct to one decimal place, the value of k for which successive loops just touch each other. [2]

Mark Scheme 4756
June 2006

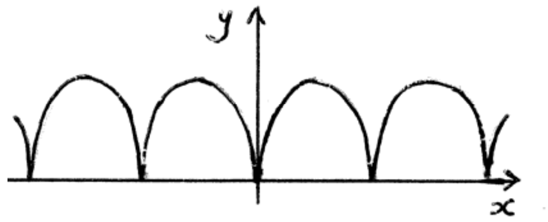
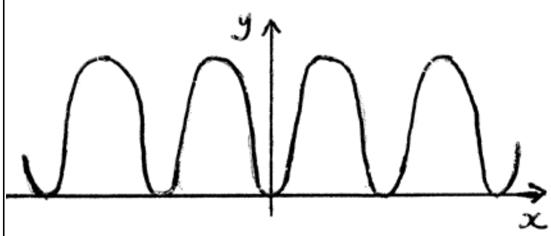
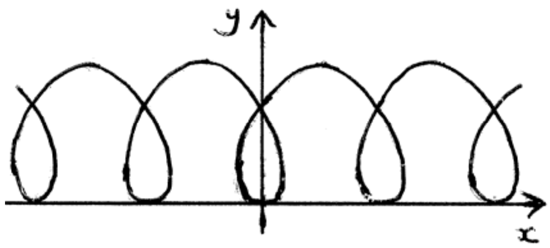
1(a)(i)		B1 B1 **2**	Correct shape for $-\frac{1}{2}\pi < \theta < \frac{1}{2}\pi$ including maximum in 1st quadrant Correct form at O and no extra sections
(ii)	Area is $\int \frac{1}{2} r^2 \, d\theta = \int_{-\frac{3}{4}\pi}^{\frac{3}{4}\pi} \frac{1}{2} a^2 (\sqrt{2} + 2\cos\theta)^2 \, d\theta$ $= \int_{-\frac{3}{4}\pi}^{\frac{3}{4}\pi} a^2 (1 + 2\sqrt{2}\cos\theta + 1 + \cos 2\theta) \, d\theta$ $= \left[a^2 (2\theta + 2\sqrt{2}\sin\theta + \frac{1}{2}\sin 2\theta) \right]_{-\frac{3}{4}\pi}^{\frac{3}{4}\pi}$ $= 3(\pi + 1) a^2$	M1 A1 B1 B1B1 ft M1 A1 **7**	For integral of $(\sqrt{2} + 2\cos\theta)^2$ For a correct integral expression including limits *(may be implied by later work)* Using $2\cos^2\theta = 1 + \cos 2\theta$ Integration of $\cos\theta$ and $\cos 2\theta$ Evaluation using $\sin\frac{3}{4}\pi = (\pm)\frac{1}{\sqrt{2}}$
(b)(i)	$f'(x) = \sec^2(\frac{1}{4}\pi + x)$ $f''(x) = 2\sec^2(\frac{1}{4}\pi + x)\tan(\frac{1}{4}\pi + x)$ $f(0) = 1, \ f'(0) = 2, \ f''(0) = 4$ $f(x) = 1 + 2x + 2x^2 + ...$	B1 B1 M1 B1A1A1 **6**	Any correct form Evaluating $f'(0)$ or $f''(0)$
	OR $g'(u) = \sec^2 u$ (where $g(u) = \tan u$) B1 $g''(u) = 2\sec^2 u \tan u$ B1 $g(\frac{1}{4}\pi) = 1, \ g'(\frac{1}{4}\pi) = 2, \ g''(\frac{1}{4}\pi) = 4$ M1 $f(x) = g(\frac{1}{4}\pi + x) = 1 + 2x + 2x^2 + ...$ B1A1A1		Condone $\sec^2 x$ etc Evaluating $g'(\frac{1}{4}\pi)$ or $g''(\frac{1}{4}\pi)$
(ii)	$\int_{-h}^{h} x^2 (1 + 2x + 2x^2 + \ ...) \, dx$ $= \left[\frac{1}{3}x^3 + \frac{1}{2}x^4 + \frac{2}{5}x^5 + ... \right]_{-h}^{h}$ $\approx (\frac{1}{3}h^3 + \frac{1}{2}h^4 + \frac{2}{5}h^5) - (-\frac{1}{3}h^3 + \frac{1}{2}h^4 - \frac{2}{5}h^5)$ $= \frac{2}{3}h^3 + \frac{4}{5}h^5$	M1 A1 ft A1 (ag) **3**	Using series and integrating (ft requires three non-zero terms) Correctly shown Allow ft from $1 + kx + 2x^2$ with $k \neq 0$

2 (a)(i)	$z^n + \frac{1}{z^n} = 2\cos n\theta$, $z^n - \frac{1}{z^n} = 2\mathrm{j}\sin n\theta$	B1B1 **2**	
(ii)	$\left(z - \frac{1}{z}\right)^4 \left(z + \frac{1}{z}\right)^2 = 64\sin^4\theta\cos^2\theta$ $= z^6 - 2z^4 - z^2 + 4 - \frac{1}{z^2} - \frac{2}{z^4} + \frac{1}{z^6}$ $= 2\cos 6\theta - 4\cos 4\theta - 2\cos 2\theta + 4$ $\sin^4\theta\cos^2\theta = \frac{1}{32}\cos 6\theta - \frac{1}{16}\cos 4\theta - \frac{1}{32}\cos 2\theta + \frac{1}{16}$ $(A = \frac{1}{32},\ B = -\frac{1}{16},\ C = -\frac{1}{32},\ D = \frac{1}{16})$	B1 M1 A1 M1 A1 ft A1 **6**	Expansion $z^6 + \ ...\ + z^{-6}$ Using $z^n + \frac{1}{z^n} = 2\cos n\theta$ with $n = 2,\ 4$ or 6. Allow M1 if used in partial expansion, or if 2 omitted, etc
(b)(i)	$\lvert 4 + 4\mathrm{j} \rvert = \sqrt{32}$, $\arg(4 + 4\mathrm{j}) = \frac{1}{4}\pi$	B1B1 **2**	Accept 5.7; $0.79,\ 45°$
(ii)	$r = \sqrt{2}$ $\theta = -\frac{3}{4}\pi,\ -\frac{7}{20}\pi,\ \frac{1}{20}\pi,\ \frac{9}{20}\pi,\ \frac{17}{20}\pi$ 	B1 B3 B2 **6**	Accept $32^{\frac{1}{10}},\ 1.4,\ \sqrt[5]{4\sqrt{2}}$ etc Accept $-2.4, -1.1, 0.16, 1.4, 2.7$ Give B2 for three correct Give B1 for one correct Deduct 1 mark (maximum) if degrees used $(-135°,\ -63°,\ 9°,\ 81°,\ 153°)$ $\frac{1}{20}\pi + \frac{2}{5}k\pi$ earns B2; with $k = -2,\ -1,\ 0,\ 1,\ 2$ earns B3 Give B1 for four points correct, or B1 ft for five points
(iii)	$\sqrt{2}\mathrm{e}^{-\frac{3}{4}\pi \mathrm{j}} = \sqrt{2}\left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\mathrm{j}\right)$ $= -1 - \mathrm{j}$ $p = -1,\ q = -1$	M1 A1 **2**	Exact evaluation of a fifth root Give B2 for correct answer stated or obtained by any other method

3 (i)	$\mathbf{M}^{-1} = \frac{1}{5-k}\begin{pmatrix} 1 & 5k-13 & 5-2k \\ 1 & 52-8k & 3k-20 \\ -1 & -12 & 5 \end{pmatrix}$	M1 A1 M1 A1 M1 A1 **6**	Evaluating determinant For $(5-k)$ *must be simplified* Finding at least four cofactors At least 6 signed cofactors correct Transposing matrix of cofactors and dividing by determinant Fully correct
	OR Elementary row operations applied to **M** (LHS) and **I** (RHS), and obtaining at least two zeros in LHS M1 Obtaining one row in LHS consisting of two zeros and a multiple of $(5-k)$ A1 Obtaining one row in RHS which is a multiple of a row of the inverse matrix A1 Obtaining two zeros in every row in LHS M1 Completing process to find inverse M1A1		or elementary column operations
(ii)	$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = -\frac{1}{2}\begin{pmatrix} 1 & 22 & -9 \\ 1 & -4 & 1 \\ -1 & -12 & 5 \end{pmatrix}\begin{pmatrix} 12 \\ m \\ 0 \end{pmatrix}$ $x = -11m-6,\quad y = 2m-6,\quad z = 6m+6$	M1 M1 M1 A2 ft **5**	Substituting $k=7$ into inverse Correct use of inverse Evaluating matrix product Give A1 ft for one correct *Accept unsimplified forms or solution left in matrix form*
	OR e.g. eliminating *x* , $3y - z = -24$ $5y - z = 4m - 36$ M2 $y = 2m - 6$ M1 $x = -11m-6,\quad y = 2m-6,\quad z = 6m+6$ A2		Eliminating one variable in two different ways Obtaining one of *x* , *y* , *z* Give M3 for any other valid method leading to one of *x* , *y* , *z* in terms of *m* Give A1 for one correct
(iii)	Eliminating *x* , $3y + 3z = -24$ $5y + 5z = 4p - 36$ For solutions, $4p - 36 = -24 \times \frac{5}{3}$	M2 A1 M1	Eliminating one variable in two different ways Two correct equations *Dependent on previous M2*
	OR Replacing one column of matrix with column from RHS, and evaluating determinant M2 determinant $12 + 12p$ or $-12 - 12p$ A1 For solutions, det $= 0$ M1		*Dependent on previous M2*

	OR Any other method leading to an equation from which p could be found M3 Correct equation A1		
	$p=-1$ Let $z=\lambda$, $x=5-\lambda,\quad y=-8-\lambda,\; z=\lambda$	A1 M1 (or M3) A1 **7**	Obtaining a line of solutions Give M3 when M0 for finding p or $x=13+\lambda,\quad y=\lambda,\; z=-8-\lambda$ or $x=\lambda,\quad y=-13+\lambda,\; z=5-\lambda$ Accept $x=5-z,\; y=-8-z$ or $x=y+13=5-z$ etc

4 (i)	$1+2\sinh^2 x = 1+2[\frac{1}{2}(e^x - e^{-x})]^2$ $= 1+\frac{1}{2}(e^{2x} - 2 + e^{-2x})$ $= \frac{1}{2}(e^{2x} + e^{-2x})$ $= \cosh 2x$	B1 B1 B1 (ag) **3**	For $(e^x - e^{-x})^2 = e^{2x} - 2 + e^{-2x}$ For $\cosh 2x = \frac{1}{2}(e^{2x} + e^{-2x})$ For completion
(ii)	$2(1+2\sinh^2 x) + \sinh x = 5$ $4\sinh^2 x + \sinh x - 3 = 0$ $(4\sinh x - 3)(\sinh x + 1) = 0$ $\sinh x = \frac{3}{4},\ -1$ $x = \text{arsinh}(\frac{3}{4}) = \ln(\frac{3}{4} + \sqrt{\frac{9}{16}+1}) = \ln 2$ $x = \text{arsinh}(-1) = \ln(-1+\sqrt{1+1}) = \ln(\sqrt{2}-1)$	M1 M1 A1A1 A1 ft A1 ft **6**	Using (i) Solving to obtain a value of $\sinh x$ or $-\ln(\sqrt{2}+1)$ *SR* Give A1 for $\pm\ln 2,\ \pm\ln(\sqrt{2}-1)$
	OR $2e^{4x} + e^{3x} - 10e^{2x} - e^x + 2 = 0$ M2 $(e^x - 2)(2e^x + 1)(e^{2x} + 2e^x - 1) = 0$ A1A1 $x = \ln 2,\ \ln(\sqrt{2}-1)$ A1A1 ft		Obtaining a linear or quadratic factor For $(e^x - 2)$ and $(e^{2x} + 2e^x - 1)$
(iii)	$\int_0^{\ln 3} \frac{1}{2}(\cosh 2x - 1)\,dx$ $= \left[\frac{1}{4}\sinh 2x - \frac{1}{2}x\right]_0^{\ln 3}$ $= \frac{1}{8}\left(9 - \frac{1}{9}\right) - \frac{1}{2}\ln 3$ $= \frac{10}{9} - \frac{1}{2}\ln 3$	M1 A1A1 M1 A1 (ag) **5**	Expressing in integrable form or $\int \frac{1}{4}(e^{2x} - 2 + e^{-2x})\,dx$ or $(\frac{1}{8}e^{2x} - \frac{1}{8}e^{-2x}) - \frac{1}{2}x$ For $e^{2\ln 3} = 9$ and $e^{-2\ln 3} = \frac{1}{9}$ M0 for just stating $\sinh(2\ln 3) = \frac{40}{9}$ etc Correctly obtained
(iv)	Put $x = 3\cosh u$ when $x = 3,\ u = 0$ when $x = 5,\ u = \text{ar}\cosh\frac{5}{3} = \ln 3$ $\int_3^5 \sqrt{x^2 - 9}\,dx = \int_0^{\ln 3} (3\sinh u)(3\sinh u\,du)$ $= 9\int_0^{\ln 3} \sinh^2 u\,du$ $= 10 - \frac{9}{2}\ln 3$	M1 B1 A1 A1 **4**	Any cosh substitution For $\ln 3$ *Not awarded for* $\text{arcosh}\frac{5}{3}$ *Limits not required*

5 (i)	 Has cusps Periodic / Symmetrical in *y*-axis / Has maxima / Is never below the *x*-axis	B2 B1 B1 **4**	At least two cusps clearly shown Give B1 for at least two arches Any other feature
(ii)	 The curve has no cusps	B2 B1 **3**	At least two minima (zero gradient) clearly shown Give B1 for general shape correct (at least two cycles) For description of any *difference*
(iii)	(*A*) 	B2 **2**	At least two loops Give B1 for general shape correct (at least one cycle)
	(*B*) $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\sin\theta}{1 - 2\cos\theta}$	M1 A1 **2**	Correct method of differentiation *Allow M1 if inverted* Allow $\frac{\sin\theta}{1 - k\cos\theta}$
	(*C*) $\frac{\mathrm{d}y}{\mathrm{d}x}$ is infinite when $1 - 2\cos\theta = 0$ $\theta = \frac{1}{3}\pi$ $x = \frac{1}{3}\pi - 2\sin\frac{1}{3}\pi$ $= -(\sqrt{3} - \frac{1}{3}\pi)$ Hence width of loop is $2(\sqrt{3} - \frac{1}{3}\pi)$ $= 2\sqrt{3} - \frac{2\pi}{3}$	M1 A1 M1 M1 A1 (ag) **5**	 Any correct value of θ Finding width of loop Correctly obtained *Condone negative answer*
(iv)	$k = 4.6$	B2 **2**	Give B1 for a value between 4 and 5 (inclusive)

4756 - Further Methods for Advanced Mathematics (FP2)

General Comments

The overall standard of work on this paper was generally good. Most candidates presented their work clearly and demonstrated their familiarity with the standard results and techniques. There were some excellent scripts, with about 15% of the candidates scoring 60 marks or more (out of 72). However, there were also a significant number who appeared to be under prepared and who failed to score marks on some straightforward parts of questions. About 20% of the candidates scored less than 30 marks.
Candidates did not always read the questions sufficiently carefully, for example the range of values of θ given in Q.1(a), 'Use the Maclaurin series' in Q.1(b)(ii), 'Integers' in Q.2(b)(iii), and 'Exact' in Q.4(ii) and (iv).
Many candidates would have done better had they seen the connections between different parts of questions, such as Q.1(b)(i) and (ii), Q.2(a)(i) and (ii), Q.2(b)(ii) and (iii), Q.4(i) and (ii), and Q.4(iii) and (iv). Parts of questions labelled (i), (ii), … are always intended to be connected in some way.
In Section A, Q.1 was the best answered question, with an average mark of about 12 (out of 18), and Q.3 was the worst answered, with an average mark of about 10. In Section B, Q.4 was chosen by almost all the candidates, and the average mark was about 11.

Comments on Individual Questions

1) **Polar curve and Maclaurin series**
In part(a)(i) the sketch of the curve was usually correct, although some candidates included an extra loop (corresponding to values of θ outside the given range). In part (a)(ii) most candidates used $\int \frac{1}{2}r^2\,\mathrm{d}\theta$ with the correct limits, and the subsequent evaluation was quite frequently carried out accurately. Most of the mistakes made were careless slips such as sign errors, or the factor a^2 being lost or ending up as *a*. However, a substantial number were unable to integrate $\cos^2\theta$.
In part (b), the methods for obtaining a Maclaurin series, and using it to evaluate an integral approximately, were very well known. However, finding the second derivative of $\tan(\frac{1}{4}\pi + x)$ caused a surprising amount of difficulty. The correct answer appeared in a great variety of forms, from the expected $2\sec^2(\frac{1}{4}\pi + x)\tan(\frac{1}{4}\pi + x)$ to $4\sin(\frac{1}{2}\pi + 2x)/(1+\cos(\frac{1}{2}\pi + 2x))^2$ and even more complicated expressions, and very many candidates failed to obtain a correct expression. Some candidates attempted integration by parts instead of applying their Maclaurin series to the final integral.

2) **Complex numbers**
In part (a)(i) most candidates were able to write down the required expressions, or find them after a few lines of working, but some had little idea of what to do, usually giving up after some manipulation of fractions.
In part (a)(ii), common errors included algebraic slips in the expansion of $(z-1/z)^4(z+1/z)^2$, replacing $z^6 + 1/z^6$ with $\cos 6\theta$ instead of $2\cos 6\theta$, and, especially, omission of the factor 64. A fair number expressed $(z-1/z)^4$ and $(z+1/z)^2$ separately in terms of multiple angles, then multiplied the results, but only a tiny fraction of these could deal successfully with the resulting $\cos 4\theta \cos 2\theta$ term.
In part (b)(i) almost all candidates found the modulus and argument correctly.
Part (b)(ii) was very often answered correctly and efficiently. The modulus ($\sqrt{2}$) of the fifth roots was given in a variety of correct forms, including $32^{0.1}$ and $\sqrt[5]{4\sqrt{2}}$. The arguments given were sometimes outside the required range, and a fairly common error was to give

the arguments as $\frac{1}{4}\pi + \frac{2}{5}k\pi$ instead of $\frac{1}{20}\pi + \frac{2}{5}k\pi$. The great majority of candidates knew that the roots should appear as the vertices of a regular pentagon on the Argand diagram. Part (b)(iii) was often omitted. Many candidates did realise that they needed to select one of the fifth roots found earlier, but very often an inappropriate root was chosen, giving values of *p* and *q* which were clearly not integers. Some ignored the connection with the previous part and expanded $(p + q\,\mathrm{j})^5$; very occasionally the correct solution $p = -1$, $q = -1$ was spotted from the resulting equations.

3) **Matrices**

In part (i) almost all candidates knew a method for finding the inverse matrix, and the process was very often completed accurately. By far the most common approach was to use cofactors; common errors included arithmetic slips, forgetting to transpose the matrix of cofactors, forgetting to change the sign of some minors to obtain the cofactors, and multiplying cofactors by their corresponding elements. A few candidates used elementary row operations.

In part (ii), those who put $k = 7$ into the inverse matrix and then used it to find the solution were usually successful. Some candidates started again and obtained a correct inverse of the matrix with $k = 7$ (possibly by calculator), even if their part (i) had been incorrect. Very many candidates worked from the three equations, eliminating variables. Careless slips were very common with this method, even when a systematic approach, starting by eliminating one variable in two different ways, was used. The work often occupied several pages, typically eliminating *x*, then *y*, then *z*, but never reaching helpful results.

Part (iii) was very often omitted. There were some very efficient solutions, in which candidates usually eliminated one variable in two different ways, compared their equations to find *p*, and then found *x*, *y* and *z* in terms of a parameter. A variety of other methods were used; the correct value of *p* was found fairly frequently, but the correct general solution was much rarer.

4) **Hyperbolic functions**

The proof in part (i) was usually fully correct. Common errors included confusing the definitions of $\sinh x$ and $\cosh x$ in terms of exponentials, and failing to expand $(\mathrm{e}^x - \mathrm{e}^{-x})^2$ correctly.

In part (ii) most candidates used the result in part (i) to obtain a quadratic in $\sinh x$, leading frequently to a fully correct solution, although some thought that the solution $\sinh x = -1$ should be rejected. Some candidates wrote the equation in exponential form, obtaining a quartic in e^x; usually no further progress was made, but a few of these spotted the factor $(\mathrm{e}^x - 2)$.

In part (iii) most candidates used the result in part (i) to obtain a form which could be integrated, although some preferred to write it in terms of exponentials, and the integration was usually performed correctly. Very many candidates did not earn the marks for the evaluation; because the answer is given, just stating $\sinh(2\ln 3) = 40/9$ is not sufficient.

In part (iv), candidates who used the substitution $x = 3\cosh u$ very often answered this correctly and efficiently. A factor of 3 often went astray; and, especially if the upper limit was left as $\operatorname{arcosh}(5/3)$ instead of $\ln 3$, the close connection with part (iii) was not always noticed.

5) **Investigation of curves**

This question was attempted by less than 2% of the candidates, and only three of these scored more than half marks.